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A Simple General Equilibrium Model of Large Excess Reserves

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A Simple General Equilibrium Model of Large Excess Reserves[1]

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Abstract

I study a non-stochastic, perfect foresight, general equilibrium model with a banking system that may hold large excess reserves when the central bank pays interest on reserves. The banking system also faces a capital constraint that may or may not be binding. When the rate of interest on reserves equals the market rate, if the quantity of reserves is large and bank capital is not scarce, the price level is indeterminate. However, for a large enough level of reserves, the bank capital constraint becomes binding and the price level moves one to one with the quantity of reserves.

1. Introduction

In September 2012, the Federal Reserves instituted a policy of buying \$40 billion of mortgage-backed securities every month for *an indefinite time*. In January 2013, \$45 billion of longer-term Treasury securities were added to the program. As a result, the total amount of reserves held by depository institutions (as reported in the Fed's H.4 statistical release) has grown at an average rate of \$78 billion per month during 2013. The difference between reserves growth and asset purchases is accounted for by increases in currency in circulation (and other minor items in the liability side of the Fed's balance sheet).

At the same time, the Federal Open Market Committee has restated consistently its long-standing commitment to price stability, targeting a 2% inflation rate. The creation of nominal reserves, then, is not intended to be accompanied with an increase in the price level that could attenuate the real effects. Rather, it is thought of as a swap of one type of liquid asset (securities) for another type of liquid asset (reserves), all in real terms.

Reserves, however, can only be held by a selected group of financial institutions; mainly, by banks. Hence, with stable prices, the growth in total reserves translates into growth of the real value of an asset that primarily resides in the balance sheets of banks. When reserves increase, either the balance sheet of banks grows or some other bank assets adjust to compensate.

The U.S. banking system is large. As of December 2012 total assets at commercial banks (as reported in the Fed's H.8 statistical release) were \$13.1 trillion, with \$1.7 trillion of cash assets (more than 90% being reserves) and \$2.7 trillion of securities (68% of which were Treasury and Agency securities – see Figure A4 in the appendix). Furthermore, the average growth rate of total assets

[1] The motivation for this work comes, in part, from my research collaboration with Alex Wolman. His thinking is likely to be (and hopefully is) reflected in these notes and I am grateful to him for sharing his ideas on this subject with me. I would also like to thank for comments the participants at the "Financial Reform and Quantitative Easing in General Equilibrium Conference" at ASU, the 2014 MMM at the University of Missouri, and the 2014 SED in Toronto. All remaining errors are exclusively my own. The views in this paper do not represent the views of the Federal Reserve Bank of Richmond, the Board of Governors of the Federal Reserve, or the Federal Reserve System. Author's email address: *huberto.ennis@rich.frb.org*.

in the banking system over the last 20 years has been close to 0.5% per month, which in December 2012 would amount to around $65 billion (see Figures 1 and 2).

While it seems plausible, given the large size of the U.S. banking system that, at least for some time, banks would be able to absorb a growing (real) value of reserves, a natural question to ask is: How long can such a process continue without any major impact on banks' lending capabilities and, eventually, on the economy's price level? To answer this question it is useful to study a general equilibrium model of an economy that includes a banking system with sophisticated balance sheets. This is the objective of this paper.

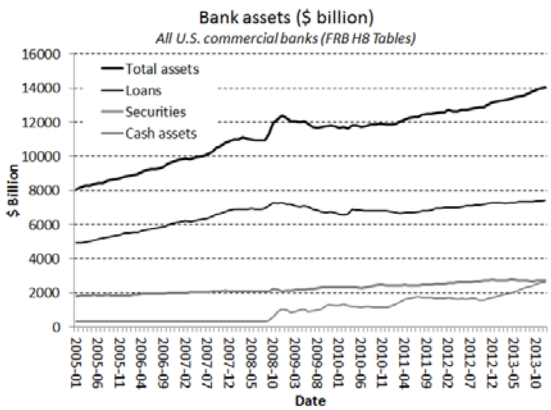

Figure 1

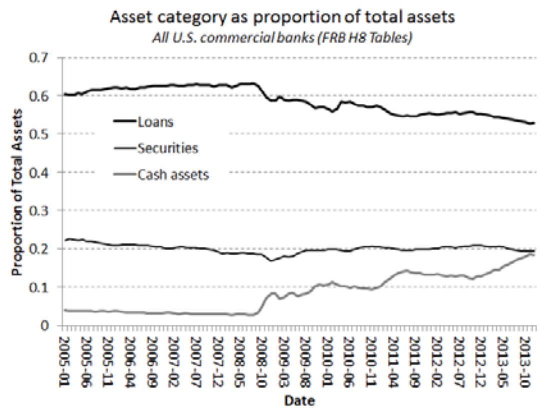

Figure 2

The model is a dynamic economy with four types of agents and a central bank. Households consume and make deposits in the banking system. Entrepreneurs take loans from banks to fund productive projects. Banking experts provide bank capital for banks and banks intermediate funding between households, expert investors, and entrepreneurs. The central bank sets the rate of growth of monetary assets in the economy, may or may not pay interest on reserves, and can impose lump-sum taxes on households and banks.

Aside from the goods produced by entrepreneurs undertaking their projects, households and experts receive an endowment of goods every period. There is also an endowment of productive assets every period which deliver goods to their holders the following period. The claims on the productive assets are securities in agents' balance sheets. All agents can, in principle, hold securities.

