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The Cyclicality of the User Cost of Labor

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Abstract

The user cost of labor is the expected difference between the present discounted value of wages paid to a worker hired in the current period and that paid to a worker hired in the next period. Analogously to the price of any long-term asset, the user cost, not wage, is the relevant price for a firm that is considering adding a worker. I construct its counterpart in the data and estimate that it is substantially more procyclical than average wages or wages of newly-hired workers. I demonstrate an application of the finding using the textbook search and matching model.

JEL: E24, E32, J30, J41, J63, J64.

Key Words: User Cost of Labor. Cyclicality. Wage Rigidity. Search and Matching.

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I. Introduction

Macroeconomists have long been interested in the business cycle behavior of the firm's real marginal cost of labor (i.e., price of labor). The literature usually considers average wage to be the measure of the price of labor.[1] Studies find that when the aggregate labor market conditions improve, the hiring wage increases while wages in ongoing employment relationships almost do not change.[2] As a result, on average, wages are only weakly procyclical. However, the wage may not be a good measure of the labor's cost. Since often a worker is employed for more than one period, wage is simply an installment payment on the implicit contract between a worker and a firm. In such a case, neither average wage nor the wage of newly hired worker captures the labor's cost. In fact, Hall (1980) writes that "[T]o see what is happening today in the labor market, one should look at the implicit asset prices of labor contracts recently negotiated, not at the average rate of compensation paid to all workers."[3]

In this paper, I provide a measure of the implicit asset price of labor – the user cost of labor and estimate its cyclicality in the data. Then, I use the estimates to address the volatility of unemployment in the textbook search and matching model (Mortensen and Pissarides (1994)).

The user cost of labor equals the increment to the expected present discounted value of costs of adding a worker now versus waiting until the subsequent period. Analogously to the price of any long-term asset, the user cost, and not wage, is the relevant price of labor for a firm that is considering adding a worker. There can be other costs associated with adding a worker besides wage payments (for example, hiring or training cost); in the paper, I estimate the cyclicality of the wage component of the user cost of labor. The wage component of the user cost of labor in period t is the difference between the expected present discounted value of wages paid to a worker hired in t and the expected present discounted value of wages paid to a worker hired in $t+1$. If the labor market is a spot market, then the difference equals

[1] See, for example, a survey in Mankiw, Rotemberg and Summers (1985).

[2] See Bils (1985), Solon, Barsky, and Parker (1994), and, among recent empirical works, Martins, Solon, and Thomas (2012), Carneiro, Guimarães and Portugal (2012), and a survey in Pissarides (2009).

[3] Barro (1977) calls sticky wages just a "façade" of the implication of the long-term labor contracts to short-term macro fluctuations.

the wage. However, if a worker is contracted for more than one period, then the difference need not be equal to the wage, as economic conditions at the time of hiring may have an impact on the future wages. The user cost takes into account both the wage at the time of hiring and the effect of the economic conditions at the time of hiring on future wages.

The paper's main empirical finding is that the wage component of the user cost of labor is substantially more procyclical than the average wages or even the wages of newly hired workers. The intuition behind the large cyclicality of the wage component of the user cost of labor as compared to the cyclicality of wages is as follows. Consider a firm hiring a worker when the unemployment rate is high. Since the unemployment rate is high, the hiring wage is low. Once a worker is hired, his wage in the employment relationship does not respond as much to the contemporaneous labor market conditions as the hiring wage does (because wages of newly hired workers are found to be more procyclical than the wages of workers in ongoing relationships (Pissarides (2009))). Hence, the stream of wages to be paid to a worker hired when the unemployment rate is high is expected to be lower than the stream of wages to be paid to a worker hired when the unemployment rate is low. Consequently, when the unemployment rate is high, the user cost of labor is lower than the already low hiring wage because the user cost also captures comparatively low future wages in this relationship. The opposite is true when a worker is hired at the peak of the business cycle, i.e., when the unemployment rate is low and is expected to rise. Then, the user cost of labor is higher than the hiring wage. Hence, the procyclical hiring wage and the relatively non-cyclical wage within employment relationship contribute to the wage component of the user cost of labor being more procyclical than average wage or even than the hiring wage.

I estimate the cyclicality of the wage component of the user cost of labor using the NLSY79 data. Since the user cost is not directly observed in the data, I construct its empirical counterpart based on the behavior of individual wages and turnover. In the construction, I discount future payments taking into account the separation rates and the real interest rates. The estimated cyclicality is the projection of the (log of the) constructed series of the user cost of labor on the unemployment rate.

I find that the constructed wage component of the user cost of labor decreases by more than 4.5% in response to a one percentage point increase in the unemployment rate, while the wages of newly hired workers decrease by 3% and average wages decrease by 1.5%.

This paper, to my knowledge, is the first attempt to directly measure the cyclicality of the price of labor taking into account the effect of economic conditions at the time of hiring on future wages. A large empirical literature exists that studies the behavior of individual wages over the business cycle.[4] However, one crucial aspect of the existing literature is that it provides evidence on the cyclical behavior of the current wage, but not on the cyclical behavior of the expected present discounted value of future wages within a match formed in the current period.

The result of the paper has an important implication for the existing models. Models often require some rigidity of the wage component of the user cost of labor to amplify and propagate the impact of a productivity shock. The weak cyclicality of wages in the data is often used as evidence of such rigidity. The results of the paper show that wage cyclicality is not a correct measure of the cyclicality of the labor's user cost and that the labor's user cost is much more procyclical than wages. Consequently, the propagation mechanism that requires rigidity of labor's user cost might lack support in the data.

In the second part of the paper, I demonstrate the implications of the estimated cyclicality of the user cost of labor for the propagation of shocks in the textbook search and matching model (Mortensen and Pissarides (1994); Pissarides (2000)). Shimer (2005) shows that the textbook search and matching model lacks amplification of the productivity shock to generate the empirical volatility of the vacancy-unemployment ratio (i.e., "unemployment volatility puzzle").[5] Shimer's work has motivated a large literature that seeks a solution for the unemployment volatility puzzle (see, for example, Rudanko (2009)). The solution that has generated considerable attention is wage rigidity (Hall (2005)).[6] While the solution works theoretically, its empirical potentiality has received much less attention (except for the contemporaneous works by Pissarides (2009) and Haefke, Sonntag, and van Rens (2009)). In this paper, I address the question whether wage data exhibit rigidity necessary to amplify the shock in the model.

In the model, the wage rigidity solution works through making the wage component of

[4] See references in footnote 2.
[5] See also Hall (2005) and Costain and Reiter (2008).
[6] Rogerson and Shimer (2011) and Diamond (2011) provide an overview of the related literature.

the user cost of labor rigid.[7] This is so because the model's free entry condition ties productivity to the wage component of the user cost of labor and the vacancy-unemployment ratio. Thus, it imposes a trade-off between the elasticity of the wage component of the user cost of labor with respect to productivity and the elasticity of the vacancy-unemployment ratio with respect to productivity. Using my estimates of the cyclicality of the wage component of the user cost of labor, I directly check the condition. Given the estimate of the cyclicality of the wage component of the user cost of labor, the free entry condition requires a much lower volatility of the vacancy-unemployment ratio than observed in the data.

Thus, the wage component of the user cost of labor is too volatile to amplify the volatility of the vacancy-unemployment ratio in the model. Consequently, the solution to the unemployment volatility puzzle cannot be explained by a wage formation because any wage formation should be able to match the empirical volatility of the wage component of the user cost of labor. Note that, conditional on the volatility of the wage component of the user cost, different wage formations deliver different volatilities of average wages or wages of newly hired workers. However, the firm's free entry condition imposes the restriction on the wage component of the user cost of labor, not wage per se. In contrast to the existing literature, the paper's estimates of the cyclicality of the wage component of the user cost of labor allow a direct comparison of the behavior of the wage component of the user cost in the model to its behavior in the data.

Furthermore, I find that there is no such a value for the unemployment benefits parameter in the set of its feasible values so that the model's free entry condition simultaneously accommodates the empirical elasticity of the vacancy-unemployment ratio and of the wage component of the user cost of labor. This contrasts with the conclusion of Hagedorn and Manovskii (2008) who argue that at the high values of the unemployment benefits the model can generate the empirical volatility of the vacancy-unemployment ratio. However, Hagedorn and Manovskii use the elasticity of the average wages rather than the elasticity of the wage component of the user cost of labor. As the estimation in this paper show, the latter is much more procyclical than the former.

[7] A related conclusion is provided by Shimer (2004) who emphasizes that what is relevant for the volatility of job creation, and, thus, of unemployment, is the rigidity of the present discounted value of wages that at the time of hiring a firm expects to pay to a worker over the course of the employment relationship.

Finally, to illustrate that the wage component of the user cost, and not wage, determines allocations, I specify and solve four search and matching models with different wage formations. The wage formations are chosen so that they allow for different degree of individual wage rigidity within an employment relationship. In particular, I consider implicit insurance contracts with different degree of commitment as in Rudanko (2009). The simulation results from the models illustrate that in the presence of contracts, a weak cyclicality of individual wages can conceal a substantial cyclicality of the wage component of the user cost. The results also show that once the cyclicality of the wage component of the user cost is calibrated to be the same across the models with different wage formations, the models generate very similar volatility of the vacancy-unemployment ratio. The cyclicality of individual wages and the wages of newly hired workers, however, differ significantly across the models. Finally, when the models are calibrated to match the estimated cyclicality of the wage component of the user cost, none of the models can generate more than half of the empirical volatility of the vacancy-unemployment ratio.

Only recently has literature turned to contrasting the wage dynamics in the search and matching model with the data to address the unemployment volatility puzzle. Pissarides (2009) compares the elasticity of wages from the textbook search and matching model to the elasticity of hiring wages in the data. Haefke, Sonntag, and van Rens (2009) use model simulations to argue that the elasticity of the wage of newly hired workers with respect to productivity can be seen as a lower bound for the elasticity of the present discounted value of wages with respect to the present discounted value of productivity. In contrast, I directly calculate the present discounted values of wages from the individual wage data, construct the wage component of the user cost of labor, and empirically estimate its cyclicality. Because the wage component of the user cost of labor is more cyclical than hiring wages, the results show that not only the wage data lack rigidity to solve the vacancy-unemployment puzzle but also the free entry condition of the model cannot simultaneously generate the empirical volatilities of the vacancy-unemployment ratio and of the wage component of the user cost of labor.

The paper is organized as follows. Section 2 introduces the user cost of labor and presents the main empirical results. Section 3 addresses the unemployment volatility puzzle in the textbook search and matching model. Section 4 presents a quantitative investigation of the cyclicality of the wage component of the user cost of labor in models with specific wage formations. Section 5 concludes.

II. Cyclicality of the Wage Component of the User Cost of Labor

A. The Wage Component of the User Cost of Labor

Consider an economy populated by a continuum of infinitely lived profit-maximizing homogeneous firms and a continuum of infinitely lived workers. Each firm operates a constant return to scale production technology, which requires only labor input. Each period, a firm decides whether to create a productive match with a worker in the current period or to postpone the creation until the next period. A firm pays according to the wage schedule agreed at the time when the worker is hired. Every period, a nonzero probability exists that a worker will exogenously separate from the firm.

The costs associated with the firm's decision to add a worker in period t versus in period $t+1$ are summarized by the user cost of labor: they are all expenses associated with creating a firm-worker match in period t that can be avoided if the creation of the match is postponed until the next period. Therefore, the user cost does not include the total payments associated with creation of a match in period t (i.e., wage payments throughout the duration of an employment relationship, hiring costs, training expenses and other); rather, it includes only the part that is expected to be in excess of what a firm will need to pay the next period. The user cost of labor is analogous to the user cost of capital, which is the difference between the purchase price and the expected price at which the remaining, un-depreciated part of the capital can sold at the end of the period.

The focus of the paper is on estimating the cyclical behavior of the wage component of the user cost of labor. The wage component of the user cost of labor is the expected difference in expenses from starting to pay wages in the current period versus the next period. Let $w_{t,\tau}$ denote the wage paid in period τ to a worker hired in period t, $\tau \geq t$, let δ denote the rate at which a worker separates from the firm, and let β denote a discount factor, $0 < \beta < 1$. The expected present discounted value of wages to be paid to a worker hired in period t is given by

$$(1) \qquad PDV_t = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} w_{t,\tau},$$

where $E_t = E(.|I_t)$ and I_t is the information set of the firm in time t.

Equation (1) states that a worker hired in period t is paid wage $w_{t,t}$. With probability $1-\delta$ the employment relationship survives until the period $t+1$ and the worker is paid wage $w_{t,t+1}$. With probability δ the relationship is terminated and the firm pays nothing.

The wage component of the user cost of labor in period t is the difference between the expected present discounted value of wages paid to a worker hired in t and the expected present discounted value of wages paid to a worker hired in $t+1$, i.e.,

$$(2) \qquad UC_t^W = E_t \left[PDV_t - \beta(1-\delta) PDV_{t+1} \right].$$

Equation (2) shows that the user cost is the expected difference of the costs of two alternatives: hiring a worker in period t, or hiring a worker in period $t+1$ with probability $(1-\delta)$. These two options differ only in how many workers the firm employes in period t, and they give the same expected employment levels in all future periods. Therefore, the difference between the costs of the two options gives the implicit price of the services of one worker in period t.

Substituting expression for PDV_t in (2) yields the following expression for the wage component of the user cost of labor:

$$(3) \qquad UC_t^W = E_t \left[w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} (w_{t,\tau} - w_{t+1,\tau}) \right].$$

Equation (3) states that the wage component of the user cost of labor in period t is the sum of the hiring wage in period t and the expected present discounted value of the differences between wages paid from the next period onward in the employment relationship that starts in period t and the employment relationship that starts in period $t+1$.[8] Thus,

[8] See Appendix A for a proof of $E_t | w_{t,t} + \sum_{\tau=t+1}^{\infty} \left(\beta^{\tau-t} \prod_{d=t}^{\tau-1} (1-\delta_d) \right) (w_{t,\tau} - w_{t+1,\tau}) | < \infty$.

the user cost of labor captures the hiring wage and the effect of the economic conditions at the time of hiring on future wages within the employment relationship. Unless the second term in equation (3) is 0, the wage component of the user cost is not equal to the wage at the time of hiring.

