The Implications of Optimal Prediction Formulae

Mark W. Watson

Marvin Goodfriend was not an econometrician, but he was a quantitative economist. He based his policy advice on the logic and quantitative implications of economic models. Like many economists who came of age in the late 1970s and early 1980s, Marvin learned how models were affected by the assumption of rational expectations. He also realized that the optimal prediction formulae used to compute rational expectations had implications for econometric practice, and he used these implications in his empirical research. Three of Marvin's papers include particularly novel applications of these insights. I'll discuss these and then conclude with some brief comments about Marvin and the research environment at the Federal Reserve Bank of Richmond.

Money demand and expected inflation

A key parameter determining the effect of money creation on prices and seigniorage is the semielasticity of demand for real balances, M_t/P_t, with respect to expected inflation, π^e_{t+1}. This is the parameter α in the celebrated Cagan money demand function[1] that is used to study periods of hyperinflation:

$$m_t - p_t = \beta + \alpha \pi^e_{t+1} \tag{1}$$

where $m_t - p_t = \ln(M_t/P_t)$. An important challenge for estimating the semielasticity α is that π^e_{t+1} is unobserved. Marvin's 1982 *Journal of Monetary Economics* paper[2] presents a method for estimating α under the rational expectations assumption:

$$\pi^e_{t+1} = \mathbb{E}(\pi_{t+1} \mid \Omega_t) \tag{2}$$

where Ω_t denotes a time t information set. In its general form, (1) represents a canonical linear model involving unobserved future

[1] Cagan (1956).

[2] Goodfriend (1982).

expectations. Estimating such models under rational expectations was an exciting and active area of research as Marvin developed his estimator for α, and Marvin, along with several other researchers, proposed estimators based on essentially the same insight. (Goodfriend references Hall [1978] and Hayashi [1979] and also discusses the important contribution by Sargent [1977].) The basic estimation insight in all of these papers is now widely appreciated: any variable X can be decomposed as $X = \mathbb{E}(X \mid \Omega) + e$, where e is the prediction error with $\mathbb{E}(e \mid \Omega) = 0$. Applied to inflation, (2) implies the decomposition $\pi_{t+1} = \pi^e_{t+1} + e_{t+1}$, where e_{t+1} is uncorrelated with any variable in the information set Ω_t. Solving for π^e_{t+1} using this decomposition and substituting into (1) yields

$$m_t - p_t = \beta + \alpha\pi_{t+1} - \alpha e_{t+1}, \qquad (3)$$

so the unobserved expectation of inflation in (1) is replaced with actual inflation and a prediction error is added to the equation. The prediction error e_{t+1} is positively correlated with π_{t+1}, so the OLS estimator of α from (3) is not consistent. What is required is an instrument. Rational expectations imply that any variable $Z_t \in \Omega_t$ is uncorrelated with e_{t+1}, so the challenge is to choose Z_t so it is correlated with π_{t+1}. From (1), $m_t - p_t$ satisfies these two requirements, leading to the IV estimator

$$\hat{\alpha}^{IV} = \frac{\text{Var}\,(m_t - p_t)}{\text{Cov}\,(m_t - p_t, \pi_{t+1})}, \qquad (4)$$

which is a version of the estimator proposed by Goodfriend. Thus, from a single equation, and without an explicit modeling of expectations, one can estimate the semielasticity that was the objective of Cagan, Sargent, and others. Notice that the structure of the model yields $m_t - p_t \propto \mathbb{E}(\pi_{t+1} \mid \Omega_t)$, so under homoskedastic i.i.d. errors, $m_t - p_t$ is the optimal instrument and $\hat{\alpha}^{IV}$ is the efficient IV estimator.

Marvin's formulation was different from (1)-(4) in three respects. First, and of no consequence, Marvin solved (3) for π_{t+1}, then regressed π_{t+1} on $m_t - p_t$ using OLS to estimate $1/\alpha$, and then inverted to find $\hat{\alpha}$. This is a "long-way-around" version of the IV estimator in (4). Second, and more interesting, Marvin decomposed $m_t - p_t$ into its m and p components, leading to a test of an overidentifying restriction in the model. Third, and most important, Marvin considered a more general version of (1) that included an additional error v, a "velocity" shock to the money demand equation. In this case, and as noted in Marvin's paper, $m_t - p_t$ is no longer a valid instrument and $\hat{\alpha}^{IV}$ is inconsistent. What is required is an instrument $Z_t \in \Omega_t$ that is correlated with π_{t+1} but uncorrelated with the velocity shock. A more complete model (as in Sargent [1977]) would yield such an instrument using, for example, an exogenous shifter in the money supply function.