Households do not like to consume their own endowment and trade goods with other households using currency (Lucas, 1990). The monetary assets issued by the central bank are endogenously divided every period between currency and bank reserves. Banks maximize per period profits and face three constraints: (1) a bank capital constraint, (2) a reserve requirement constraint, and (3) a liquidity constraint. There is free entry in banking and, hence, banks make zero profits in equilibrium.[2]

I study the model's stationary competitive equilibrium. The main result is that when the central bank is paying interest on reserves at the market rate, if the quantity of reserves is large and bank capital is not scarce, the price level is indeterminate and belongs to a close interval.[3] For a large enough level of reserves, the bank capital constraint becomes binding and the price level becomes determinate and moves one to one with the quantity of reserves. This result suggests that there is

[2] Ireland (2013) and Hornstein (2010) are two recent papers closely related to mine. See the discussion in Part 2 of the Appendix.

[3] In their seminal contribution, Sargent and Wallace (1985) show a similar result using an overlapping generations endowment economy with money (see also Smith, 1991). The logic behind their result, however, is very different from the one studied here. The real rate of interest is pinned down in my model, but it is indeterminate in Sargent and Wallace's analysis. Also, Sargent and Wallace do not consider the endogenous division of monetary assets between currency and reserves, which plays an important role in this paper. See Part 2 of the Appendix for a more detailed discussion of the connection between this paper and Sargent and Wallace (1985).

a limit to central bank purchases of securities financed with reserves when the intention is to not induce increases in the price level: After some point, if excess reserves become large enough, more reserves are associated with higher price levels.

The main components that drive the results in my paper are the ability of the central bank to pay interest on reserves and the fact that the central bank only controls the total amount of monetary assets (currency plus reserves) but does not control the split between the two (an endogenous variable in the model). When the central bank pays interest on reserves at the market rate and the capital, reserves, and liquidity constraints are not binding, banks are indifferent both between holding securities and reserves and between funding their operations with more deposits or more capital. This indifference creates the indeterminacy: Different equilibria result from banks holding more or less reserve funded by more or less bank deposits. Given Lucas-type price level determination, more reserves in the banking system imply less currency in circulation and a lower price level.

The questions in this paper mainly involve long-run trends and large, relatively persistent, changes in the levels of the various components of a monetary policy. For this reason, I choose to work with a non-stochastic, perfect foresight model. However, as it will become clear in the analysis, a stochastic extension of the model may be useful for understanding, for example, the short-run effects of central bank asset purchases. This subject is left for future research.[4]

Even though there are multiple moving parts, I call the model "simple" because I keep the problems of the agents and their interactions as simple as possible, while still being able to obtain some useful insights about the possible answers of the questions that I am concerned with. For example, in the model banks do not solve an explicit information problem. Instead, they are assumed to be the necessary intermediaries of funds between entrepreneurs and other agents. Furthermore, the reason for banks to hold reserves is kept very simple: either reserves are not dominated in rate of return or the exogenously imposed reserve requirement is binding. Similarly, bank capital is either a non-dominated source of funding or the exogenously imposed bank capital constraint is binding. These are strong assumptions, and various alternatives to them already exist in the literature. However, the idea here is that the simplicity associated with these assumptions makes them an acceptable compromise in a first attempt to answer some of the basic questions addressed in this paper.[5]

The paper is organized as follows. The next section describes the baseline model. Section 3 defines equilibrium. Section 4 studies stationary equilibrium, first when the central bank does not pay interest on reserves and then when it does. We use the case of no interest on reserves to build up some understanding of the workings of the model. Payment of interest on reserves is crucial for addressing the issues described above, and it is where the main contribution of the paper can be found. Section 5 considers an extension of the model to the case when deposits provide a transaction-based convenience yield to households. Section 6 concludes.

2. The model

Time is discrete and goes on forever. Let $t = 0, 1, 2...$ denote time. There is a central bank and four types of agents in the economy: private households, expert investors, entrepreneurs, and bank

[4] Ireland (2013) studies quantitatively the short-term response to shocks of an economy where the central bank pays interest on reserves. Examples of papers addressing the general equilibrium impact of central bank asset purchases are Curdia and Woodford (2011), Gertler and Karadi (2011), and Foerster (2011). Prominent precursors to this general line of research are Diaz Gimenez et al. (1992) and Goodfriend and McCallum (1997).

[5] Lacker (1997) studies a model where reserves play a fundamental role in the payment system and investigates the consequences of paying interest on reserves in such a model. Bianchi and Bigio (2013) also provide a micro-founded role for bank reserves and a quantitative analysis of some of the issues involved. Williamson (2012) is a prominent example of a paper that studies the recent experience of U.S. monetary policy in a general equilibrium model with explicit microfoundations for the demand of monetary assets and the role of the banking system. Berentsen and Monnet (2008) also present a model within the tradition of the money search literature and study a channel system for the implementation of monetary policy. Naturally, the floor of the channel involves a central bank deposit facility that pays interest on reserves. Goodfriend (2002) is a common reference for a discussion of monetary policy implementation using interest on reserves.

managers. Private households are dynasties that live forever. Expert investors, entrepreneurs, and bank managers live for two periods, and a new generation of them is born every period. There is a measure one of each type of agent. There is a perishable good every period and a stock of monetary assets that becomes reserves when and if a bank deposits some of these assets at the central bank.

Before moving into the specific details associated with the economic decisions of each of the different types of agents in the model, let us briefly describe how these agents will interact in equilibrium. All transactions take place in competitive markets. Every period, each member of the new generation of bank managers has the ability to form a two-period-lived bank that uses deposits and bank capital to finance entrepreneurs. Expert investors are able to provide bank capital to bankers, relying on some endowed expertise that allows them to invest in banking effectively. Households make deposits with banks, and entrepreneurs take loans only from banks. In other words, by assumption, households cannot directly make loans to entrepreneurs and all lending to entrepreneurs must be intermediated by banks.