Consider the conditions under which the second term in (3) is 0. One such example is the case in which the wage is bargained each period and does not depend on the history of the labor market conditions from the start of the job. Then, if the productivity in two matches is the same, these matches pay the same wages, independently when the matches were formed, i.e., $w_{t,\tau} = w_{t+1,\tau} = w_\tau \ \forall \tau \geq t+1$. In such a case, the wage component of the user cost of labor equals the wage at the time of hiring and also equals the average wage. Nash bargaining in the textbook search and matching model is an example of such a wage formation. Another example of a wage formation in which the wage component of the user cost is equal to the wage at the time of hiring is the situation when the wage is rigid and is not responsive to changes in economic conditions, i.e., $w_{t,\tau} = w_{t+1,\tau} = w \ \forall \tau \geq t+1$.

In general, if wages depend on the economic conditions from the start of the job, the wage component of the user cost of labor does not equal wage. Note that there could be different sources of such a dependence. For example, the dependence can arise as a result of insurance contracts between workers and firms against fluctuations in productivity (for example, Thomas and Worrall (1988)). Another source of the dependence of wages on the economic conditions from the start of the job can be due to the systematically different match quality of newly created and ongoing matches over the business cycle.[9] Importantly, however, for the measurement of the cyclicality of the user cost of labor the source of the differences in the present discounted values of wages to be paid to a worker hired in t and to the worker's replacement hired in $t+1$ is irrelevant. This is so because the user cost of labor is concerned only with the cost side of adding a worker. A firm that is considering adding a worker compares the user cost of such a decision to its benefits. Suppose, for example, that when the unemployment rate increases, the quality of newly created matches

[9] Gertler and Trigari (2009) and Hagedorn and Manovskii (2010) argue that in recessions the wages of newly hired workers are lower than the wages of workers in ongoing relationships because matches created in recessions are of lower productivity than the matches created in better times. Recently, however, Carneiro, Guimarães and Portugal (2012) use matched employer-employee data, which enable controlling for firm-worker match effects, and find that wages of new hires are more procyclical than wages of existing workers.

decreases and, as a result, wages of newly hired workers are lower than wages of workers in ongoing relationships. Then, the differential wages in the new and ongoing relationships will be reflected in the user cost of labor and constitute a part of the effective price of labor faced by a firm that is considering adding a worker. The differential productivity in the new and ongoing relationships should be reflected in the benefits side of the decision of adding a worker. This paper provides an estimate of the cyclicality of one side of such a decision – the cost side. In this sense, the present paper is similar in spirit to the earlier literature that is concerned with the measurement exercise of the cyclical dynamics of wage movements (Bils (1985), Shin (1994), among others) rather than with the cyclical dynamics of wage net of productivity.

B. Estimation Procedure

The cyclicality of the wage component of the user cost of labor is the expected proportional change in the wage component, UC_t^W, in response to a unit change in the unemployment rate, u_t, i.e., the semielasticity of UC_t^W with respect to u_t. This measure of cyclicality is typically used in the literature on the cyclicality of wages (Pissarides (2009)). It can be estimated as the projection of $\ln UC_t^W$ on u_t:

$$\gamma = \frac{cov(\ln UC_t^W, u_t)}{var(u_t)}.$$

Let UC_t^{WR} be the realized, ex post value of the wage component of the user cost of labor such that $UC_t^W = E_t(UC_t^{WR})$. Given the standard rational expectations argument[10], the cyclicality of the wage component can be calculated as

$$\gamma = \frac{cov(\ln UC_t^{WR}, u_t)}{var(u_t)}. \tag{4}$$

Now, the task is to construct an empirical counterpart of UC_t^{WR} and to estimate the

[10] Define a random variable ε_t such that $UC_t^{WR} = UC_t^W \varepsilon_t$, where ε_t is independent of the variables in the information set of a firm in t. Then $cov(\ln UC_t^{WR}, u_t) = cov(\ln UC_t^W, u_t) + cov(\ln \varepsilon_t, u_t)$. Because the information set of a firm in t contains the contemporaneous unemployment rate, u_t, the last term is 0. Then $cov(\ln UC_t^{WR}, u_t) = cov(\ln UC_t^W, u_t)$. This yields expression (4) for the cyclicality of the wage component of the user cost of labor.

cyclicality in (4).

The wage component of the user cost of labor is not directly observed in the data; hence, I construct an empirical counterpart of UC_t^{WR}, $\widehat{UC_t^{WR}}$, from individual wages and turnover. Calculation of the wage component requires two sequences of wages for each t in the sample: a sequence of wages to be paid to a worker hired in t and a sequence of wages to be paid to a worker hired in $t+1$. To construct these expected sequences, I clean the wage data from the individual-specific effects. To obtain a series of wages free of individual-specific effects, I estimate the following wage equation using panel data on individual wages that covers period from year 1 to year T:

$$\ln w_{t,\tau}^i = c + \alpha^i + \varsigma\tau + \Psi X_\tau^i + \sum_{d_0=1}^{T}\sum_{d=d_0}^{T} \chi_{d_0,d} D_{d_0,d}^i + \varepsilon_\tau^i, \tag{5}$$

where $w_{t,\tau}^i$ is the wage of worker i in year τ hired in year t, X_τ^i is the vector of individual- and job-specific characteristics, α^i is the worker-specific individual fixed effect, and ε_τ^i is the error term such that $\varepsilon_\tau^i \sim N(0, \sigma_\varepsilon^2)$. $D_{d_0,d}^i$ is the dummy variable that takes value 1 if $d_0 = t$ and $d = \tau$, and 0 otherwise. That is, $D_{d_0,d}^i$ takes value 1 for all wage observations in year τ in the employment relationships that started in year t, where $t \in [1, T]$ and $\tau \in [t, T]$.

Importantly, equation (5) does not impose any particular wage formation on the wage data. The estimates of the coefficients $\{\chi_{d_0,d}\}$ allow constructing the expected wage for each pair of $\{t, \tau\}$ in the sample period, conditional on worker characteristics.

The detailed steps that I use to construct the wage component of the user cost of labor and calculate its cyclicality are as follows.

Step 1.

I estimate equation (5) using the NLSY79 data that span the period from 1978 to 2004. The vector of individual- and job- specific characteristics, X_τ^i consists of education, a quadratic in tenure, and a quadratic in potential labor market experience. Equation (5) is estimated weighting each observation by sampling weights and controlling for worker-specific fixed effects. The standard errors are clustered by time.

Using the coefficient estimates on the set of dummies, $\{\chi_{d_0,d}\}$, I calculate the fitted values for wages, $\widehat{w_{t,\tau}}$, for all t and $\tau : t, \tau = \{1, T; t \leq \tau\}$, i.e.,

$$\widehat{w_{t,\tau}} = \exp\left(\widehat{c} + \widehat{\varsigma}\overline{\tau} + \widehat{\Psi}\overline{X} + \widehat{\chi_{t,\tau}}\right),$$

where $\overline{\tau}$ and $\overline{X}$ are the sample means. Note that $\widehat{w_{t,\tau}} = E_t\left(w_{t,\tau}\right)/\exp\left(-\frac{\sigma_\varepsilon^2}{2}\right)$. Assuming that $\sigma_\varepsilon^2 = const$, the cyclicality of the wage component of the user cost of labor does not depend on the actual values of $\bar{t}$, $\overline{X}$ and σ_ε^2.

Step 2.

I set discount factor, β, to $1/(1.045)$ and annual separation rate, δ, to 0.295, which is obtained from the monthly separation rate of 0.029 in the NLSY79 data. Then, I use equation (3) to calculate an empirical counterpart of the realized wage component of the user cost of labor using the constructed series $\{\widehat{w_{t,\tau}}\}$, β and δ.

The expression for the wage component of the user cost of labor, (3), assumes infinitely lived firms and workers, while the data allow a sample of a finite size. Thus, I truncate the calculations of the sum at different time horizons and check the sensitivity of the estimated cyclicality to the truncation horizon. In the benchmark case, I truncate the horizon at 7 years.[11] Truncation of the time horizon for calculation of UC_t^W can be justified by two considerations. First, the discount factor, which includes the turnover rate and the real interest rate, increases. This decreases the weight of the terms far in the future. Second, if, for example, the model behind the dependence of wages on the history of unemployment rates is as in Thomas and Worrall (1988) and the unemployment rate follows the mean-reverting process, then the wages in the employment relationships that started in different years but that have lasted long enough to experience similar episodes of minimum and maximum unemployment rates will be the same. In such a case, the terms in the brackets in equation (3) will be equal to 0 for all τ higher than some τ'.

Step 3.

Finally, to obtain the cyclicality of the wage component of the user cost of labor, I regress

[11] Given the truncation at 7 years and the sample period from 1978 to 2004, the wage component of the user cost of labor is calculated for 20 years, from 1978 to 1997. This number of observation is typical for the papers on the cyclicality of wages that employ a two-step estimation procedure (Solon, Barsky and Parker (1994), Devereux (2001)). For example, Devereux (2001) reports 22 observations in the second-stage regression.

the logarithm of the constructed realized wage component of the user cost of labor on the unemployment rate and a time trend. The cyclicality is the coefficient on the unemployment rate multiplied by 100%. I also report bootstrapped standard errors.

C. Data

The data in the study are from the NLSY79, 1978—2004. The survey collects information on work histories of a nationally representative sample of young individuals who were between 14 and 21 years of age in 1979, when the first interview was taken.

I focus on the cross-sectional sample that represents the non-institutionalized civilian population and further restrict my analysis to males. This restriction is typical for empirical studies of wage cyclicality (Beaudry and DiNardo (1991); Shin (1994)). Hence, I work with the following sub-samples, as defined in the NLSY79: 1 (cross-sectional white males), 3 (cross-sectional black males), and 4 (cross-sectional Hispanic males).

The data set is particularly suited for the purposes of this study because it separately records wages and other job characteristics for up to five jobs that an individual might hold between two consecutive interviews. By tracking individuals over the years, I can isolate the individual-specific fixed effects. In addition, if a worker simultaneously held more than one job, the NLSY79 keeps a separate record for each job, as opposed to the PSID data that report the average wage in such cases. The NLSY79 data contain information on individuals at the early stages of their labor market experience. Because jobs taken at the early stages of an individual's labor experience may be predominantly seasonal or temporary, this may disproportionately affect the wage cyclicality. To alleviate this problem, I restrict the observations included in the wage equation to the observations of individuals who started a job at the age of 16 or older, and who were 21 years old or older at the time of the observation. Because I use workers' fixed effects in the estimation, the sample is restricted to the workers having more than one observation.

The wage is an hourly pay variable constructed by the NLSY. I deflate wages using the annual CPI index of the year the observation refers to. The unemployment rate is the annual, national, civilian unemployment rate for ages 16 and older obtained from the Bureau of Labor Statistics.

D. Main Empirical Result

Using the data, I construct the wage component of the user cost of labor and estimate its cyclicality as described in the steps above. The main empirical result is presented in Table 1. The first row of the table shows the estimate of the cyclicality of the wage component of the user cost of labor. The estimate shows that when the unemployment rate goes up by a one percentage point, the wage component of the user cost of labor, on average, goes down by more than 4.50%.

For comparison, I also estimate the cyclicality of individual wages. Column 1 of Table 2 contains the estimation results from the full sample. Column 2 of Table 2 contains the estimation results from the sample of new hires only, i.e., the sample of workers with tenure less than 1 year.[12] The regressions are estimated controlling for workers' fixed effects; thus, each sample is restricted to at least two observations per worker. The results show that a one percentage point increase in the unemployment rate is associated with a 3% decrease in wages of newly hired workers and a 1.78% decrease in wages of all workers. The results in Table 2 are consistent with the findings of the numerous studies on the cyclicality of wages (see a survey in Pissarides (2009)).

Rows 2 and 3 of Table 1 replicate the cyclicality of wages of newly hired workers and the cyclicality of wages of all workers, respectively, from Table 2. Comparing Rows 1—3 reveals that the wage component of the user cost of labor is substantially more procyclical than average wages, and also more procyclical than wages of newly hired workers.

The different cyclicality of the wages of newly hired workers (Row 2) and the wages of all workers (Row 3) implies that wages in ongoing employment relationships do not respond to the changes in the aggregate unemployment rate at the same rate as the wages of newly hired workers. Consequently, if a firm hires a worker when the unemployment rate is high, the stream of wages to be paid to the worker is expected to be lower than the stream of wages to be paid to a worker hired when unemployment is low. By analogy with the user

[12] In the regressions, the dependent variable is the natural logarithm of wages. The explanatory variables are the contemporaneous unemployment rate and the individual- and job-specific controls that are typically used in estimating the cyclicality of wages (i.e., the number of years of education, a quadratic in tenure, a quadratic in experience, a dummy for union status, and a linear time trend). The standard errors are clustered by time. Columns 3 and 4 of Table 2 show the results from the regressions with the controls for 14 industry dummies.

cost of any asset that can be used for more than one period, the firm's user cost of labor takes into account the future capital gains (or losses) from hiring a worker. Thus, when the unemployment rate is high, the wage component of the user cost of labor is even lower than the already low hiring wage. The opposite is true when a worker is hired at the peak of the cycle, when the unemployment rate is low but is expected to rise: the wage component of the user cost is higher than the hiring wage. Consequently, the wage component of the user cost of labor is more procyclical than wages of newly hired workers. Thus, the differential cyclical response of wages of newly hired workers and wages of workers in ongoing relationships contribute to the substantial procyclicality of the user cost of labor.

E. Robustness: Estimation with Time-Varying Separation Rates

The expression for the wage component of the user cost of labor in equation (3) is based on the assumption of the constant separation rate. However, in the data, the separation rate may vary with the contemporaneous labor market conditions or the labor market conditions at the start of the job.

To understand the effect that the time-varying separation rates might have on the cyclicality of the wage component of the user cost of labor, suppose that the separation rate depends positively on the unemployment rate at the time of hiring.[13] In such a case, the workers who are hired when the unemployment rate is high tend to separate sooner than the workers hired when the unemployment rate is low. Once a worker is separated, a firm must hire a new one to fill the position. If the labor market conditions have improved, a new worker is offered a present discounted value of wages that is expected to be higher than the value that would have been paid to the worker hired earlier. Thus, the higher separation rate might weaken the lock-in to the initial labor market conditions.

To examine whether this effect is quantitatively important for the cyclicality of the wage component of the user cost, I construct and estimate the cyclicality of the two alternative measures of the wage component of the user cost of labor: (1) a measure that allows the separation rate to depend on the contemporaneous period, and (2) a measure that allows

[13] Bowlus (1995) provides evidence that the matches created when the unemployment rate is high usually separate earlier than the matches created when the unemployment rate is lower.

the separation rate to depend both on the contemporaneous period and on the period when a worker is hired.