The estimator proposed by Goodfriend in this paper and the related estimators proposed by several others during the late 1970s and early 1980s were important drivers of the study of GMM estimators.[3] This analysis has largely been carried out for stationary (or $I(0)$) time series. Far less work has been done on the properties of these estimators (and related inference procedures) in models with the explosive and/or nonstationary data generated by hyperinflations — these were the data of interest in Goodfriend (1982). Marvin's use of the rational expectations assumption yielded valid moment conditions and an associated IV estimator, but statistical inference with locally explosive data remains an understudied challenge, even 40 years after Marvin's contribution. There is still work to do.

Invoking the properties of rational forecast errors to develop estimators is a direct implication of optimal prediction formulae. Marvin's other two papers use optimal prediction formulae in more subtle ways.

Money demand and partial adjustment

Marvin continued his study of money demand in Goodfriend (1985) but in a stationary (non-hyperinflation) environment. A standard formulation expresses the demand for real balances as a function of a vector of variables, x_t, that includes real income and the nominal rate of interest:

$$m_t - p_t = \delta + x_t'\beta + \text{ error}. \qquad (5)$$

An empirical puzzle emerged when (5) was estimated using data from countries like the United States during the 1950s through the

[3] E.g., Hansen (1982) and Hayashi and Sims (1983).

1970s and early 1980s: the model fit the data rather poorly, but the fit improved substantially after augmenting the model with a lagged value of *m-p*, say

$$m_t - p_t = \delta(1-\lambda) + x_t'\beta(1-\lambda) + \lambda(m_{t-1} - p_{t-1}) + \text{ error} \qquad (6)$$

A popular rationale for (6) is that the demand for real balances adjusts slowly toward its target value given by $\delta + x_t'\beta$,[4] with a partial adjustment parameter given by $(1-\lambda)$. A problem with this rationale is that the estimated value of λ turned out to be large, implying an unreasonably long adjustment process. For example, Goldfeld (1973) reports $\hat{\lambda}$ =0.72 from a benchmark specification estimated using quarterly data from the US over 1952:Q2-1972:Q4. This implies an adjustment of only 28 percent $(= 1-\hat{\lambda})$ within the quarter and only 70 percent within a year. Does money demand really adjust that slowly?

Marvin suggested that money demand might, in fact, adjust quite rapidly, and he suggested that the OLS estimator of λ in (6) suffers from errors-in-variables bias. Specifically, he asked: What if the measured value of x is a noisy version of the relevant measures of income and nominal interest rates, say x^*? Could the resulting errors-in-variables bias lead to large estimated values of λ, even though $\lambda = 0$ when using the true value of x^*? Marvin uses optimal prediction formulae to buttress the case for this clever solution to the puzzle about the apparent sluggish adjustment of money demand.

Classical errors-in-variables lead to well-known attenuation bias, so the OLS estimators of the coefficients in (5) are biased toward zero. But Marvin asked the more interesting question: What are the implications of errors-in-variables for estimating the coefficients in (6)? Answering this question requires specifying a joint stochastic process for $m_t - p_t$, the true value of income and interest rates relevant for money demand, x_t^*, and the measurement error, $x_t - x_t^*$.

In practice, empirical researchers use proxies for the income (or expenditure) and interest rates relevant for money demand. For example, Goldfeld used real GDP for income together with interest rates on

[4] For example, see Goldfeld (1973).

commercial paper and time deposits; these were Goldfeld's x-measurements. These are arguably sensible proxies, but they are not perfect measurements of the expenditure and opportunity cost variables determining money demand. Marvin used a variety of sensible calibrations for the $(m_t - p_t, x_t^*, x_t)$ stochastic process, imposing $\mathbb{E}(m_t - p_t \mid x_t^*, m_{t-1} - p_{t-1}) = \delta + x_t^{*\prime}\beta$ so that there is complete adjustment of money demand within the period. He then replaced the true value of x^* with the noisy measurement x and computed $\mathbb{E}(m_t - p_t \mid x_t, m_{t-1} - p_{t-1})$, yielding the population values of (δ, β, λ) in (6). Interestingly, these calibrations yield values of λ that are large and in line with those estimated in the empirical money demand literature.

Marvin's explanation for this dynamic errors-in-variables finding is enlightening: from (5), $m_{t-1} - p_{t-1}$ is positively correlated with $x_{t-1}^{*\prime}\beta$ (and highly so, if the error in (5) is small), x_t^* is likely to be highly serially correlated, so $m_{t-1} - p_{t-1}$ has important predictive power for $x_t^{*\prime}\beta$, even after controlling for the proxy measurements in x. In Marvin's explanation, money demand adjusts rapidly to the fundamentals x_t^*, and the large value of λ in the estimated regression (6) is not structural but instead captures the predictive power of lags of $m-p$ for the correctly measured fundamental x^*.