2.1 Private households

In every period t, each household receives an endowment of goods w_{ht} and an endowment of one-period zero-coupon securities of size s_t. Each of these securities pays one unit of the good next period and they represent private productive assets in the economy.

Households cannot consume their own endowment. Following Lucas (1990), we can think of the household as having two members. One member of the household trades securities s_{ht}, makes nominal deposits d_t, pays lump-sum taxes τ_{ht} to the government, and buys the consumption good c_t. The other member of the household sells the endowment in exchange for (and only for) cash m_t.[6] The cash obtained from selling the endowment cannot be used until the following period.

Utility from household consumption in period t is given by a strictly increasing and strictly concave function $u(c)$. All households discount the future at factor β.

Let us denote the nominal rate of return on deposits by i_{dt} and the price of private securities by q_t. The real value of money (the inverse of the price level) at time t is given by ϕ_t. Then, the problem of the private households is:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to

$$\begin{aligned} c_t + \phi_t d_t + \phi_t n_t + q_t s_{ht} &= \phi_t (1 + i_{dt-1}) d_{t-1} + s_{ht-1} + q_t s_t + \phi_t m_{t-1} - \phi_t \tau_{ht}, \\ \phi_t m_t &= w_{ht} + \phi_t n_t, \\ c_t, n_t, d_t &\geq 0, \end{aligned}$$

where n_t represents excess cash that the household decides not to deposit or spend in consumption and securities. Using the first order condition with respect to n_t it is easy to see that whenever $i_{dt} > 0$ we have that $n_t = 0$ is optimal. We will restrict attention to this case in what follows.

The other first order conditions from the household optimization problem can be summarized as:

$$-\phi_t u'(c_t) + \phi_{t+1}(1 + i_{dt})\beta u'(c_{t+1}) \leq 0, \tag{1}$$

$$-q_t u'(c_t) + \beta u'(c_{t+1}) = 0, \tag{2}$$

and the first inequality should hold with equality whenever $d_t > 0$ is satisfied. We can interpret the inverse of q_t as the gross real return on households savings at time t; that is, $1/q_t \equiv 1 + r_{qt}$ is the (gross) real rate of interest available to the household in period t.

[6] Lucas' (1990) household has three members, one buying consumption goods, one selling the endowment, and one trading securities. We choose a simpler setup as a first step, but it would be interesting to extend the analysis to consider three-member households and study, as Lucas does, the possibility of liquidity effects in the securities market.

Whenever d_t is positive in equilibrium a Fisher-type equation holds:

$$1 + i_{dt} = (1 + r_{qt}) \frac{\phi_t}{\phi_{t+1}}, \tag{3}$$

which tells us that the equilibrium gross nominal interest rate on deposits equals the gross real interest rate multiplied by the gross inflation rate. In Section 5 we extend the model to allow for a situation where deposits provide a convenience yield to households, and hence their real return in the market can be lower than the real return on securities.

2.2 Expert investors

Every period t, each expert investor has an endowment of goods w_{et} and some expertise that allows him to lend resources to the banks in the form of bank capital. Experts live for two periods and their period utility function is given by $v(x_t)$, where $x_t \in \mathbb{R}_+$ and $v(x)$ is a strictly increasing, strictly concave, smooth function. Experts can save by holding securities s_{et} (but cannot short them) or by allocating resources to bank capital. They discount the future at factor $\beta < 1$.

We assume that at each time t there is a competitive market for bank capital. Let r_{kt} be the real rate of return that is set in that market at time t. Experts, then, take as given q_t and r_{kt} and solve the following optimization problem:

$$\max v(x_{1t}) + \beta v(x_{2t+1}),$$

subject to

$$\begin{aligned} x_{1t} + q_t s_{et} + k_{et} &= w_{et}, \\ x_{2t+1} &= s_{et} + (1 + r_{kt}) k_{et}, \\ s_{et} &\geq 0, \quad 0 \leq k_{et} \leq w_{et}. \end{aligned}$$

In general, we will restrict attention to situations where $1+r_{kt} \geq 1/q_t$ in equilibrium, so that $k_{et} > 0$ for all t. If $1+r_{kt} > 1/q_t$ then we will have that $s_{et} = 0$. This is the case when bank capital is scarce in the economy. If instead, $1+r_{kt} = 1/q_t$ then experts are indifferent between holding securities and bank capital and the supply of bank capital is infinitely elastic as long as $k_{et} \in (0, w_{et})$.

A particularly simple version of the experts' problem is the one in which they are risk neutral and, hence, $v(.)$ is linear. In that case, we have that whenever $1 + r_{kt} > 1/q_t \geq 1/\beta$ the experts dedicate all the endowment to bank capital and $k_{et} = w_{et}$. This will be a situation in which the economy confronts an inelastic supply of bank capital.

More generally, define $z_{et} = q_t s_{et} + k_{et}$. From the first order conditions we have that when $1 + r_{kt} \geq 1/q_t$, the following equation must hold:

$$-v'(w_{et} - z_{et}) + \beta (1 + r_{kt}) v'((1 + r_{kt}) z_{et}) = 0,$$

and when $1+r_{kt} > 1/q_t$, then $s_{et} = 0$ and $z_{et} = k_{et}$. This expression defines a correspondence $k_e(r_{kt})$ for $1+r_{kt} \geq 1/q_t$. If, for example, experts have a constant-relative-risk-aversion utility function with a coefficient of relative risk aversion between zero and one, then $k_e(r_k)$ is an increasing function of r_{kt} for all $r_{kt} > 1/q_t - 1$. In other words, experts supply more bank capital when the return on bank capital increases.