Let δ_τ denote the separation rate in period τ (i.e., the separation rate depends on the contemporaneous period). Then, equation (2) can be rewritten to allow the separation rate to depend on the contemporaneous period as follows:

$$UC_t^W = PDV_t - \beta(1-\delta_t)E_t PDV_{t+1},$$

where $PDV_t = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} \left(\beta^{\tau-t} \prod_{d=t}^{\tau-1}(1-\delta_d)\right) w_{t,\tau}$. Substituting the expression for PDV_t yields

$$UC_t^W = w_{t,t} + E_t \left[\sum_{\tau=t+1}^{\infty} \left(\beta^{\tau-t} \prod_{d=t}^{\tau-1}(1-\delta_d) \right) (w_{t,\tau} - w_{t+1,\tau}) \right]. \tag{6}$$

Equation (6) is the expected difference in the costs between two alternatives: hiring a worker in period t, or hiring a worker in period $t+1$ with probability $(1-\delta_t)$. These two options differ only in how many workers the firm employs in period t; they give the same expected employment levels in all future periods. Therefore, the difference between them gives the implicit price of the services of one worker in period t.

Alternatively, consider the separation rate that may depend both on the contemporaneous period and on the period when the worker is hired. Then, equation (6) should be modified to ensure that in every period after t, the employment level in the relationship that starts in t and the employment level in the relationship that starts in $t+1$ are equal. One way to incorporate the modification is to assume that, whenever a worker separates, a firm must rehire a worker to replace the separated one at a new wage agreement. Then, the wage component of the user cost of labor in period t can be thought of as the difference between the expected present discounted value of wages paid at the position opened in period t and the one opened in period $t+1$. These two options give the same expected employment level – one – in all future periods. Therefore, the difference between them gives the implicit price of the services of one worker in period t. The exact expression for the wage component of the user cost of labor with the separation rate that might depend on the contemporaneous period and on the period when the worker is hired is derived in Appendix A.

To estimate the cyclicality of the wage component of the user cost of labor with time-varying separation rates, I construct the realized wage component of the user cost of labor using a procedure similar to the three-step procedure described above. The difference consists of using the corresponding equations for the wage component of the user cost of labor with time-varying separation rates and using the estimated series of the separation rates from the NLSY79 data instead of the constant separation rate.

To obtain the series of time-varying separation rates, I, first, estimate a linear probability model of the monthly probability of separation as a function of a quartic in the monthly trend and use the estimates to de-trend the fitted probabilities. Then, to obtain the series that depend only on the contemporaneous period, I estimate a linear model of the detrended monthly separation rate as a function of the set of the contemporaneous time dummies. Using the coefficients estimates, I obtain fitted projections, $\widehat{\delta_t}$, for all t in the sample period. To obtain the series of the separation rates that depend on the contemporaneous period and on the period when the job starts, I estimate a linear probability model of the detrended monthly separation rates on two sets of time dummies: a set of time dummies that corresponds to the year the job starts and a set of dummies that corresponds to the contemporaneous year. Using the coefficients estimates, I obtain fitted projections, $\widehat{\delta_{t,\tau}}$, for all $\{t,\tau\}, t \leq \tau$, in the sample period. Finally, I use monthly fitted projections to obtain annual separation rates.

The results of the estimation of the cyclicality of the wage component of the user cost of labor with different treatment of separation rates are presented in Table 3. Rows 2 and 3 of the table contain the estimated cyclicality of the wage component of the user cost of labor with the separation rate that depends on the contemporaneous period and the wage component of the user cost of labor with the separation rate that depends both on the contemporaneous period and the period when the worker is hired, respectively. For comparison, Row 1 contains the cyclicality of the wage component of the user cost of labor with the constant separation rate. The columns of the table show the cyclicality at different truncation horizons (5, 7, and 9 years). As can be seen from the table, in all specifications, the absolute value of the cyclicality of the wage component of the user cost of labor exceeds 4.5%, which is substantially higher than the cyclicality of individual wages of all workers and also noticeably higher than the cyclicality of wages of newly hired workers.

III. The User Cost of Labor with Search and Matching

In this section, I use the estimates of the cyclicality of the wage component of the user cost of labor to examine the quantitative behavior of the textbook search and matching model (Pissarides (2000), Shimer (2005)).

A. Environment

The environment closely follows the one in Shimer (2005) except that workers can be risk averse and no particular wage formation is specified. The economy is populated by a continuum of infinitely lived profit-maximizing homogeneous firms and a continuum of measure 1 of homogeneous infinitely lived workers. Time is discrete. Workers maximize the present discounted value of utility, $u(c)$, with $u'(c) > 0$, $u''(c) \leq 0$. They do not have access to credit markets and cannot save or borrow. Firms and workers discount the future with a common discount factor β, $0 < \beta < 1$.

A firm can choose to remain inactive or to start production. Production requires only labor input. To start production, a firm must enter the labor market and hire a worker. Upon entering the labor market, a firm opens vacancies and searches for a worker. The entry is free; however, a firm must pay a per vacancy cost, c, measured in units of the consumption good. Workers in the economy can be employed or unemployed. An unemployed worker receives a per period unemployment benefit, b, and costlessly searches for a job. Given the number of unemployed workers, u, and the number of vacancies, v, the number of newly created matches in the economy is determined by a matching function, $m(u, v) = Ku^{\alpha}v^{1-\alpha}$, where $\alpha \in [0, 1]$ (Petrongolo and Pissarides (2001)) and $K > 0$. Given $\theta \equiv \frac{v}{u}$, the labor market tightness, the probability of filling a vacancy for a firm is $q(\theta) \equiv K\theta^{-\alpha}$ and the probability of finding a job for an unemployed worker is $\mu(\theta) \equiv K\theta^{1-\alpha}$. While matched, each firm-worker match produces per period output z. The stochastic process for z is governed by a stationary first-order Markov process. Workers cannot search while employed.

Each period τ, an employed worker receives wage $w_{t,\tau}$, where t is the period when the worker is hired, $t \leq \tau$. One can think of the wage as the result of a surplus division agreement between the firm and the worker, which may or may not entail history dependence in wages. In the textbook search and matching model (Shimer (2005)), wages are negotiated each period in all matches according to Nash bargaining rule, in which case $w_{t,\tau} = w_{\tau,\tau} = w_\tau$ $\forall \tau \geq t$. To carry the analysis in this section, however, I do not need to specify a particular wage formation. It suffice to assume that wages take a general form of $w_{t,\tau}$ and are the result of an optimal decision between worker and a firm.[14]

The economy operates according to the following time-line:[15] 1) At the beginning of a period, a firm decides whether to create a job or to stay inactive; if the decision is to create a job, the firm posts vacancies and incurs the vacancy posting cost. Workers who were unemployed for at least one period costlessly search for jobs. 2) When a firm with an open vacancy encounters an unemployed worker, a new match is created. 3) Production takes place in both newly created matches and matches that were carried over from the previous period. Employed workers receive wages and unemployed workers receive unemployment benefit. 4) At the end of a period, a fraction δ of productive matches is randomly selected and exogenously destroyed: the workers who were employed in those matches become unemployed and the firms who operated those matches return to the pool of inactive firms. 5) Surviving matches are carried over to the next period.

B. The User Cost of Labor

The only nontrivial economic decision in this environment is a firm's decision to create a match with a worker in the current period versus to postpone the creation until the next period. The costs of such a decision are summarized by the user cost of labor. The user cost of labor is the expected present discounted value of the costs associated with creating and maintaining a productive match with a worker that starts in period t less the expected present discounted value of the costs of replacing the worker in period $t + 1$.

In the model, a firm faces two sources of costs associated with a match: wage payments to a worker and costs associated with vacancy opening. Thus, the user cost of labor, UC_t,

[14] In Section 4, I provide examples with specific wage formations.

[15] The value functions for the economy are summarized in Appendix B.

can be decomposed into its two components: the wage component, UC_t^W, and the vacancy component, UC_t^V, i.e.,

$$UC_t = UC_t^W + UC_t^V.$$

The wage component of the user cost of labor, UC_t^W, is given in equation (3). The vacancy component is associated with fixed cost on vacancy opening, c, and the probabilities of filling a vacancy in t, q_t, and in $t+1$, q_{t+1}. Given that to create one match in period t, a firm opens $1/q_t$ vacancies, each at cost c, the vacancy component is

$$UC_t^V = \frac{c}{q_t} - \beta(1-\delta)E_t\frac{c}{q_{t+1}}.$$

The vacancy component of the user cost of labor takes into account the different probabilities of filling a vacancy in period t and period $t+1$, the real interest rate associated with paying cost in t instead of delaying it until $t+1$, and a worker turnover cost (due to the possibility of separation in period t, which decreases the expected number of matches surviving until period $t+1$). If, for example, a worker is always available for hire, i.e., $q_t = q_{t+1} = 1$, then the vacancy component equals $(1-\beta(1-\delta))c$. However, with search and matching frictions, the probability of filling a vacancy may differ between t and $t+1$.

C. *Free Entry Condition*

The key equilibrium condition in the search and matching model is the free entry condition for firms. The condition implies that firms enter the labor market and create vacancies until the cost of creating a vacancy equals the benefits,

$$\frac{c}{q(\theta_t)} = J_{t,t}, \tag{7}$$

where $J_{t,\tau}$ is the value of a firm with a worker at time τ given that the productive match starts at time t, i.e., $J_{t,\tau} \equiv z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(z_\tau - w_{t,\tau})$.

Note that the free entry condition specifies that equation (7) is required to hold only for newly created matches, i.e., for $J_{t,\tau} : \tau = t$. The free entry condition per se does not impose restriction (7) on the firm's value in the ongoing matches (i.e., for $J_{t,\tau} : \tau > t$).[16]

The following proposition obtains.

Proposition 1. *Given the free entry condition for firms, the marginal productivity of a firm-worker match equals the user cost of labor, $z_t = UC_t$ $\forall t$, i.e.,*

$$z_t = \left(w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(w_{t,\tau} - w_{t+1,\tau}) \right) + \left(\frac{c}{q(\theta_t)} - \beta(1-\delta)E_t \frac{c}{q(\theta_{t+1})} \right). \tag{8}$$

Proof.

The expected difference between the firm's value of a newly created match in period t and the discounted value of a newly created match in period $t+1$ is

$$J_{t,t} - \beta(1-\delta)E_t J_{t+1,t+1} = z_t - \left[w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} E_t(w_{t,\tau} - w_{t+1,\tau}) \right],$$

where the equality follows from the definition of $J_{t,\tau}$.

Substituting free entry condition (7) into the left-hand side of the above equation yields

$$\frac{c}{q(\theta_t)} - \beta(1-\delta)E_t \frac{c}{q(\theta_{t+1})} = z_t - \left[w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} E_t(w_{t,\tau} - w_{t+1,\tau}) \right].$$

Using $UC_t^W \equiv w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(w_{t,\tau} - w_{t+1,\tau})$ and $UC_t^V \equiv \frac{c}{q(\theta_t)} - \beta(1-\delta)E_t\frac{c}{q(\theta_{t+1})}$, and rearranging yields

$$z_t = UC_t^W + UC_t^V.$$

■

Equation (8) is intuitive: firms create jobs in period t as long as the marginal benefit from adding a worker exceeds the user cost of labor. With free entry, the firms will enter the labor market until the net benefit is driven to 0. At that point, the decision to add a worker exactly equates the current benefit from a worker with the current cost and the present value of the expected future cost resulting from the current decision.[17]

[16] In the model with Nash bargaining in all matches each period, $J_{t,\tau} = \frac{c}{q(\theta_\tau)}$ holds for all $\tau \geq t$ because wage formation ensures $J_{t,\tau} = J_{\tau,\tau}$.

[17] The model assumes that newly formed and ongoing matches are homogeneous, i.e., they are equally productive in every period. As is standard in the literature, the benefits side of the firm's job creation

Equation (8) is crucial to understanding the concept of allocational price of labor. The equation shows that the free entry condition imposes the restriction on the volatility of the vacancy unemployment ratio, θ, and on the volatility of the wage component of the user cost of labor, UC^W, with respect to the volatility of productivity. The equation also shows that the free entry condition does not impose a direct restriction on individual wages. Conditional on the dynamics of the wage component of the user cost, the dynamics of individual wages do not have a direct impact on the dynamics of vacancies and unemployment.

To examine the quantitative restrictions imposed by the free entry condition, I rewrite equation (8) in terms of elasticities with respect to productivity. I, first, consider a steady state. Total differentiation of equation (8) and rearrangement yields

$$1 = \varepsilon_{UC^W,z}\left(1 - \frac{UC^V}{z}\right) + \varepsilon_{UC^V,z}\frac{UC^V}{z},$$

where $\varepsilon_{UC^W,z}$ and $\varepsilon_{UC^V,z}$ are the elasticities of the wage component and of the vacancy component, respectively, with respect to productivity. Rearranging yields

$$\frac{UC^V}{z} = \frac{1 - \varepsilon_{UC^W,z}}{\varepsilon_{UC^V,z} - \varepsilon_{UC^W,z}}. \tag{9}$$

Since in the steady state $UC^V > 0$, $UC^W > 0$ and $z = UC^V + UC^W$, it must be that $0 < \frac{UC^V}{z} < 1$. Using the steady state expression for UC^V, $UC^V = \frac{c}{K\theta^{-\alpha}}(1 - \beta(1 - \delta))$, yields $\varepsilon_{UC^V,z} = \alpha\varepsilon_{\theta,z}$, where $\varepsilon_{\theta,z}$ is the elasticity of the vacancy-unemployment ratio with respect to productivity. Then, combining equation (9), $0 < \frac{UC^V}{z} < 1$ and $\varepsilon_{UC^V,z} = \alpha\varepsilon_{\theta,z}$, the following must hold:

$$0 < \frac{1 - \varepsilon_{UC^W,z}}{\alpha\varepsilon_{\theta,z} - \varepsilon_{UC^W,z}} < 1. \tag{10}$$

Condition (10) holds if (1) either $\varepsilon_{UC^W,z} < 1 < \alpha\varepsilon_{\theta,z}$, or (2) $\alpha\varepsilon_{\theta,z} < 1 < \varepsilon_{UC^W,z}$. Given

decision in the textbook search and matching model, productivity, is calibrated to the average productivity in the data. Recently, the findings in Gertler and Trigari (2009) and Hagedorn and Manovskii (2010) suggest that the productivity of the newly created matches systematically differs over the business cycle. In such a case, the benefits side of the job creation condition should be modified accordingly to reflect these differences in productivity in new and ongoing matches. Such a modification is beyond the scope of the paper.

the value of the elasticity of the vacancy-unemployment ratio, $\varepsilon_{\theta,z}$, of 7.56[18] and a range of values for α that can be found in the literature, $[0.235, 0.72]$[19], one obtains $\alpha\varepsilon_{\theta,z} > 1$. Thus, for (10) to hold, one should have $\varepsilon_{UC^W,z} < 1$.