Consumption and income

The final contribution that I highlight is Goodfriend (1992). The substantive question Marvin addressed in this paper is an apparent failure of the rational expectations version of the life-cycle model for consumption when applied to economy-wide aggregate measures of consumption and income. Specifically, Marvin considered a version of the Hall (1978) random walk model of consumption that implies (under a set of assumptions) that consumption, c_t, is a martingale, so that consumption changes are unpredictable. Marvin's paper studies the robustness of the martingale property under aggregation: he postulates a model in which each individual's consumption is a martingale, and he asks whether the martingale property carries over to aggregate consumption.

Using generic notation, write the model as

$$Y = X + e \quad (7)$$

where $X = E(Y \mid \Omega)$ so that $\mathbb{E}(e \mid \Omega) = 0$. Marvin's paper considers a version of (7) with $Y = c_t$ and $X = c_{t-1}$. Equation (7) implies that the regression of Y onto X and any variable $Z \in \Omega$ will have a unit coefficient on X and a zero coefficient on Z. This is the insight underlying the well-known Mincer-Zarnowitz test for optimal forecasts and the related tests of efficient markets in finance.

Marvin considers a case in which (7) holds for each of *n* members of a population, so $X_i = E(Y_i \mid \Omega_i)$ for $i = 1, \ldots, n$. He then studies the implications for the aggregates, say $Y^{Agg} = \sum_i^n Y_i$ and $X^{Agg} = \sum_{i=1}^n X_i$. Will a Mincer-Zarnowitz regression of Y^{Agg} onto (X^{Agg}, Z^{Agg}) share the same properties as the regression of Y_i onto (X_i, Z_i)? Marvin shows that the aggregates will obey the optimal forecasting relationship if individuals in the economy share the same information set, that is $\Omega_i = \Omega$ for all i, but as general matter, not otherwise. As he notes, $\mathbb{E}(e_i \mid \Omega_i) = 0$ does not imply that $\mathbb{E}\left(e_i \mid \Omega_i, \Omega_j\right) = 0$ because Ω_j may contain useful information about Y_i not contained in Ω_i. Goodfriend (1992) uses this insight to discuss Mincer-Zarnowitz regressions using aggregates and panel data models involving many individuals (large n), but over short time periods (small T). The results are interesting and insightful.

A consultant's view

I became a regular consultant in FRB Richmond's research department in 1995. Marvin was research director at the time, and I came at his invitation. The research department was, and remains, a small, friendly, and very serious place to work. Seminars are great, lunchtime conversation is always focused, and a lot gets done. I learn something, or better yet, get puzzled by something, during every visit. I can't know for sure how much of the department's culture is because of Marvin, or how much of Marvin was because of the department's culture. I suspect there was feedback.

Marvin's research will have a lasting effect on economics, and his collegiality and friendship will have a lasting effect on those of us who were lucky enough to work with him.

References

Cagan, P. 1956. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money*, edited by Milton Friedman, 25-117. Chicago: University of Chicago Press.

Goldfeld, S.M. 1973. "The Demand for Money Revisited." *Brookings Papers on Economic Activity* 3: 577-638.

Goodfriend, M. 1982. "An Alternate Method of Estimating the Cagan Money Demand Function In Hyperinflation Under Rational Expectations." *Journal of Monetary Economics* 9, no. 1: 43-57.

Goodfriend, M. 1985. "Reinterpreting Money Demand Regressions." *Carnegie-Rochester Conference Series on Public Policy* 22: 207-242.

Goodfriend, M. 1992. "Information-Aggregation Bias." *American Economic Review* 82, no. 3 (June): 508-519.

Hall, R.E. 1978. "Stochastic Implications of the Life Cycle - Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86, no. 6 (December) 971-987.

Hansen, L.P. 1982. "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica* 50, no. 4 (July): 1029-1054.

Hayashi, F. 1979. "A New Estimation Procedure Under Rational Expectations." *Economics Letters* 4, no. 1: 41-43.

Hayashi, F., and C. Sims. 1983. "Nearly Efficient Estimation of Time Series Models with Predetermined, But Not Exogenous, Instruments." *Econometrica* 51, no. 3 (May): 783-798.

Mincer, J., and V. Zarnowitz. 1969. "The Evaluation of Economic Forecasts." In *Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance*, edited by Jacob A. Mincer, 3-46. Cambridge: National Bureau of Economic Research.

Sargent, T.J. 1977. "The Demand for Money During Hyperinflations Under Rational Expectations." *International Economic Review* 18, no. 1 (February): 59-82.