We have assumed that experts cannot short-sell private securities. Given this, when $1+r_{kt} > 1/q_t$, experts adjust their supply of bank capital smoothly in response to changes in the rate of return r_{kt}. This is not the case when short-selling is allowed.[7] The short-selling constraint, then, is particularly helpful when we want to consider a situation with an elastic supply of bank capital.

[7] Even if we restrict bank capital to be less than or equal to experts endowments, when experts can short sell securities, they use securities trading to smooth consumption over time and the sensitivity of bank capital to its return would be extreme: completely inelastic and equal to w_{et} for all $r_{kt} > 1/q_t - 1$ and infinitely elastic when $r_{kt} = 1/q_t - 1$.

2.3 Entrepreneurs

There is a measure one continuum of entrepreneurs in the economy, each with an indivisible project and no endowment of goods. Entrepreneurs live for two periods, are risk neutral, and consume only in the second period. Each project requires the investment of one unit of resources (goods) in the first period and gives a return of goods in the second period. Projects are heterogeneous in rate of return. Let $G(r) \geq 0$ be the measure of entrepreneurs with project return less than or equal to $1+r$ where $r \in [0, \overline{r}]$ and, hence, $G(\overline{r}) = 1$. We will consider values of $\overline{r}$ large enough so that there are always projects that are profitable enough to deserve funding.

An entrepreneur j that receives at time t a loan from a bank with interest rate r_{lt} must repay the loan at time $t+1$ using the returns from the project. If $r_j > r_{lt}$, then entrepreneur j can consume $r_j - r_{lt}$. Then, given a loan rate r_{lt}, all entrepreneurs with project return greater than r_{lt} will take a loan from a bank and undertake their project. The total demand for loans in the economy at time t is, then, a function of the loan rate at time t and is given by:

$$L^D(r_{lt}) = \int_{r_{lt}}^{\overline{r}} dG(r) \equiv 1 - G(r_{lt}).$$

Note that $L^D(r_{lt})$ is decreasing in r_{lt} because $G(r_{l,t})$ is a distribution function and, hence, is increasing in r_{lt}.

2.4 Banks

We assume that bank managers are risk neutral and, hence, maximize bank profits. Each generation of bank managers have the ability to form new banks. At time t, each newly formed bank j takes deposits d_{jt} from households and capital k_{jt} from expert investors. With the proceeds, banks make one-period loans to entrepreneurs l_{jt}, hold securities s_{jt} and reserves f_{jt}. Let r_{lt} be the real return on loans extended at time t (and paid back at time $t+1$). Each bank has a cost $\chi(l)$ that represents the managing cost of providing l loans to entrepreneurs. The function χ is strictly convex, twice continuously differentiable, and with $\lim_{l \to 0} \chi(l) > 0$ (that is, there is a fixed cost of providing loans). Banks also pay a lump-sum nominal tax/fee $\tau_{bt} \geq 0$ to the central bank.

The bank can make deposits at the central bank to increase its reserves holdings. The nominal interest paid by the central bank at time t on those reserves is equal to i_{iort}. If a bank needs to increase its holdings of reserves after all depositors have made their deposits in banks, the bank can access a competitive interbank market and borrow reserves b^f_{jt} from other banks at rate i_{ft}.

Banks must hold enough reserves to satisfy a regulatory reserve requirement ρ per unit of deposit and enough bank capital to satisfy a regulatory capital requirement κ per unit of asset (i.e., a leverage ratio requirement). Aside from funding, bank capital serves no special role in the model. However, a simple extension in which banks perform a monitoring function (Holmstrom and Tirole, 1967) could be used to justify the imposition of regulatory capital requirements. In the model here, when the cost of capital is higher than other financing means, banks will hold only enough capital to satisfy the capital requirement. If we think that capital requirements are binding in the real world, then this simple model may be an acceptable approach to study the issues that interest us here.

Securities and reserves can also be a source of "liquidity" for banks. First, assume that when a bank gives a loan for a project, it commits to providing extra funds if the project needs them. In other words, the bank gives an entrepreneur a loan and a loan commitment (a line of credit).[8] Also assume that, at the beginning of the second period of the bank's life and before loans have been repaid, a proportion $\theta \in [0,1]$ of depositors withdraw their deposits and a proportion $\xi \in [0,1]$ of the entrepreneurs financed by the bank need an extra unit of resources to complete their project and, hence, draw on their line of credit with the bank. Finally, assume that banks have to also

[8] As it will become clear later, this assumption is intended to capture the need for banks to hold liquidity not just to satisfy depositors' potential demand for cash but also to satisfy liquidity needs that come associated with the standard lending activities of the bank (Kashyap, Rajan, and Stein, 2002). This is not an essential assumption for the results, and most of the paper deals with the case when the loan commitment is not present (i.e., $\xi = 0$).

repay outstanding interbank loans at the beginning of the second period. These assumptions induce a demand for bank liquidity in the form of securities and reserves.[9]

There is free entry in the business of banking and the population of potential bank managers is large enough to drive bank profits to zero in equilibrium. We denote by Γ_t the measure of active bank managers in the economy.