An analogous argument carries over to the stochastic case. Specifically, assume that $\ln z_t$ follows an $AR(1)$ process with autocorrelation coefficient ρ, $\rho < 1$, and normal innovations. Then, the elasticity of the vacancy-unemployment ratio with respect to productivity takes the form $\varepsilon_{UC_t^V,z_t} = \alpha\varepsilon_{\theta,z}x_t$,where $x_t > 1$ provided $\rho < \alpha\varepsilon_{\theta,z}$.[20]

Similarly as above, I obtain

$$0 < \frac{1 - \varepsilon_{UC^W,z}}{\alpha\varepsilon_{\theta,z}x_t - \varepsilon_{UC^W,z}} < 1. \tag{11}$$

Condition (11) holds if (1) either $\varepsilon_{UC^W,z} < 1 < \alpha\varepsilon_{\theta,z}x_t$, or (2) $\alpha\varepsilon_{\theta,z}x_t < 1 < \varepsilon_{UC^W,z}$. Since $x_t > 1$ and $\alpha\varepsilon_{\theta,z} > 1$, for (11) to hold, it should be the case that $\varepsilon_{UC^W,z} < 1$.

Equation (11) demonstrates the trade-off between the elasticity of the wage component of the user cost of labor and the elasticity of the vacancy-unemployment ratio imposed by the free entry condition of the model. If both the wage component of the user cost of labor and the vacancy-unemployment ratio covary positively with productivity, there is a trade-off in the magnitude of the response of UC^W and θ to changes in productivity.

The restrictions summarized by equation (11) allow bringing together the data on the unemployment-vacancy ratio and the statistics from wage data that are relevant for the firm's job creation decision. The equation allows examining whether the search and matching model can simultaneously generate empirical elasticities $\varepsilon_{\theta,z}$ and $\varepsilon_{UC^W,z}$. Since, as mentioned above, the conventional values for α and empirical estimates of $\varepsilon_{\theta,z}$ deliver the value of $\alpha\varepsilon_{\theta,z}$ that exceeds 1, the answer depends on the value for the elasticity of the wage component of the user cost of labor, $\varepsilon_{UC^W,z}$. Importantly, note that equation (11) is derived without evoking a particular surplus division rule or a wage formation.

[18] See, for example, Rudanko (2009) and Pissarides (2009).

[19] See, for example, Hall (2005) and Shimer (2005).

[20] Appendix B contains the derivation of the expression for $\varepsilon_{UC_t^V,z_t}$. It also shows that $\Pr(UC_t^V > 0) > 0.99$, $\Pr(UC_t^W > 0) > 0.99$, which implies $0 < \frac{UC_t^V}{z_t} < 1$.

D. *Implications of the Cyclicality of the Wage Component of the User Cost of Labor for the Search and Matching Model*

Since, by definition, the cyclicality is the semi-elasticity with respect to the unemployment rate, $\frac{d\ln UC^W}{du}$, I use the estimates of the cyclicality of UC^W obtained in Section 2 to calculate the elasticity of UC^W with respect to productivity, i.e.,

$$\varepsilon_{UC^W,z} \equiv \frac{d\ln UC^W}{d\ln z} = \frac{d\ln UC^W}{du}\frac{du}{d\ln z},$$

where $\frac{du}{d\ln z}$ is the change in the unemployment rate in response to a percentage change in productivity. Pissarides (2009) provides the following estimates of $\frac{du}{d\ln z}$: -0.34 for the period 1948-2006 and -0.49 for the period 1970-1993. Combining the estimate of the cyclicality of the wage component of the user cost of labor of -4.5% with these estimates yields the elasticity of the wage component with respect to productivity of 1.530 and 2.205, respectively.

For the free entry condition in the search and matching model to hold, provided that $\varepsilon_{\theta,z} = 7.56$, the elasticity of the wage component of the user cost of labor should be less than 1 (equation (11)). The results of the estimation show that $\varepsilon_{UC^W,z} > 1$. Thus, the restrictions imposed by the model's free entry condition on the data do not hold.

Since both $\alpha\varepsilon_{\theta,z}$ and $\varepsilon_{UC^W,z}$ exceed 1, the model cannot simultaneously generate the empirical elasticities of the vacancy-unemployment ratio and the wage component of the user cost of labor. This leads to the conclusion that if the model is to simultaneously match the volatility of quantities (vacancies and unemployment) and the relevant measure of prices (the wage component of the user cost of labor), then the solution for the unemployment volatility puzzle cannot be explained by a wage formation. This is so because, even if a particular wage formation delivers wage rigidity theoretically, any wage formation should be able to generate the elasticity of the wage component of the user cost of labor observed in the data. However, as the estimates show, the wage component of the user cost of labor is too volatile to amplify the fluctuations of the vacancy-unemployment ratio in the search and matching model.

Importantly, using the volatility of the wage component of the user cost of labor as opposed to the volatility of average wage or the wage of the newly hired workers helps to isolate the quantitative test of the search and matching framework from the issue of the wage formation in the model. Consider, for example, a search and matching model in which wages are bargained by Nash bargaining in all matches in every period. In the model with such a wage formation, the wage component of the user cost of labor equals average wages and wages of newly hired workers. When bringing the dynamics of wages from such a model to the data, a researcher faces three calibration targets: the dynamics of individual wages of all workers, the dynamics of wages of newly hired workers, and the dynamics of the wage component of the user cost of labor. The choice would not be crucial for the test if in the data the dynamics of the three statistics were the same. However, this is not the case. As the results in Section 2 reveal, the wage component of the user cost of labor is economically and statistically more procyclical than the wages of newly hired workers, and wages of newly hired workers are more procyclical than wages of all workers. Calibration of the dynamics of wages from such a model to the dynamics of individual wages in the data is the joint test of the wage formation and the search and matching framework and, thus, may lead to inaccurate conclusions about the quantitative performance of the search and matching framework versus the performance of the assumed wage formation.

E. Illustrative Example

To illustrate the findings, I consider the search and matching model as described in Section 3.A with two additional assumptions: (1) workers are risk neutral, and (2) at the time the match is formed, the surplus between a worker and a firm is divided by a generalized Nash bargaining with constant bargaining shares. Note that there are different wage formations that encompass this surplus division rule at the beginning of the match. One example of such a wage formation is Nash bargaining period by period in all matches. Another example is a constant wage within the employment relationship.

It can easily be shown that, given linear utility functions for a worker and a firm, models with different wage formations within the match but in which the surplus at the beginning of the match is divided using a constant shares Nash bargaining rule, deliver identically equal wage components of the user cost of labor. Consequently, following Proposition 1, such models deliver the same allocations, i.e., the vacancy-unemployment ratio. Thus, to demonstrate the implications of Proposition 1 for models with such a surplus division rule, it is sufficient to analyze a model with one of the wage formations with such a surplus division rule at the beginning of a match. A convenient model to analyze is the model in which wages are set by Nash bargaining in every period in all matches, which is widely used in the literature. Thus, in addition to the two assumptions above, I add the third one: every period the wage is set by Nash bargaining between a worker and a firm with a constant bargaining share of a worker equal η.

With Nash bargaining period by period, the wage depends only on the contemporaneous economic conditions, $w_{t_1,\tau} = w_{t_2,\tau} = w_\tau \ \forall t_1, t_2 \leq \tau$. Then, the last term in brackets in equation (3) is 0. It implies that with Nash bargaining period by period, the wage component of the user cost of labor equals wage, i.e., $UC_\tau^W = w_\tau$ for all τ. This conveniently allows deriving the closed-form expression for UC_τ^W in the model, i.e., I, first, derive w_τ and, then, set $UC_\tau^W = w_\tau$.

Two parameters are crucial for the volatilities of the vacancy-unemployment ratio and wages in the model: the unemployment benefit, b, and a worker's bargaining power, η (see, for example, a detailed discussion in Hornstein, Krusell and Violante (2005)). I, first, derive the expression for b/z as a function of η and the elasticity of the vacancy-unemployment ratio with respect to productivity[21], $\varepsilon_{\theta,z}$, i.e.,

$$\frac{b}{z} = (1 - \frac{1}{\varepsilon_{\theta,z}} \frac{1 - \beta(1 - \delta - \eta\mu)}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)}), \tag{12}$$

where μ is the steady state value of the job finding rate.

I, then, derive the expression for b/z as a function of η and $\varepsilon_{w,z}$, i.e.,

$$\begin{aligned} \frac{b}{z} &= \frac{\eta}{1-\eta} \frac{1}{\varepsilon_{w,z}} \frac{1}{1-\beta(1-\delta)} ((1 - \beta(1 - \delta - \eta\mu))(1 - \varepsilon_{w,z}) + \\ &\quad \frac{(1-\eta)(1-\alpha)\beta\mu(1-\beta(1-\delta))}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)}) \text{ if } \frac{b}{z} \neq 0, \end{aligned} \tag{13}$$

Now I can plot the two functions: $\frac{b}{z}(\eta|\varepsilon_{\theta,z})$ and $\frac{b}{z}(\eta|\varepsilon_{w,z})$ given values for $\varepsilon_{\theta z}$ and ε_{wz} and a set of parameters $(\alpha, \beta, \delta, \mu)$. The intersection of the two functions, if one exists, gives

[21] See Appendix B for the expressions for $\varepsilon_{\theta,z}$ and $\varepsilon_{w,z}$.

a pair of $(\frac{b}{z}, \eta)$ that deliver targeted values of $\varepsilon_{\theta,z}$ and $\varepsilon_{w,z}$.

I obtain the following parametrization for the quarterly model: $\beta = 1/1.012; \delta = 0.10; \mu = 1.35$ (Hornstein, Krusell and Violante (2005), Shimer (2005)). Since literature provides a range of values for the elasticity of the matching function with respect to unemployment, α, I present results for three different values of α: $\alpha = 0.235$ (Hall 2005), $\alpha = 0.72$ (Shimer 2005), and $\alpha = 0.5$ (the value in the range proposed by Petrongolo and Pissarides (2000)). I set $\varepsilon_{\theta,z} = 7.56$ (Rudanko (2009), Pissarides (2009)).

It remains to specify the value of $\varepsilon_{w,z}$. As shown above, in the model with wage bargaining period by period, wages are equal across all matches in each period. Thus, the average wage equals wages of newly hired workers and equals the wage component of the user cost of labor. However, in the data those three statistics from wages are different. In particular, Rudanko (2009) and Hagedorn and Manovskii (2008) summarize the elasticity of averages wages with respect to productivity at 0.5 and 0.47, respectively. Pissarides (2009), based on the cyclicality of -3%, summarizes the implied elasticity of wages of newly hired workers between 1.02 and 1.47, depending on the value of $\frac{du}{d\ln z}$. In this paper, I find the elasticity of the wage component of the user cost of labor to be between 1.53 and 2.20, based on the cyclicality of -4.5%.

Thus, it matters how the empirical counterpart for $\varepsilon_{w,z}$ is chosen in the data. Since in the model with Nash bargaining period by period all three responses above are the same, this wage formation cannot be used to describe the behavior of individual wages in the data. Instead, the empirical counterpart of $\varepsilon_{w,z}$ in this model is the elasticity of the wage component of the user cost of labor.

Figure 1 plots $\frac{b}{z}(\eta|\varepsilon_{\theta,z} = 7.56)$ and $\frac{b}{z}(\eta|\varepsilon_{w,z} = 1.5)$. The figure illustrates two points. First, two functions $\frac{b}{z}(\eta|\varepsilon_{\theta,z} = 7.56)$ and $\frac{b}{z}(\eta|\varepsilon_{w,z} = 1.5)$ do not have points in common. Thus, the model cannot generate both the elasticity of the vacancy-unemployment ratio of 7.56 and the elasticity of the wage component of the user cost of labor in excess of 1. Second, the model can generate the elasticity of the wage component of the user cost of labor equal to 1.5 for only a small set of $\frac{b}{z}$ and η values. In particular, the case of $\alpha = 0.72$ is not plotted because there are no admissible values of the pair $(\frac{b}{z}, \eta)$ that can deliver $\varepsilon_{w,z} = 1.5$, given the values for (β, δ, μ) and $\alpha = 0.72$.

Figure 1 also demonstrates that (1) the elasticity of the vacancy-unemployment ratio is very sensitive to the value of $\frac{b}{z}$ and less sensitive to η, and (2) the empirical value of $\varepsilon_{\theta,z}$ requires a high value of $\frac{b}{z}$, which is the conclusion reached in Hornstein, Krusell and Violante (2005) and Hagedorn and Manovskii (2008).

The conclusion that the textbook search and matching model cannot generate empirical volatility of the vacancy-unemployment ratio contrasts with the conclusion of Hagedorn and Manovskii (2008) who argue that at the high values of the unemployment benefits the model can generate the empirical volatility of the vacancy-unemployment ratio. Hagedorn and Manovskii consider the same model as in this illustrative example, i.e., the textbook search and matching model with Nash bargaining of wages in each period in all matches. They calibrate $\varepsilon_{w,z}$ in the model to the elasticity of averages wages in the data, 0.5. Figure 2 shows such calibration, i.e., it shows $\frac{b}{z}(\eta|\varepsilon_{w,z} = 0.5)$. As in Hagedorn and Manovskii (2008), Figure 2 demonstrates that there exists a pair of $(\frac{b}{z}, \eta)$ that can simultaneously deliver $\varepsilon_{\theta,z} = 7.56$ and $\varepsilon_{w,z} = 0.5$. However, the calibration of $\varepsilon_{w,z}$ to 0.5 assumes a particular wage formation, Nash bargaining in every period in all matches. In the model with the wage formation, the average wage equals the wage component of the user cost of labor by construction, while in the data average wage is much less volatile than the wage component of the user cost of labor. This result illustrates that focusing on the cyclicality of individual wages might lead to a misleading assessment of the quantitative behavior of the model if the wage formation (which is not a central feature of the model) is specified incorrectly.

IV. Example: Cyclicality of the User Cost of Labor in Models with Specific Wage Formations

In this section, I illustrate the conclusions reached above by examining the quantitative behavior of the wage component of the user cost of labor and wages in a few search and matching models with different wage formations commonly used in the literature.