A bank j formed at time t takes as given prices $\left(\phi_t, \phi_{t+1}, r_{lt}, i_{dt}, r_{kt}, i_{ft}, q_t\right)$, the tax τ_{bt+1}, and the policy parameters (i_{iort}, κ, ρ), and solves the following optimization problem:

$$\max \; (1+r_{lt})\, l_{jt} + s_{jt} + \phi_{t+1}\,(1+i_{ior,t})\, f_{jt} - \phi_{t+1}\,(1+i_{dt})\, d_{jt} - (1+r_{kt})\, k_{jt} - \phi_{t+1}\,(1+i_{ft})\, b^f_{jt}$$
$$-\chi\,(l_{jt}) - \phi_{t+1}\tau_{bt+1},$$

subject to

$$\begin{aligned}
a_{jt} &\equiv l_{jt} + q_t s_{jt} + \phi_t f_{jt} = \phi_t\left(d_{jt} + b^f_{jt}\right) + k_{jt}, \\
k_{jt} &\geq \kappa a_{jt}, \\
f_{jt} &\geq \rho d_{jt}, \\
s_{jt} + \phi_{t+1}\,(1+i_{iort})\, f_{jt} &\geq \theta\phi_{t+1}\,(1+i_{dt})\, d_{jt} + \xi l_{jt} + \phi_{t+1}\,(1+i_{ft})\, b^f_{jt},
\end{aligned}$$

and non-negativity constraints on l_{jt}, s_{jt}, and d_{jt}.[10] The first constraint defines bank assets and gives a formal statement of the bank's balance sheet identity (that is, total assets equal total liabilities plus capital). The second constraint reflects the fact that the bank must satisfy capital regulations. The third constraint is due to an exogenously imposed regulatory reserve requirement, and the fourth constraint is a liquidity requirement that can have technological origins associated with the regular business of banking (holding deposits and making loans) or can reflect further regulation by a banking authority. Note that the non-negativity constraints on l_{jt}, s_{jt}, and d_{jt}, together with the other constraints, imply that f_{jt} and k_{jt} are also non-negative in the solution to the bank's problem.

The details of the liquidity constraint deserve some discussion. Basically, it reflects a timing assumption: The payoffs produced by securities and reserves become available before the bank needs to have access to liquid funds to satisfy the liquidity demands of depositors and borrowers. Borrowed funds also become due at the same time and, hence, also absorb liquidity. This implies that borrowed funds can be used to finance bank assets but, at the same time, they increase the demand for liquid assets when liquidity is scarce. We make these assumptions here mainly for concreteness, but it could be a productive activity to investigate alternative, and potentially more flexible, specifications of the liquidity constraint.

For some values of the rates of return faced by the bank, this decision problem allows for the possibility of an optimal banking arrangement fully funded by bank capital (i.e., a bank with no deposits). While this could be considered an interesting theoretical possibility, deposits are a distinguishing characteristic of banks in the United States and for the practical questions of this paper it is reasonable to restrict attention to equilibrium situations where $d_t > 0$. In Part 4 of the appendix, we discuss the case of $d_t = 0$ for completeness.

Assuming that $d_t > 0$ and, to simplify notation, defining the real return on activity s as $1 + r_{st} = \left(\phi_{t+1}/\phi_t\right)(1+i_{st})$ with $s = d, f, ior$ the problem of a banker j at time t is:

$$\max \; (r_{lt} - r_{dt})\, l_{jt} + (r_{qt} - r_{dt})\, q_t s_{jt} + (r_{ior,t} - r_{dt})\, \phi_t f_t - (r_{kt} - r_{dt})\, k_{jt}$$
$$- (r_{ft} - r_{dt})\, \phi_t b^f_{jt} - \chi\,(l_{jt}) - \phi_{t+1}\tau_{bt+1}, \qquad \text{(Problem A)}$$

subject to

$$k_{jt} - \kappa\,(l_{jt} + q_t s_{jt} + \phi_t f_{jt}) \geq 0,$$

[9] The extra unit of loan given by the bank to the proportion ξ of the entrepreneurs is paid back to the bank later in the period at zero interest rate. For this reason, these extra loans do not appear in the objective function (profits) of the bank.

[10] Note that we assume that banks cannot, or are not allowed to, short securities (i.e., $s_{jt} \geq 0$ for all j and all t).

$$\phi_t f_{jt} - \rho \left(l_{jt} + q_t s_{jt} + \phi_t f_{jt} - \phi_t b^f_{jt} - k_{jt} \right) \geq 0,$$

$$s_{jt} + (1 + r_{ior,t}) \phi_t f_{jt} - \theta (1 + r_{dt}) \left(l_{jt} + q_t s_{jt} + \phi_t f_{jt} - \phi_t b^f_{jt} - k_{jt} \right) - \xi l_{jt} - (1 + r_{ft}) \phi_t b^f_{jt} \geq 0,$$

and $s_{jt} \geq 0$.

Let λ_t, μ_{1t}, and μ_{2t} be the Lagrange multipliers for the three constraints in Problem A, respectively. All three multipliers are greater than or equal to zero, by definition. Then, the first order conditions for the bank's problem are:

$$\begin{aligned}
l_{jt} &: \quad (r_{lt} - r_{dt}) - \chi' (l_{jt}) - \kappa \lambda_t - \rho \mu_{1t} - [\theta (1 + r_{dt}) + \xi] \mu_{2t} = 0, \\
q_t s_{jt} &: \quad (r_{qt} - r_{dt}) - \kappa \lambda_t - \rho \mu_{1t} + [(1/q_t) - \theta (1 + r_{dt})] \mu_{2t} \leq 0, \\
\phi_t f_{jt} &: \quad (r_{iort} - r_{dt}) - \kappa \lambda_t + (1 - \rho) \mu_{1t} + [(1 + r_{iort}) - \theta (1 + r_{dt})] \mu_{2t} = 0, \\
k_{jt} &: \quad - (r_{kt} - r_{dt}) + \lambda_t + \rho \mu_{1t} + \theta (1 + r_{dt}) \mu_{2t} = 0, \\
b^f_{jt} &: \quad - (r_{ft} - r_{dt}) + \rho \mu_{1t} + [\theta (1 + r_{dt}) - (1 + r_{ft})] \mu_{2t} = 0.
\end{aligned}$$

We will use these conditions in the characterization of equilibrium presented below. It is clear from the first order condition with respect to capital that these equations are consistent only with the case in which $r_{kt} \geq r_{dt}$. If $r_{kt} < r_{dt}$, it is easy to show that the bank would finance itself all with capital (that is, $d_t = 0$). So, assuming that $d_t > 0$ implies that we are considering only equilibrium situations where $r_{kt} \geq r_{dt}$, but the reverse is not true. Even if $r_{kt} > r_{dt}$, the bank may choose to finance all its operations with capital. That is, the assumptions that result in $d_t > 0$ are stronger. We will provide conditions that guarantee that deposits are positive in all the equilibrium situations we consider.

2.5 The central bank

At every time t, the central bank controls the total supply of monetary assets MA_t in the economy. The sum of cash and reserves $M_t + F_t$ must equal MA_t for all t. The central bank can also buy and sell private securities in the market, but it does not create its own bonds (although this may be an interesting possibility to study). Finally, the central bank charges taxes to households and banks for a total of T_t at each time t.[11] The central bank's budget constraint, then, is given by:

$$\phi_t [M_{t-1} + (1 + i_{ior,t-1}) F_{t-1}] + q_t S_{c,t} = \phi_t MA_t + S_{c,t-1} + \phi_t T_t,$$

for all t and with M_0, F_0, and $S_{c,0}$ given. Note here that negative values of S_{ct} would correspond to the case when the central bank can issue bonds.

To fix ideas, it is interesting to consider a few special cases of this constraint. For example, if the central bank is paying interest on reserves and does not tax agents in the economy, nor does it own any securities, then it must be the case that the monetary assets in the economy are growing. This can be seen by using the fact that $MA_t = M_t + F_t$ for all t to substitute in the expression of the central bank's budget constraint when S_{ct} and T_t are equal to zero for all t. In that case, we have that:

$$MA_t - MA_{t-1} = i_{ior,t-1} F_{t-1} > 0.$$

Another interesting case is when the central bank holds no securities and is paying interest on reserves while keeping the stock of monetary assets constant. In that case, we have that:

$$T_t = i_{ior,t-1} F_{t-1} \quad \text{for all } t,$$

[11] Here, we are considering a consolidated budget constraint for the central bank and the fiscal authority. In principle, we could set up some notation to deal with the case when the central bank sends and receives remittances from the fiscal authority. The key assumptions here are that the fiscal authority has the power to charge lump-sum taxes/subsidies to agents and that those taxes are adjusted to accommodate the decisions of the central bank. Monetary and fiscal policy coordination is an important issue that has received considerable attention in the literature. For a recent treatment see Cochrane (2014), where the payment of interest on reserves is explicitly discussed.

that is, the central bank pays its interest expense with revenue from taxation.

Finally, consider the case when the central bank is not paying interest on reserves, nor taxing any agent, and is also not holding any securities at time $t-1$. Suppose now that the central bank conducts a one-time open market operation at time t to purchase S_{ct} securities financed with monetary assets. Then, the central bank's budget constraint implies that:

$$\phi_t \left(MA_t - MA_{t-1} \right) = q_t S_{ct} > 0,$$

that is, the increase in (central bank issued) monetary assets will exactly equal the nominal amount the central bank pays for the securities. In the following period, the central bank's budget constraint will be:

$$\phi_{t+1} \left(MA_{t+1} - MA_t \right) = -S_{ct} < 0,$$

so that monetary assets will decrease one period after the open market operation. To keep the size of the monetary assets from decreasing, the central bank would need to conduct a new open market operation (that is, the central bank needs to roll over its holdings of maturing assets).[12]

3. Equilibrium

We use a standard definition of competitive equilibrium, with free entry in banking. An equilibrium is a feasible allocation of resources and assets, and a set of prices and interest rates such that, given those prices and interest rates, the quantities associated to the equilibrium allocation solve the optimization problems for the corresponding agents, the central bank's budget constraint is satisfied, and banks make zero profits. We use capital letters to denote aggregate, economy-wide values of the relevant quantities in the equilibrium allocation.

We restrict attention to symmetric equilibrium in which all operating banks make identical decisions. Denote by $\left(l_{bt}, s_{bt}, f_{bt}, d_{bt}, k_{bt}, b^f_{bt} \right)$ the solution to the optimization problem of the representative bank. Then, for example, since we have that all banks provide the same amount of loans to entrepreneurs, the total supply of loans in the economy is given by:

$$L_t = \Gamma_t l_{bt}.$$

In the same way, we can aggregate across banks to compute banks' total demand for securities, $S_{bt} = \Gamma_t s_{bt}$, reserves, $F_t = \Gamma_t f_{bt}$, deposits, $D^D_t = \Gamma_t d_{bt}$, capital, $K_{bt} = \Gamma_t k_{bt}$, and the total demand for interbank loans, $B^f_{bt} = \Gamma_t b^f_{bt}$.

Since there is a measure one of households, the supply of deposits in the economy is given by $D_t = d_t$, where d_t is the level of deposits that corresponds to the individual household optimization problem. Similarly, $C_t = c_t$, $S_{ht} = s_{ht}$, $M_t = m_t$, and $S_t = s_t$.