A. Description of the Models

The environment is described in Section 3.A. In addition, assume that workers are risk averse, i.e., $u'(c) > 0$, $u''(c) < 0$. Consider four different wage formations in this environment. The first three models are the models with implicit insurance contracts as in Thomas and Worrall (1988).[22] In these models, the wage component of the user cost of labor in (3) has a non-zero second term. For comparison, the fourth model is a model with Nash bargaining every period in all matches.

In the first three models, wages are the outcome of the implicit self-enforcing contracts between a worker and firm. In the models, risk-neutral firms insure risk-averse workers, who do not have access to capital markets, against fluctuations in consumption due to fluctuations in earnings. Three types of contracts are distinguished based on different degrees of commitment: full commitment contracts, contracts with lack of commitment from the worker's side and full commitment from the firm's side, and contracts with lack of commitment from both the worker's and firm's sides. If there is a lack of commitment from any side of the contract, the contract should be self-enforcing for that side to prevent reneging. In the original Thomas and Worrall (1988) environment, workers who renege on the contract are prohibited from entering any contractual arrangements in the future and are bound to trade their labor services at the spot market wage. In the current environment, workers who renege on the contract become unemployed. Once unemployed, workers search and enter contractual arrangements as soon as they find a new match. Both firms and workers face search and matching frictions. These frictions influence the value of the outside option through the probability of finding a new match.

Firms open vacancies with associated employment contracts and workers direct their search to the contracts. The vacancies opened with the associated contract σ and the unemployed workers searching for contract σ constitute a labor market with a market tightness θ_σ. A contract is a state-contingent sequence of wages that delivers a promised value to the worker. Equilibrium contracts are limited to efficient optimal contracts. To ensure a unique contract in equilibrium, I follow Rudanko (2009) and impose the following equilibrium re-

[22]See Rudanko (2009) for an excellent treatment of Thomas and Worrall (1988) contracts in the search and matching model.

finement: there does not exist an efficient self-enforcing contract $\sigma'(z)$ and an associated labor market with tightness $\theta_{\sigma'}(z)$ such that the net surpluses from search for a worker and a firm are at least as much as under $\sigma(z)$ and $\theta_\sigma(z)$ and, for one party, it is strictly more.

Thomas and Worrall (1988) and Rudanko (2009) show that in such an environment, for any history (z^t, z_{t+1}), there exists a $w_{\min}(z_{t+1})$ and $w_{\max}(z_{t+1})$, $w_{\min}(z_{t+1}) \leq w_{\max}(z_{t+1})$, such that the contract wage at $t+1$ is:

1) in the contract with full commitment: $w(z^t, z_{t+1}) = w(z^t)$;

2) in the contract with lack of commitment from the worker and full commitment from the firm:

$$w(z^t, z_{t+1}) = \begin{cases} w(z^t) & \text{if } w_{\min}(z_{t+1}) \leq w(z^t) \\ w_{\min}(z_{t+1}) & \text{if } w(z^t) < w_{\min}(z_{t+1}) \end{cases};$$

3) in the contract with two-sided lack of commitment:

$$w(z^t, z_{t+1}) = \begin{cases} w_{\max}(z_{t+1}) & \text{if } w(z^t) > w_{\max}(z_{t+1}) \\ w(z^t) & \text{if } w_{\min}(z_{t+1}) \leq w(z^t) \leq w_{\max}(z_{t+1}) \\ w_{\min}(z_{t+1}) & \text{if } w(z^t) < w_{\min}(z_{t+1}) \end{cases}.$$

Thus, in the optimal contract with commitment, the wage remains constant. In the contracts with lack of commitment, the wage remains constant until the value of the outside option for the party without commitment exceeds the value under the contract, in which case the wage is adjusted to prevent reneging.

In the fourth model, wages are determined period by period in new and existing matches by the following rule: $\frac{W(z)-U(z)}{u'(w(z))}/J(z) = \frac{\eta}{1-\eta} \forall z \in Z$, where $W(z)$, $U(z)$ and $J(z)$ are values for an employed worker, an unemployed worker, and a firm with filled vacancy, respectively, and η is a worker's bargaining power. This rule is well known in the literature: the share of the surplus that the agent obtains from a match corresponds to his bargaining power. If workers are risk neutral, then it describes generalized Nash bargaining period by period over total surplus as in the canonical search and matching model (Pissarides (2000)). In the appendix, I specify the firm's optimization problem and define the equilibrium in the models described above.[23]

[23] See Appendix C for more details.

B. The Dynamics of the User Cost and Wages in the Models

The parameters of the stochastic process for productivity shocks can be calibrated outside of the models.[24] The only parameter that requires calibration within a model is the cost of posting a vacancy, c, which I calibrate to match the mean monthly job-finding rate, $E(\mu) = 0.45$. The model period is one month. The discount factor is 0.9960, which corresponds to the annual discount rate of 4.88%. The monthly separation rate is set to 0.034 (Shimer 2005). I set the bargaining power of workers to equal α to preserve the mathematical equivalence of the competitive search and random search equilibria (Rudanko (2009)). The parameters are summarized in Table 4.

I obtain corresponding statistics for the models by simulating economies with each of the four different wage formations as follows. First, a common vector of aggregate shocks, z, is generated. For the panel of $10,000$ individuals, an initial employment status is drawn. Then, each period, the separation shock is drawn for each employed individual and his employment status is updated, and for each unemployed individual the job finding shock is drawn and his unemployment status is updated. Given the employment histories, individual wages are generated according to a model-specific wage formation. The first $4,000$ periods of the simulated series are discarded; the statistics are based on the series from the last 636 periods. The cyclicality of series x is measured as $cov(\ln(x), u)/var(u) * 100$, which is the semi-elasticity of the series with respect to unemployment.

Table 5 reports the cyclicality of the individual wages of all workers, the cyclicality of the wages of newly hired workers, the cyclicality of the wage component of the user cost of labor and the cyclicality of the vacancy component of the user cost of labor in the model with the workers' logarithmic utility function. As can be seen from the table, the cyclicality of individual wages varies across the four economies. Wages are only mildly procyclical in

[24] To calibrate a stochastic process for productivity, I consider a three-state symmetric Markov process as in Rudanko (2009), $\mathbf{z} = [z_0 - \Delta, z_0, z_0 + \Delta]$, $\Delta > 0$, with the transition matrix (by row): $[\lambda, 1-\lambda, 0; 0.5(1-\lambda), \lambda, 0.5(1-\lambda); 0, 1-\lambda, \lambda]$. The variance of this process, σ_z^2, is $\frac{\Delta^2}{2}$ and the autocorrelation, ρ, is λ. $E(z)$ is normalized to 1. Δ and λ are calibrated to match the standard deviation, 0.02, and autocorrelation, 0.878, of productivity per worker, obtained from Shimer (2005), Table 1. To find Δ and λ, I draw the initial shock from a stationary distribution of z and, using the initial values for Δ and λ, generate monthly series of length $12T$, where T is the length of the time series in the data in years (from 1951 to 2003); aggregate by summing to obtain quarterly data; calculate the standard deviation and the autocorrelation of the logged quarterly series; and iterate until matching the targets.

the implicit contract models and are as cyclical as the wage component of the user cost in Nash bargaining model. Importantly, in the models with contracts, the wage component of the user cost of labor is much more procyclical than the wages of newly hired workers, and the wages of newly hired workers are approximately 3 times as cyclical as the wages of all workers.[25]

In the model with implicit contracts and full two-sided commitment, wages within employment relationships are rigid by construction. Thus, the cyclicality of the average wage is due to new hires entering employment relationships, constant separation rates and a positive autocorrelation in the productivity process. In the model with full commitment on the firm's side and lack of commitment on the worker's side, in addition to the composition effect, the wages in the existing employment relationships are bid up whenever the worker's outside option value becomes more attractive than the value from the contract. In the model with lack of commitment on both the firm's and the worker's sides, the wages can also be bid down whenever the value from the match for a firm falls below 0.

To understand why the cyclicality of the wage component of the user cost in the contract models is higher than the cyclicality of wages at the time of hiring, recall the workings of this wage formation. The implicit contracts offer wages that are rigid during the employment relationship to insure workers against fluctuations in consumption. The wages of new hires adjust to reflect the worker's outside option value. Consequently, the wages of newly hired workers are more cyclical than the wages of all workers. For example, when the job finding rate is low, the hiring wage is relatively low. In addition, the wages in all subsequent periods in the employment relationship are relatively lower than the wages in the contracts, initiated under more favorable economic conditions. The wage component of the user cost takes into account both the lower hiring wage and lower future wage payments. Hence, it is more procyclical than the wages of newly hired workers.

Note from Table 5 that for $b = 0.70$, the implicit contract model generates a standard deviation of the vacancy-unemployment ratio of approximately 0.0620, while the empirical counterpart is 0.382 (see Shimer (2005)).

[25] As is typical in such calibrations, the cyclicality of individual wages in the models with contracts is sensitive to the number of the states of the discrete stochastic process used to approximate the productivity process in the data. However, it does not have an impact on the main results.

C. The Allocational Role of the Wage Component of the User Cost of Labor

Next, I use the value of unemployment benefits, b, to calibrate the models to match the cyclicality of the wage component of the user cost estimated in Section 2, i.e., to match $cov(\ln(UC^W), u)/var(u) = -0.045$. Then, I examine how much volatility of the vacancy-unemployment ratio the calibrated models generate. Table 6 shows the results for all four wage formations. As can be seen from the table, all four models generate less than half of the empirical volatility of the vacancy-unemployment ratio, 0.382. This so because the strong procyclicality of the wage component of the user cost dampens the response of the job creation to changes in productivity. Alternatively, Table 7 shows the results when the models are calibrated to match the empirical volatility of the vacancy-unemployment ratio. As can be seen from the table, the wage component of the user cost is too rigid as compared to its empirical counterpart. Tables 6 and 7 contain the results for different functional forms of the worker's utility function: a linear utility, the logarithmic utility and the utility function with the CRRA parameter equal 3; all the results carry through.[26]

As the results in Table 6 show, once the cyclicality of the wage component of the user cost is calibrated across different models to its empirical counterpart, the economies that are hit by the same sequence of productivity shocks generate very similar dynamics of vacancies and unemployment, regardless of the wage formation. In fact, in the case when both firms and workers are risk neutral, in each period the economies are observationally equivalent in terms of allocations.[27] Simultaneously, however, the dynamics of the individual wages and of the wages of newly hired workers across economies differ substantially and are determined by a specific wage formation.

The results demonstrate that, when wages depend on the history from the start of the job, individual wages or wages of newly hired workers are not allocational for employment. With wage smoothing, the dynamics of wages are not directly related to the dynamics of the wage component of the user cost. In this case, a weak procyclicality of hiring wages can

[26]Note that the implicit contracts do not have a micro-foundation when workers are risk-neutral.

[27]In the case of the logarithmic utility or the CRRA utility, when all four economies are calibrated to the same cyclicality of the wage component of the user cost, there is a slight difference in the volatilities of the vacancy-unemployment ratio across the four wage formations. This difference is due to the curvature of the worker's utility function.

conceal a substantial procyclicality of the wage component of the user cost.

The magnitudes of the cyclicality of individual wages in Table 6 provide some insight into the relevance of implicit contracts for modeling individual wage dynamics. In particular, as the estimates in Section 2 show (also see Pissarides (2009)), the empirical studies report the cyclicality with respect to unemployment in the range -1 to -1.5% for wages of all workers and -3% for wages of newly hired workers. In Table 6, the model with implicit contracts and two-sided lack of commitment generates individual wage dynamics with the cyclicality comparable to their empirical counterparts. Note that in this table I do not calibrate the cyclicality of the individual wages (wages of newly hired workers or average wages) but only the cyclicality of the wage component of the user cost of labor.

V. Conclusion

This paper contributes to the broad and long-standing debate in macroeconomics on the allocational price of labor and its cyclicality (Barro (1977); Hall (1980); Kydland and Prescott (1980)). The paper shows that it is the user cost of labor, which captures the hiring wage and the effect of the economic conditions at the time of hiring on future wages, that is weighed against the marginal revenue product of a worker at the time of hiring.

The main contribution of the paper is to provide an estimate of the cyclicality of the wage component of the user cost of labor. In contrast to the existing literature on the cyclicality of the firm's labor cost, which focuses on the cyclicality of current wage, the paper takes into account the cyclical behavior of the expected present discounted value of future wages to be paid within a match formed in the current period. Importantly, I find that the wage component of the user cost of labor is more procyclical than average wages or even wages of newly hired workers.

I estimate the cyclicality of the wage component of the user cost from the NLSY data. Because it is not directly observed in the data, I construct the wage component of the user cost based on the behavior of individual wages and turnover. I find that a one percentage point increase in unemployment generates more than 4.5% decrease in the wage component of the user cost.

Using the estimate, I analyze the quantitative behavior of the textbook search and matching model. In a model with search and matching, the user cost of labor can be decomposed into two components: the vacancy component and the wage component. With free entry of firms, the marginal productivity of a worker equals the user cost of labor, the sum of the vacancy component and the wage component. This condition allows for testing the quantitative behavior of the search and matching model. The test examines the model's ability to jointly replicate the elasticity of the vacancy-unemployment ratio and the elasticity of the wage component of the user cost of labor observed in the data in response to productivity shocks. To perform the test requires an estimate of the cyclicality of the wage component of the user cost of labor. This paper provides such an estimate. The cyclicality of the wage component of the user cost of labor translates into elasticity with respect to productivity of above 1.5. Using the free entry condition, I show that the search and matching model cannot simultaneously generate empirical elasticities of the vacancy-unemployment ratio and the wage component of the user cost of labor. The conclusion does not depend on a surplus division rule at the beginning of the match or individual wage dynamics within employment relationships.

Consequently, the results show the solution to the unemployment volatility puzzle in the textbook search and matching model cannot be explained by a wage formation alone. Furthermore, the model's free entry condition imposes a trade-off between the elasticity of the wage component of the user cost of labor and the elasticity of the vacancy-unemployment ratio; however, in the data these elasticities are higher than the trade-off in the model allows.