Aggregating across experts we have that $X_{1t} = x_{1t}$, $X_{2t} = x_{2t}$, and $S_{et} = s_{et}$. We also have that the aggregate endowment of goods in the economy in period t is $W_t = W_{ht} + W_{et} = w_{ht} + w_{et}$.

The total output obtained from projects at time t is given by:

$$Y_t = \int_{r_{lt-1}}^{\overline{r}} (1+r)\, dG(r),$$

and total consumption of entrepreneurs at time t is given by:

$$Z_t = \int_{r_{lt-1}}^{\overline{r}} (r - r_{lt-1})\, dG(r)$$

[12] Since we are going to concentrate attention only on stationary equilibria, we do not discuss here any policy rule potentially followed by the central bank in the short run. The properties of those policy rules, however, are important for the stability of stationary equilibria. See Hornstein (2010) for a detailed treatment of this issue.

There are seven market clearing conditions. Securities market clearing is given by $S_{ht}+S_{bt}+S_{et}+S_{ct}=S_t$. Loans market clearing is given by $L_t = L^D(r_{lt}) = 1-G(r_{lt})$. Deposit market clearing is $D_t^D = D_t$, and bank capital market clearing is $K_{bt} = K_{et} = k_{et}$. The interbank market clears when $B_{bt}^f = 0$ because interbank loans are in zero net supply; and market clearing for monetary assets imply that $M_t + F_t = MA_t$.[13]

Total (nominal) revenue from taxes is given by $T_t = \tau_{ht} + \Gamma_t \tau_{bt}$. Using Walras Law, we have that, in equilibrium, the goods market clears; that is:

$$C_t + X_{1t} + X_{2t} + Z_t + L_t + \Gamma_{t-1}\chi(l_{bt-1}) = W_t + S_{t-1} + Y_t.$$

Recall from the household problem that restricting attention to equilibria with $d_t > 0$ for all t, implies that $r_{dt} = r_{qt}$ for all t in those equilibria.

3.1 Price level determination

As is common in monetary models, the demand for currency M_t is pinned down whenever i_{dt} is (strictly) positive. As we saw when introducing the households' problem, if $i_{dt} > 0$, then $n_t = 0$ and $\phi_t M_t = W_{ht}$ in equilibrium.[14] From the money market clearing condition we have that $M_t = MA_t - F_t$. Hence, we have that:

$$\phi_t (MA_t - F_t) = W_{ht}.$$

After a change in MA_t, if F_t changes in the same amount (given W_{ht}), the price level could remain unchanged. In other words, if all the extra monetary assets are held by the banks in the form of reserves, then the price level in the model does not need to adjust to changes in the supply of monetary assets. It is important, however, to understand that increases in reserve holdings F_t when the price level is constant (when ϕ_t constant) imply that banks are holding a stock of reserves with a higher total real value. So, in that case, the question would be whether banks can accommodate the real value of the extra reserves, and what are the implications of that adjustment, if any, for the real allocation of resources.

4. Stationary equilibrium

Even though we try to keep the model as simple as possible, there are still several moving parts that interact in general equilibrium. To understand this interaction better, it is useful to restrict attention to stationary equilibrium. Since our interest is in the long-run level of prices consistent with equilibrium, rather than their short-run behavior, studying stationary equilibrium is a natural first step. For this purpose, assume that $W_{ht} = W_h$, $W_{et} = W_e$, and $S_t = S$ for all t. Furthermore, let $MA_t = (1+\gamma) MA_{t-1}$ for all t with $\gamma \geq 0$ and MA_0 given. Also, let the real value of taxes $\phi_t \tau_{bt} \equiv \widehat{\tau}_b$, $\phi_t \tau_{ht} \equiv \widehat{\tau}_h$ and the nominal interest paid on reserves $i_{ior,t}$ be constant over time. Finally, assume that there is a unique solution to the equation:

$$\chi'(l) = \frac{\chi(l) + \widehat{\tau}_b}{l},$$

and call that solution $l^o(\widehat{\tau}_b) > 0$.

Even under these assumptions, depending on parameter values, stationary outcomes can display several possible equilibrium configurations. In this section, we study the cases that appear most relevant. We start by considering the case when the central bank pays no interest on reserves (as the Fed did before October 2008). We show that at least one of the constraints in Problem A must

[13] For a macroeconomic model where the interbank market plays an active role see Gertler and Kiyotaki (2010).

[14] If $i_{dt} = 0$, then n_t could be greater than zero and a continuum of possible values of M_t are consistent with equilibrium. While this is an interesting monetary phenomenon that has attracted some attention in the literature, we do not discuss it here (see, for example, Woodford, 1994). If we want to consider the case when $i_{dt} = 0$, then assuming that $n_t = 0$ is consistent with equilibrium and allows us to abstract from this well-known monetary indeterminacy issue.

be binding and consider the case of binding reserve requirements and the case of binding capital constraints. Then, we turn attention to the case when the central bank pays interest on reserves and consider the case when the capital constraint is binding and the case when none of the constraints are binding. In this last case is where price level indeterminacy arises.