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Figure 1. Pairs of $(\frac{b}{z}, \eta)$ that generate $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 1.5$

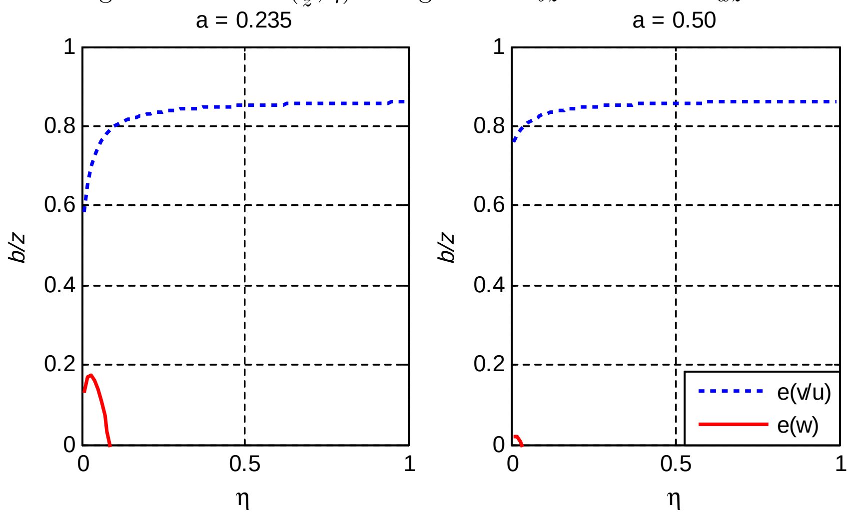

Figure 2. Pairs of $(\frac{b}{z}, \eta)$ that generate $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 0.5$

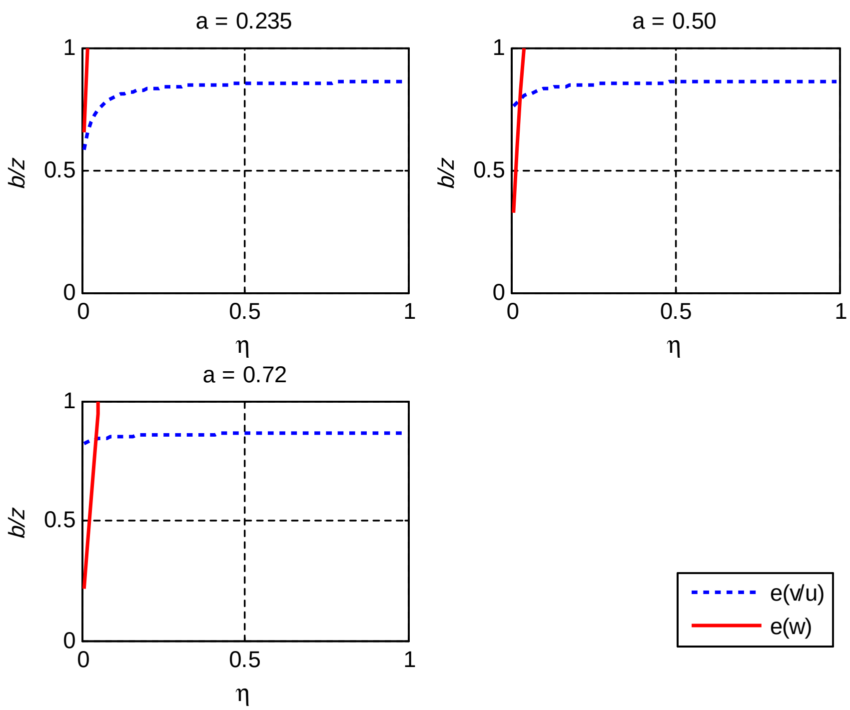

Table 1: Cyclicality of the Wage Component of the User Cost of Labor

	Semi-Elasticity with respect to Unemployment	
	Estimate	95% Confidence Interval
Wage component of the user cost of labor, UC_t^W	-5.20***	-6.69...-3.71
	(0.76)	
Wage of new hires	-3.00***	-4.61...-1.40
	(0.78)	
Average wage	-1.78**	-3.26...-0.30
	(0.72)	

Note – The semi-elasticity is the coefficient on the unemployment rate from the regression of the (natural logarithm of the) respective series on the contemporaneous unemployment rate and other controls. The estimates for the average wage and for the wage of newly hired workers are from the regressions in Columns 1 and 2 of Table 2, respectively. The estimates for the wage component of the user cost of labor are from the regression of the (natural logarithm of the) wage component of the user cost of labor on the unemployment rate and a time trend (annual). There are 20 observations in the regression of the wage component of the user cost - from 1978 to 1997. The bootsrapped standard errors are in parentheses (1000 replications); p-values: ***p<0.01, ** p<0.05, * p<0.1. All coefficients and standard errors are multiplied by 100.

Table 2: THE CYCLICALITY OF INDIVIDUAL WAGES

			With industry controls	
	Full sample	New hires only	Full sample	New hires only
	(1)	(2)	(3)	(4)
$u_{current}$	-1.78**	-3.00***	-2.02**	-2.99***
	(0.72)	(0.78)	(0.93)	(0.92)
u_{start}	x	x	x	x
$u_{\min}$	x	x	x	x
Grade	7.98***	12.52***	7.42***	11.67***
	(1.52)	(1.65)	(1.55)	(1.98)
Experience	4.22**	8.28***	3.71*	7.77***
	(1.66)	(1.75)	(1.84)	(2.10)
Experience^2	-0.13***	-0.14***	-0.14***	-0.15***
	(0.02)	(0.02)	(0.02)	(0.02)
Tenure	3.55***	4.02	3.71***	7.57
	(0.23)	(4.60)	(0.29)	(4.95)
Tenure^2	-0.11***	3.29	-0.13***	-0.29
	(0.02)	(4.30)	(0.02)	(4.64)
Trend	1.03	-3.52**	1.55	-2.95
	(1.74)	(1.70)	(1.86)	(2.04)
Union dummy	0.19***	0.17***	0.16***	0.15***
	(0.01)	(0.02)	(0.01)	(0.02)
Industry dummies	x	x	yes	yes
Observations	52593	19406	46753	16963
R-squared	0.529	0.472	0.558	0.507

Note – The data are from NLSY79, men only, 1978 - 2004. The sample of new hires is restricted to observations with tenure less than 1 year. The dependent variable is the natural logarithm of real hourly wage. All regressions are estimated with fixed effects using sampling weights. Unemployment rate is the annual unemployment rate. Columns 3 and 4 include controls for 14 industries and are estimated on 1978 - 2002. The estimated standard errors are in parentheses, clustered by time. The coefficients and standard errors are multiplied by 100. P-values: ***p<0.01, ** p<0.05, * p<0.1.

Table 3: ROBUSTNESS RESULTS ON THE CYCLICALITY OF THE WAGE COMPONENT OF THE USER COST OF LABOR

	# of years in calculating UC_t^W		
	5 years	7 years	9 years
UC_t^W, $\delta_t = const$	-5.03	-5.24	-5.33
	(0.77)	(0.81)	(0.83)
UC_t^W, δ_t	-5.02	-5.19	-5.27
	(0.80)	(0.76)	(0.81)
UC_t^W, $\delta_{t_0,t}$	-4.79	-4.91	-4.89
	(0.16)	(0.59)	(0.70)

Note - The estimates are from the regression of the natural logarithm of the wage component of the user cost of labor on the unemployment rate and a time trend (annual). There are 18 observations in each regression - from 1978 to 1995. The bootstrapped standard errors are in parentheses (1000 replications). All coefficients and standard errors are multiplied by 100. The three rows reflect different ways of treating the separation rates in the construction of the wage component of the user cost of labor: 1) constant separation rate, $\delta_t = const$; 2) separation rate that depends on the current period, δ_t; and 3) separation rate that depends both on the current period and the period when the job started, $\delta_{t_0,t}$.

Table 4: PARAMETERS

Parameter	Value	Comment
Discount rate, β	0.9960	
Separation rate, δ	0.0340	Shimer (2005)
Matching function elasticity ($Ku^\alpha v^{1-\alpha}$), α	0.5 - 0.7	Petrongolo and Pissarides (2001)
Matching function constant ($Ku^\alpha v^{1-\alpha}$), K	0.5	Normalization
Worker's bargaining power, η	α	Hosios (1990), Rudanko (2009)

Table 5: Cyclicality of the User Cost of Labor and Its Components

		Log utility, $\alpha = 0.60$, $b = 0.70$			
		Commitment Models			Re-
		Full	1-sided lack of	2-sided lack of	bargain
		Cyclicality			
1	Individual wages (all)	-1.47	-1.47	-1.73	-9.47
2	Individual wages (new hires only)	-4.77	-4.77	-4.99	-9.41
3	Wage component of user cost	-11.15	-11.15	-11.07	-9.47
4	Vacancy component of user cost	-55.06	-55.06	-54.96	-55.14
5	User cost of labor	-11.89	-11.89	-11.82	-10.24
		θ statistics			
6	$\sigma_{\ln(\theta)}$, quarterly	0.0622	0.0622	0.0611	0.0704
		Calibrated parameter			
7	Vacancy creation cost, c	0.2675	0.2675	0.2676	0.2674

Note - Results are from simulating the models with risk averse workers (log utility). The vacancy creation cost, c, is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component for the models are -11.15, -11.15, -11.08, and -9.4723, respectively.

Table 6: THE VOLATILITY OF THE V-U RATIO GIVEN THE CALIBRATED CYCLICALITY OF THE WAGE COMPONENT OF THE USER COST OF LABOR

	Linear utility				Logarithmic utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
Cyclicality												
Wages (all)	-0.58	-0.90	-1.65	-4.50	-0.58	-0.86	-1.56	-4.50	-0.58	-0.80	-1.36	-4.50
Wages (new hires only)	-1.94	-2.25	-2.85	-4.50	-1.94	-2.24	-2.77	-4.50	-1.94	-2.21	-2.59	-4.50
Wage component of UC	-4.50				-4.50				-4.50			
Vacancy component of UC	-57.26				-57.20	-57.29	-57.25	-57.02	-57.06	-57.32	-57.25	-56.53
User cost of labor	-4.81				-4.82	-4.83	-4.83	-4.85	-4.87	-4.88	-4.89	-4.99
θ statistics												
$\sigma_{\ln(\theta)}$, quarterly	0.1480				0.1475	0.1473	0.1472	0.1468	0.1463	0.1458	0.1453	0.1431
Calibrated parameters												
Consumption of un-ed, b	0.8702				0.8710	0.8698	0.8681	0.8616	0.8715	0.8683	0.8644	0.8381
Vacancy creation cost, c	0.0984				0.1041	0.1051	0.1069	0.1123	0.1176	0.1211	0.1262	0.1565

Note - Results from simulating the models with risk averse workers. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and the cyclicality of the wage component of the user cost. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component for the models are equal to the ones reported in the table (to the decimal points reported).

Table 7: THE CYCLICALITY OF THE USER COST AND ITS COMPONENTS GIVEN THE CALIBRATED VOLATILITY OF THE V-U RATIO

	Linear utility				Logarithmic utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
Cyclicality												
Wages (all)	-0.21	-0.51	-1.09	-1.62	-0.21	-0.51	-1.06	-1.62	-0.21	-0.50	-1.01	-1.62
Wages (new hires only)	-0.70	-1.05	-1.39	-1.62	-0.70	-1.04	-1.38	-1.62	-0.70	-1.03	-1.35	-1.62
Wage component of UC	-1.62				-1.62	-1.62	-1.62	-1.62	-1.62	-1.62	-1.62	-1.62
Vacancy component of UC	-76.89				-76.76	-76.80	-76.57	-76.24	-76.47	-76.60	-75.98	-74.98
User cost of labor	-1.75				-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.76
Calibration targets												
$\sigma_{\ln(\theta)}$, quarterly						0.3820						
$E(\mu(\theta))$						0.4500						
Calibrated parameters												
Consumption of un-ed, b	0.9434				0.9438	0.9432	0.9428	0.9424	0.9444	0.9427	0.9417	0.9402
Vacancy creation cost, c	0.0415				0.0424	0.0427	0.0432	0.0435	0.0443	0.0454	0.0471	0.0481

Note - Results from simulating the models with workers with different utility functions as indicated. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and $\sigma_{\ln(\theta)}$= 0.0382. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component for the four models and different utility functions are equal with respect to the precision in the tables, except for the log utility, where the statistics are -1.63 for all four models.

A Appendix A

A. Existence of UC^W

Lemma 1. $E_t|w_{t,t}+\sum_{\tau=t+1}^{\infty}\left(\beta^{\tau-t}\prod_{d=t}^{\tau-1}(1-\delta_d)\right)(w_{t,\tau}-w_{t+1,\tau})|<\infty$.

Proof.

Notice that $0\leq 1-\delta_t\leq 1\ \forall t$. Let $1-\delta\equiv\sup_t(1-\delta_t)$. Then $\beta^{\tau}\prod_{k=0}^{\tau-1}(1-\delta_{t+k})\leq(\beta(1-\delta))^{\tau}$.

Suppose that $\exists\upsilon<\beta(1-\delta)$ and $\overline{w}<\infty$ s.t. $\forall\tau\geq t\ w_{t,\tau}\leq\overline{w}\upsilon^{\tau-t}$. In other words, wages do not grow faster than $\beta(1-\delta)$. Then, given $w_{t,\tau}\geq 0$,

$$\left|w_{t,t}+\sum_{\tau=t+1}^{\infty}\left(\beta^{\tau-t}\prod_{d=t}^{\tau-1}(1-\delta_d)\right)(w_{t,\tau}-w_{t+1,\tau})\right|\leq$$

$$\left|\sum_{\tau=t+1}^{\infty}\left(\beta^{\tau-t}\prod_{d=t}^{\tau-1}(1-\delta_d)\right)(w_{t,\tau}-w_{t+1,\tau})\right|\leq$$

$$\sum_{\tau=t+1}^{\infty}\left(\beta^{\tau-t}\prod_{d=t}^{\tau-1}(1-\delta_d)\right)|w_{t,\tau}-w_{t+1,\tau}|\leq$$

$$\sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}\max\{w_{t,\tau},w_{t+1,\tau}\}.$$

Then

$$\begin{aligned}E_t\left|w_{t,t}+\sum_{\tau=t+1}^{\infty}\left(\beta^{\tau-t}\prod_{d=t}^{\tau-1}(1-\delta_d)\right)(w_{t,\tau}-w_{t+1,\tau})\right| &\leq\\ E\left(\sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}\,\overline{w}\upsilon^{\tau-t}\right) &=\\ \frac{\overline{w}}{1-\beta(1-\delta)\upsilon} &< \infty.\blacksquare\end{aligned}$$

B. The Wage Component of the User Cost of Labor with Time-Varying Separation Rates

The expected present discounted value of wages paid in a position, created in t, from t onwards is given by

$$\begin{aligned}PDV_t' = w_{t,t}+E_t[\beta((1-\delta_{t,t})w_{t,t+1}+\delta_{t,t}w_{t+1,t+1})+\\ \beta^2((1-\delta_{t,t})(1-\delta_{t,t+1})w_{t,t+2}+\delta_{t,t}(1-\delta_{t+1,t+1})w_{t+1,t+2}+\\ ((1-\delta_{t,t})\delta_{t,t+1}+\delta_{t,t}(1-\delta_{t+1,t+1}))w_{t+2,t+2}+...]=\\ w_{t,t}+E_t[\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\sum_{k=t}^{\tau-1}(\Lambda_{t,k,\tau-1}w_{k+1,\tau})],\end{aligned}\tag{A1}$$

where w_{t_1,t_2} is the wage paid in t_2 to a worker hired in t_1; δ_{t_1,t_2} is the separation rate at the end of t_2 for a worker hired in t_1, conditional that there is no separation between $t_1$and t_2; and $\Lambda_{t,k,\tau}$ is the probability that a separation takes place at the end of period k at the position that a firm opened in t and a new worker is hired in $k+1$ and continues working on that position in τ; and $E_t=E(.|I_t)$ where I_t is the firm's information set at time t. Both wage payments and separation rates are allowed to depend on the history of the labor market conditions from the period a worker is hired.

Equation (A1) states that a worker hired in period t is paid a wage $w_{t,t}$. With probability $1-\delta_{t,t}$ the firm-worker relationship survives until the period $t+1$ and the worker is paid wage $w_{t,t+1}$. With probability

$\delta_{t,t}$ the relationship is terminated and the firm hires a new worker at a wage $w_{t+1,t+1}$ to fill the position. By analogy, in period $t+2$ a firm retains a worker hired in period t with probability $(1-\delta_{t,t})(1-\delta_{t,t+1})$ and pays a wage $w_{t,t+2}$. With probability $(1-\delta_{t,t})\delta_{t,t+1}$ that worker is separated and the firm replaces the worker with another at wage $w_{t+2,t+2}$. Also, in period $t+2$ a worker hired in $t+1$ is retained with probability $\delta_{t,t}(1-\delta_{t+1,t+1})$ and receives wage $w_{t+1,t+2}$. In case of separation, with probability $\delta_{t,t}\delta_{t,t+1}$ this worker is replaced with a new one at wage $w_{t+2,t+2}$.

The wage component of the user cost of labor in period t is the difference between the expected present discounted value of wages paid at the position opened in period t and $t+1$:

$$UC_t^W = PDV_t' - \beta E_t PDV_{t+1}'.$$

Substituting from (A1), I obtain the following expression for the wage component of the user cost of labor:

$$UC_t^W = w_{t,t} + E_t[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(w_{t,\tau}\prod_{k=t}^{\tau-1}(1-\delta_{t,k}) - w_{t+1,\tau}(1-\delta_{t,t})\prod_{k=t+1}^{\tau-1}(1-\delta_{t+1,k}))+$$

$$\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}(\sum_{k=t}^{\tau-1}(\Lambda_{t,k,\tau-1} - (1-\lambda_{t,t})\Lambda_{t+1,k,\tau-1})w_{k,\tau})]. \tag{A2}$$

If separation depends only on the contemporaneous labor market conditions, $\delta_{t_0,t} = \delta_t$ for all t and t_0, then (A2) simplifies to the following expression:

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(\prod_{k=t}^{\tau-1}(1-\delta_k))(w_{t,\tau} - w_{t+1,\tau}). \tag{A3}$$

If the separation rate is constant, $\delta_{t_0,t} = \delta$, equation (A2) simplifies to

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(w_{t,\tau} - w_{t+1,\tau}).$$

B Appendix B

A. Value Functions

The values in the economy that is described in Section 3 can be summarized by the following functions. Let Ω_t denote a vector of state variables at time t, including aggregate productivity z_t, and let $\Omega^t \equiv \{\Omega_\tau\}_{\tau=0}^t$. To save on notation, I suppress the dependence of the value functions on corresponding histories. In the economy, wage may depend on the history of the labor market conditions from the start of the job. Thus, the wage is indexed by the contemporaneous period and the period a worker is hired.

The option value of an inactive firm is assumed to be equal to 0. The value function of a firm with a worker at time τ, given that a firm-worker match started at time t, $\tau \geq t$, is

$$J_{t,\tau} = z_t - w_{t,\tau} + \beta(1-\delta)E_\tau J_{t,\tau+1}. \tag{B1}$$

The value function of an opened vacancy at τ is

$$V_\tau = -c + q_\tau J_{\tau,\tau} + \beta(1-q_\tau)E_\tau V_{\tau+1}. \tag{B2}$$

The value function of an employed worker at time τ, given that a firm-worker match started at time t, $W_{t,\tau}$, is

$$W_{t,\tau} = u(w_{t,\tau}) + \beta E_\tau\left[(1-\delta)W_{t,\tau+1} + \delta U_{\tau+1}\right]. \tag{B3}$$

The value function of an unemployed worker at time τ, U_τ, is

(B4) $$U_\tau = u(b) + \beta E_{\tau+1}\left[\mu_{\tau+1} W_{\tau+1,\tau+1} + (1-\mu_{\tau+1})U_{\tau+1}\right].$$

B. $\Pr(UC_t^V > 0)$

Lemma 2. $\Pr(UC_t^V > 0) > 0.99$, given the empirical volatility and autocorrelation of z_t.
Proof.
$UC_t^V > 0$ can be rewritten as

$$\left(\frac{c}{q(\theta(z_t))} - \beta(1-\delta)E_t\frac{c}{q(\theta(z_{t+1}))}\right) > 0$$

or

(B5) $$\frac{\theta(z_t)^\alpha}{E_t(\theta(z_{t+1})^\alpha)} > \beta(1-\delta).$$

Equation (B5) imposes restrictions on the volatility of the stochastic process of $\theta(z_{t+1})$ conditional on $\theta(z_t)$. One can check whether these restrictions hold in the data.

Since $0 < \alpha < 1$, by Jensen's inequality:

$$E_t\left(\theta(z_{t+1})^\alpha\right) \leq \left(E_t\theta(z_{t+1})\right)^\alpha.$$

It implies

$$\frac{\theta(z_t)^\alpha}{E_t(\theta(z_{t+1})^\alpha)} \geq \frac{\theta(z_t)^\alpha}{\left(E_t\theta(z_{t+1})\right)^\alpha}.$$

Thus, to show (B5), it is suffice to show

(B6) $$\frac{\theta(z_t)}{(E_t\theta(z_{t+1}))} > \left(\beta(1-\delta)\right)^{1/\alpha}.$$

Log-linearization of $\theta(z_{t+1})$ around $\theta(z_t)$ yields

$$\theta(z_{t+1}) \simeq \theta(z_t)(1 + \varepsilon_{\theta(z_t),z_t}\ln\frac{z_{t+1}}{z_t}).$$

Then, (B6) can be rewritten as

$$\frac{1}{(1+\varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t})} > \left(\beta(1-\delta)\right)^{1/\alpha}$$

or, noting that $1+\varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t} > 0$ since $\theta(z_t), \theta(z_{t+1}) > 0$:

(B7) $$1-\left(\beta(1-\delta)\right)^{1/\alpha} > \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t}.$$

The stochastic process for z_{t+1} can be specified as

(B8) $$\ln z_{t+1} = (1-\rho)\ln\overline{z} + \rho\ln z_t + \iota_{t+1},$$

where $\iota_{t+1} \sim N(0,\sigma_\iota^2)$.

Then, inequality (B7) can be rewritten as

$$1-\left(\beta(1-\delta)\right)^{1/\alpha} > \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}((1-\rho)\ln\overline{z} + \rho\ln z_t) - \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}\ln z_t,$$

which, given $\varepsilon_{\theta(z_t),z_t} > 0$, after simplification yields as

$$\ln\frac{z_t}{\overline{z}} > \frac{(\beta(1-\delta))^{1/\alpha}-1}{(\beta(1-\delta))^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}}.$$

Given the stochastic process for z_t,(B8), quarterly values $\beta = 1/(1+0.012)$ and $\delta = 0.01$ (Shimer (2005), Hornstein, Krusell and Violante (2005)), $\rho_z = 0.878$ and $\sigma_z = 0.02$ for quarterly log deviations of z from an HP trend (Shimer (2005)), and a high value of $\alpha = 0.72$ found in the literature, it yields

(B9)

$$\Pr\left(\ln\frac{z_t}{\overline{z}} > \frac{(\beta(1-\delta))^{1/\alpha}-1}{(\beta(1-\delta))^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}}\right) = \Pr\left(\frac{\ln\frac{z_t}{\overline{z}}}{\sigma_z} > \frac{0.89^{1/\alpha}-1}{0.89^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}\sigma_z}\right) = 1-\Phi\left(\frac{-72.00}{\varepsilon_{\theta(z_t),z_t}}\right),$$

where $\Phi(.)$ is a c.d.f. of the standard normal distribution.

For $\varepsilon_{\theta,z} = 7.56$, the right hand side of (B9) is $1-\Phi(-9.52) >> 0.99$. When the value of $\varepsilon_{\theta(z_t),z_t}$ more than doubles, say, $\varepsilon_{\theta(z_t),z_t} = 20$, then $1-\Phi\left(\frac{-72.00}{\varepsilon_{\theta(z_t),z_t}}\right) = 1-\Phi(-3.6) > 0.99$. Thus, given $\rho_z = 0.878$, $\sigma_z = 0.02$, and $\varepsilon_{\theta,z} = 7.56$, $\Pr(UC_t^V > 0) > 0.99$.■

C. $\Pr(UC_t^W > 0)$

Lemma 3. $\Pr(UC_t^W > 0) > 0.99$, given the empirical volatility and autocorrelation of z_t.

Proof.

$UC_t^W > 0$ can be rewritten:

$$PDV^W(z_t) - \beta(1-\delta)E_t PDV^W(z_{t+1}) > 0.$$

or

$$\frac{PDV^W(z_t)}{E_t PDV^W(z_{t+1})} > \beta(1-\delta). \tag{B10}$$

Log-linearization of $PDV(z_{t+1})$ around z_t yields

$$PDV^W(z_{t+1}) \simeq PDV^W(z_t)(1+\varepsilon_{PDV^W(z_t),z_t}\ln\frac{z_{t+1}}{z_t}),$$

where $\varepsilon_{PDV^W(z_t),z_t}$ is the elasticity of $PDV^W(z_t)$ at z_t. Note that $1-\varepsilon_{PDV^W(z_t),z_t}(1-\rho)\ln\frac{z_t}{\overline{z}} > 0$ because $PDV^W(z_{t+1}) > 0$, which holds true if all wages are non-negative and at least one is positive.

Equation (B10) can be rewritten:

$$\frac{1}{1-\varepsilon_{PDV^W(z_t),z_t}E_t\ln\frac{z_t}{\overline{z}}} > \beta(1-\delta). \tag{B11}$$

Using the stochastic process for z_t, (B8), inequality (B11) can be rewritten as follows:

$$\ln\frac{z_t}{\overline{z}} > \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)\varepsilon_{PDV^W(z_t),z_t}},$$

if $\varepsilon_{PDV^w(z_t),z_t} > 0$, and

$$\ln\frac{z_t}{\overline{z}} < \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)\varepsilon_{PDV^W(z_t),z_t}},$$

if $\varepsilon_{PDV^w(z_t),z_t} < 0$.

Given the quarterly parameter values discussed in the appendix above and the stochastic process for z_t, these two cases can be combined as follows:

$$\Pr\left(\ln\frac{z_t}{\overline{z}} > \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)|\varepsilon_{PDV^w(z_t),z_t}|}\right) = 1-\Phi\left(\frac{-50.65}{|\varepsilon_{PDV^w(z_t),z_t}|}\right). \tag{B12}$$

To obtain a bound on $\varepsilon_{PDV^w(z_t),z_t}$, consider free entry condition:

$$\frac{c}{\theta^{-\alpha}} = PDV^Z(z_t) - PDV^W(z_t).$$

Differentiating and rearranging yields

$$\alpha\varepsilon_{\theta(z_t),z_t} J(z_t) = \varepsilon_{PDV^Z(z_t),z_t} PDV^Z(z_t) - \varepsilon_{PDV^w(z_t),z_t} PDV^W(z_t),$$

where $J(z_t) \equiv PDV^Z(z_t) - PDV^W(z_t) \geq 0$, given free entry, and $\varepsilon_{PDV^Z(z_t),z_t} > 0$ (see below). Rearranging, it follows:

$$\text{(B13)} \qquad \varepsilon_{PDV^w(z_t),z_t} = \varepsilon_{PDV^Z(z_t),z_t} \frac{PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} J(z_t)}{PDV^Z(z_t) - J(z_t)}.$$

It can be shown that the following holds:

$$\text{(B14)} \qquad \left| \frac{PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} J(z_t)}{PDV^Z(z_t) - J(z_t)} \right| < 1.$$

To see this, note, that if $\varepsilon_{PDV^w(z_t),z_t} > 0$, then $PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} J(z_t) > 0$ because $PDV^Z(z_t) - J(z_t) = PDV^W(z_t) > 0$. Then, equation (B14) can be rewritten: $PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} J(z_t) < PDV^Z(z_t) - J(z_t)$, which holds when $\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} > 1$.

Alternatively, if $\varepsilon_{PDV^w(z_t),z_t} < 0$, then $\frac{PDV^Z(z_t)}{J(z_t)} < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$. Then equation (B14) can be rewritten: $-(PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} J(z_t)) < PDV^Z(z_t) - J(z_t)$, which can be rewritten as

$$\text{(B15)} \qquad \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} + 1 < 2\frac{PDV^Z(z_t)}{J(z_t)}.$$

Since $\frac{PDV^Z(z_t)}{J(z_t)} < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$, equation (B15) holds if $1 < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$.

Thus, $\varepsilon_{PDV^W(z_t),z_t} = \varepsilon_{PDV^Z(z_t),z_t} x_t$,where $|x_t| < 1$ if $1 < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$.

Given the stochastic process for z_t, $PDV^Z(z_t)$ can be written:

$$PDV^Z(z_t) = z_t + \sum_{\tau=t+1} (\beta(1-\delta))^{\tau-t} \exp\left((1-\rho)\sum_{k=0}^{\tau-t} \rho^k \ln \overline{z} + \rho^{\tau-t} \ln z_t + \frac{\sigma_i^2}{2} \sum_{k=0}^{\tau-t-1} \rho^k \right).$$

Note the following:

$$\varepsilon_{PDV^Z(z_t),z_t} = \frac{dPDV^Z(z_t), z_t}{dz_t} \frac{z_t}{PDV^Z(z_t), z_t} = \frac{z_t + \sum_{\tau=t+1} (\rho\beta(1-\delta))^{\tau-t} E_t z_\tau}{z_t + \sum_{\tau=t+1} (\beta(1-\delta))^{\tau-t} E_t z_\tau},$$

which delivers $0 < \varepsilon_{PDV^Z(z_t),z_t} < 1$ since $0 < \rho < 1$ and $z_t + \sum_{\tau=t+1} (\beta(1-\delta))^{\tau-t} E_t z_\tau \equiv PDV^Z(z_t) > 0$.

Note that $\alpha\varepsilon_{\theta(z_t),z_t} > 1$ given the values for α and $\varepsilon_{\theta(z_t),z_t}$ as described in Section 3.C. Thus, from $\alpha\varepsilon_{\theta(z_t),z_t} > 1$ and $0 < \varepsilon_{PDV^Z(z_t),z_t} < 1$, it follows that $\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} > 1$. Hence, $|\varepsilon_{PDV^w(z_t),z_t}| = |\varepsilon_{PDV^w(z_t),z_t} x_t| < 1$.

Using $|\varepsilon_{PDV^w(z_t),z_t}| < 1$ in expression (B12) delivers $\Pr\left(UC^W > 0\right) > 0.99$.■

D. *Derivation of* $\varepsilon_{UC_t^V, z_t} = \alpha \varepsilon_{\theta,z} x_t$

Using (B8), the probability density function for z_{t+1} given z_t is:

$$f(z_{t+1}|z_t) = \frac{1}{z_{t+1}\sigma_{\ln z_{t+1}}\sqrt{2\pi}} \exp(-\frac{\ln(z_{t+1}) - ((1-\rho)\ln \overline{z} + \rho \ln z_t)}{2\sigma^2_{\ln z_{t+1}}}).$$

The elasticity of the vacancy component of the user cost of labor with respect to productivity is:

$$\varepsilon_{UC_t^V, z_t} = \frac{d\left(\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)\int \frac{c}{K\theta_{t+1}^{-\alpha}} f(z_{t+1}|z_t)dz_{t+1}\right)}{dz_t} \frac{z_t}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} =$$

$$\frac{\alpha\varepsilon_{\theta,z}\frac{c}{K\theta_t^{-\alpha}} - \rho\beta(1-\delta)\int \frac{c}{K\theta_{t+1}^{-\alpha}} f(z_{t+1}|z_t)dz_{t+1}}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} =$$

$$\frac{\alpha\varepsilon_{\theta,z}\left(\frac{c}{K\theta_t^{-\alpha}} - \frac{\rho}{\alpha\varepsilon_{\theta,z}}\beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}\right)}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} = \alpha\varepsilon_{\theta,z}x_t,$$

where $x_t = \frac{\frac{c}{K\theta_t^{-\alpha}} - \frac{\rho}{\alpha\varepsilon_{\theta,z}}\beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}}$. Since $\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}} > 0$ (see proof above) and $\rho < \alpha\varepsilon_{\theta,z}$ (for $\rho < 1$, $\varepsilon_{\theta,z} = 7.56$ and $\alpha \in [0.235; 0.72]$), one obtains $x_t > 1$.■

E. *Expressions for* $\varepsilon_{\theta z}$ *and* ε_{wz}

In the steady state the elasticity of the vacancy-unemployment ratio with respect to productivity is

$$\varepsilon_{\theta z} = \frac{1}{1-b/z}\frac{1-\beta(1-\delta-\eta\mu)}{\alpha-\beta(\alpha-\delta\alpha-\eta\mu)}$$

and the elasticity of wages is

$$\varepsilon_{wz} = \frac{\eta}{\eta(1-\beta(1-\delta-\mu)) + (1-\eta)\frac{b}{z}(1-\beta(1-\delta))} \cdot ((1-\beta(1-\delta-\mu)) + \frac{(1-\eta)(1-\alpha)\beta\mu(1-\beta(1-\delta))}{\alpha-\beta(\alpha-\delta\alpha-\eta\mu)}),$$

where η is the worker's bargaining power, b is the unemployment benefit, and μ is the steady state value of the job-finding rate.

C Appendix C

A. *Models with Implicit Contracts*

The value an employed worker receives in period t from a contract that started in period t_0, $W_\sigma(t_0, z^t)$, is

$$W_\sigma(t_0, z^t) = u(w_\sigma(t_0, z^t)) + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(1-\delta)^{\tau-(t+1)}\left[(1-\delta)u(w_\sigma(t_0, \{z^{\tau-1}, z_\tau\})) + \delta U(z_\tau)\right].$$

The value of a newly unemployed worker or a worker who did not find a match in the current period is a sum of the current utility, obtained from consuming an unemployment benefit, b, and the expected discounted value from searching:

$$U(z_t) = u(b) + \beta E_t \left[\mu(\theta_\sigma(\{z^{t+1}, z_t\}))W_\sigma(t+1, \{z^t, z_{t+1}\}) + (1 - \mu(\theta_\sigma(\{z^{t+1}, z_t\})))U(z_{t+1}) \right].$$

The value a firm obtains in period t given the aggregate state z_t from a contract σ that started in period t_0 is

$$J_\sigma(t_0, z^t) = z_t - w_\sigma(t_0, z^t) + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(z_\tau - w_\sigma(t_0, \{z^{\tau-1}, z_\tau\})).$$

Equilibrium contracts are limited to efficient optimal contracts. A contract is efficient if there exists no other contract that offers each party at least as much expected utility and one party strictly more. A contract is optimal if it maximizes the total welfare given the initial promise of a value to one of the parties. An efficient contract cannot be Pareto dominated after any history. Hence, after any history it can be rewritten as a maximization problem. The Pareto frontier is traced by varying the value promised by the contract to the worker and maximizing the value of the firm given the worker's promised value. As in Thomas and Worrall (1988), the history of the productivity realizations from the start of the match can be summarized by the worker's promised value. Given the assumption that z_t follows a first order Markov process, it is sufficient to keep track of the current value of z to determine the expectations. In the presentation that follows the time subscripts are suppressed: z denotes the current value of productivity and z' denotes the value next period.

Let W be the value promised to a worker under the contract. Let $U(z)$ be the value of an unemployed worker given aggregate state z and let $f(z, W, U(z))$ denote a value of a firm from a contract on a Pareto frontier given z, W, $U(z)$, and the evolution of $U(z)$. Then $f(z, W, U(z))$ solves the following dynamic programming optimization problem for all $z \in Z$:

(C1) $$f(z, W, U(z)) = \max_{w, \{W(z')\}_{z' \in Z}} z - w + \beta E_z (1-\delta) f(z', W(z'), U(z'))$$

s. t.

(C2) $$W = \varphi(w) + \beta E_z \left[(1-\delta)W(z') + \delta U(z')\right]$$

(C3) $$W(z') \geq U(z') \; \forall \; z' \in Z$$

(C4) $$f(z', W(z'), U(z')) \geq 0 \; \forall \; z' \in Z.$$

An efficient contract maximizes the value of a firm, f, given the aggregate state, z, the promised value for the worker, W, and the worker's outside option, $U(z)$. The first constraint is a promise-keeping constraint that specifies that a worker gets exactly value W from the contract that pays wage w and promises values $W(z')$ for all states $z' \in Z$ where there is no exogenous separation. The second and third constraints are self-enforcing constraints for the worker and the firm, respectively. By omitting self-enforcing constraints, contracts with different degrees of commitment are obtained: 1) full commitment (by omitting (C3) and (C4)); 2) lack of commitment from the worker's side and full commitment from the firm's side (by omitting (C4)); and 3) two-sided lack of commitment (when both (C3) and (C4) are present).

I study equilibria of this economy which consist of a contract $\sigma(z)$, value functions for the firm from a contract $\sigma(z)$, f_σ, values promised to the worker at the time of hiring, $W_{h,\sigma}(z)$, values of an unemployed worker, $U(z)$, and a market tightness, $\theta_\sigma(z)$, associated with the contract $\sigma(z)$ for each $z \in Z$, such that

1. (Optimization) Given a vector U, the list of functions $f(z, W_{h,\sigma}(z), U(z))$ solves the dynamic programming problem (C1)-(C4).

2. (Free entry) Firms enter a labor market and post vacancies with the associated contract σ until the value of posting a vacancy is driven to 0:

(C5) $$q(\theta_\sigma(z)) f(z, W_{h,\sigma}(z), U(z)) = c.$$

3. The value of an unemployed worker evolves according to the following rule:

$$U(z) = u(b) + \beta E_z \left[\mu(\theta_\sigma(z'))W_{h,\sigma}(z') + (1 - \mu(\theta_\sigma(z')))U(z')\right]. \tag{C6}$$

In addition, I impose the following equilibrium refinement:

4. (Pareto efficiency) There does not exist an efficient self-enforcing contract $\sigma'(z)$ and an associated labor market with tightness $\theta_{\sigma'}(z)$ such that the net surpluses from search for a worker, $\mu(\theta_{\sigma'}(z))(W_{h,\sigma'}(z)-U(z))$,and for a firm, $-c+q(\theta_\sigma(z))f(z, W_{h,\sigma'}(z), U(z))$, are at least as much as under $\sigma(z)$ and $\theta_\sigma(z)$ and for one party it is strictly more.

This refinement of the set of equilibrium contracts follows Rudanko (2009), who motivates it from the competitive search, in which competitive market-makers specify the set of the efficient self-enforcing contracts that can be posted in the economy. Each contract is offered in a separate market with an associated labor-market tightness, and in equilibrium each market must offer the same surplus from search for firms and the same surplus for workers. Because of competition between market-makers, only markets in which the offered contract is on the Pareto frontier will be opened in equilibrium. Condition 2 combined with Condition 3 determines equilibrium values of the promised value for the worker at the time of hiring, $W_{h,\sigma}(z)$, and an equilibrium value of the market tightness in the market with σ, $\theta_\sigma(z)$.

In this economy unemployment evolves according to the following law, given $u(z_{t_0})$:

$$u(\{z^t, z_{t+1}\}) = u(z^t) + (1 - u(z^t))\delta - \mu(\theta(\{z^t, z_{t+1}\}))u(z^t).$$

The pool of unemployed in the current period consists of unemployed workers from the previous period and those who became unemployed because of the exogenous separations in the previous period, net of the unemployed workers who find jobs in the current period.

Thomas and Worrall (1988) and Rudanko (2009) prove that the optimization problem described above is a concave problem, so the first-order conditions are necessary and sufficient. The first-order conditions for an arbitrary z read:

$$-\lambda_z = -\frac{1}{\varphi'(w)}. \tag{C7}$$

$$-\lambda_z = (1 + \zeta(z'))f_V(z', W(z'), U(z')) + \phi(z') \ \forall z' \in Z, \tag{C8}$$

where λ_z is the Langrange multiplier on the promise-keeping constraint; $\beta\pi(z'|z)\phi(z')$, are Langrange multipliers on the self-enforcing constraints for a worker, and $\beta\pi(z'|z)\zeta(z')$ are Langrange multipliers on self-enforcing constraints for a firm $\forall z' \in Z$. Complimentary slackness conditions: $\lambda_z \geq 0$, $\zeta(z'), \phi(z') \geq 0 \ \forall z'$, and (C3) and (C4). The envelope condition:

$$f_V(z, W(z), U(z)) = -\lambda_z. \tag{C9}$$

Combining the envelope condition, (C9), with the first order conditions gives the following condition, which links current and next period wage:

$$\frac{1}{\varphi'(w(z, W, U(z)))} = (1 + \zeta(z'))\frac{1}{\varphi'(w(z', W(z'), U(z')))} + \phi(z') \ \forall z' \in Z$$

Because of free entry and Pareto optimality, $W_h(z)$ and $\theta(z)$ solve the following maximization problem given $V_u(z)$:[28]

$$\max_{\{\theta(z)\},\{V_h(z)\}} \left\{ \ \mu(\theta(z))(W_h(z) - U(z)) \ \right\} \tag{C10}$$
$$\text{s.t. } q(\theta(z)f(z, W_h(z), U(z)) = c$$

[28] Rudanko (2009) proves that given fairly mild conditions there is a unique Pareto-efficient contract offered in equilibrium.

Combining the first order condition for Pareto optimality problem, (C10), the free entry condition, (C5), the envelope condition, (C9), the first order condition for wages, (C7), and the law of motion for the value of unemployed workers, the following system of equations characterizes the equilibrium objects f, $U(z)$, $W_h(z)$ and $\theta(z)$ $\forall z \in Z$, given the optimal contract.

(C11)
$$\frac{\alpha}{1-\alpha} f(z, W_h(z), U(z)) = \frac{W_h(z) - U(z)}{\varphi'(w(z, W_h(z), U(z)))}.$$

$$\theta(z) = \left(\frac{c}{f(z, W_h(z), U(z))} \frac{1}{K} \right)^{-\frac{1}{\alpha}}.$$

$$U(z) = u(b) + \beta E_z \left[\mu(\theta_\sigma(z')) W_h(z') + (1 - \mu(\theta_\sigma(z'))) U(z') \right].$$

B. Model with Bargaining Period by Period

An equilibrium in the economy with bargaining period by period consists of the set of the value functions for a firm, $J(z)$, (B1), and $V(z)$, (B2), and a worker, $W(z)$, (B3) and $U(z)$, (B4), and a market tightness $\theta(z)$, such that

1. (Free entry) The value of a vacancy is 0:

$$q(\theta(z)) J(z) = c.$$

2. (Surplus division) Each period during an employment relationship, the firm and the worker bargaining over the match surplus. At the time of bargaining the outside option value for a worker is the value of unemployment, while the outside option for a firm is 0 (the value of an inactive firm). A matched worker-firm pair divides the total surplus from the match by solving the following maximization problem:

(C12)
$$\max_{V_e(z) - V_u(z), J_F(z)} \left(W(z) - U(z) \right)^\eta J(z)^{1-\eta}$$
$$\text{s.t. } \frac{W(z) - U(z)}{u'(w(z))} + J(z) = S(z)$$

where η is a bargaining power of the worker, $u'(w)$ is the marginal utility of income, and $S(z)$ is a total surplus.

3. The value of an unemployed worker evolves according to the following rule:

$$U(z) = u(b) + \beta E_z \left[\mu(\theta(z')) W(z') + (1 - \mu(\theta(z'))) W(z') \right].$$

■