To simplify the analysis, we will assume that experts are risk neutral and consume only in their second period of life. As we discussed when introducing the bank's problem, we will concentrate attention only on equilibrium with positive bank deposits. From the consumer's problem, if $d_t = d > 0$ for all t, then $1 + r_d = 1 + r_q = 1/\beta$ for all t. This implies that $r_d > 0$ and hence $n_t = 0$ for all t. Furthermore, in a stationary equilibrium M_t/F_t is constant and hence both (nominal) variables M_t and F_t are growing at rate γ. Since $n_t = 0$ for all t, we have that

$$\phi_t M_t = W_h = \phi_{t+1} M_{t+1},$$

which implies that $\phi_{t+1}/\phi_t = 1/(1+\gamma)$.[15]

4.1 No interest on reserves

If the central bank is not paying interest on reserves, then $i_{ior,t} = 0$ for all t and $1+r_{ior} = 1/(1+\gamma) \leq 1$ so the net real interest on reserves is negative whenever γ is positive (that is, whenever inflation is positive). To hold reserves, banks have to fund them with either deposits or capital. If the return on reserves is lower than the cost of funding them, then it cannot be that banks hold more reserves than the ones they require to satisfy reserve requirements. The following lemma formalizes this logic.

Lemma 1. If $i_{ior} = 0$, then in any stationary equilibrium with $d > 0$ at least one of the constraints in Problem A is binding.

Proof. Suppose not. Then, from the first order conditions of the bank problem, we have that $r_{ior} = r_d = r_k = r_f$. But we know that $1 + r_d = 1/\beta$ and $1 + r_{ior} = 1/(1+\gamma) \leq 1 < 1/\beta$, so we reach a contradiction. ■

In principle, any one of the constraints in Problem A can be binding in equilibrium. Here we will study the case when the reserve requirement and the bank capital constraint are binding. We relegate to the appendix the study of the case when the liquidity constraint is binding.

Binding reserve requirements

Consider first the case when *only* the reserve requirement constraint is binding.

Proposition 1. Let $i_{ior} = 0$. Given a value of $\rho \in (0,1)$, there are threshold values $\overline{\kappa}(\rho) \in (0,1), \overline{\theta}(\rho) \in (0,1)$ and a boundary $\overline{\xi}(\theta;\rho) > 0$, such that for all $\kappa < \overline{\kappa}(\rho)$, $\theta < \overline{\theta}(\rho)$ and $\xi < \overline{\xi}(\xi;\rho)$ there exists a unique stationary (monetary) equilibrium where the bank reserve requirement constraint is binding and the bank capital and bank liquidity constraints are not binding.

Proof. From the bank's problem we have that in equilibrium:

$$r_k^* = r_d + \frac{\rho}{1-\rho}(r_d - r_{ior}) = r_f^*,$$

that is, the cost for the bank of funding its lending with capital or fed funds is the same as that of funding its lending with deposits. Given this, the objective of the bank can be expressed as $(r_l - r_k)\, l - \chi(l) - \widehat{\tau}_b$. Using the first order condition with respect to l and the zero profit condition we have that the optimal value of l, denoted l^*, equals $l^o(\widehat{\tau}_b)$. Then, $r_l^* = r_k^* + \chi'(l^*)$ and $\Gamma^* = [1 - G(r_l^*)]/l^*$.

[15] The value of money in this economy is not just determined by the demand for money by households. Every period, banks demand reserves to (at least) satisfy their reserve requirements. If households believe that money will not have value next period, they may not sell their endowment. In general, this opens the door to the possibility of a non-monetary equilibrium. Here, since banks always demand reserves when deposits are positive, the inverse of the price level in equilibrium is always positive.

Since experts are risk neutral and $r_k^* > r_d^*$, we have that $\Gamma^* k_b^* = W_e$. Furthermore, $r_k^* > r_d^*$ implies that banks do not hold securities and $S_b = 0$. Now using the banks' balance sheet conditions and aggregating across banks we have that:[16]

$$\phi_t^* F_t^* = \rho \phi_t^* D_t^* = \frac{\rho}{1-\rho}\left(L^* - W_e\right), \tag{4}$$

and total assets in the banking system are given by:

$$L^* + \phi_t^* F_t^* = L^* + \frac{\rho}{1-\rho}\left(L^* - W_e\right),$$

where we are using that $L^* = \Gamma^* l^*$. Define κ_l^* the equilibrium capital to loan ratio in the banking system. Since the real value of total deposits is positive in equilibrium, then $L^* > W_e$ and $\kappa_l^* = W_e/L^* < 1$. Now make $\overline{\kappa}(\rho)$ equal to the equilibrium capital to assets ratio. That is:

$$\overline{\kappa}(\rho) = \frac{W_e}{L^* + \frac{\rho}{1-\rho}\left(L^* - W_e\right)} = \frac{(1-\rho)\kappa_l^*}{1-\rho\kappa_l^*}.$$

Since $\kappa_l^* < 1$ we have that $\overline{\kappa} \in (0,1)$. Clearly, then, as asserted in the statement of the proposition, whenever $\kappa < \overline{\kappa}$ the bank capital constraint is not binding.

Similarly, define $\overline{\theta}(\rho)$ as the ratio:

$$\overline{\theta}(\rho) = \frac{\rho\left(1+r_{ior}\right)}{\left(1+r_d\right)}.$$

Clearly $\overline{\theta}(\rho) \in (0,1)$. Now, for each pair (ρ, θ) let

$$\overline{\xi}(\theta;\rho) = \frac{\left[\left(1+r_{ior}\right)\rho - \theta\left(1+r_d\right)\right]}{1-\rho}\left(1-\kappa_l^*\right).$$

Clearly if $\theta < \overline{\theta}(\rho)$ and $\xi < \overline{\xi}(\theta;\rho)$, then the bank liquidity constraint is not binding in equilibrium. ■

Corollary 1.1 (Price level determination). Given a sequence for MA_t set by the central bank, the price level in the equilibrium described in Proposition 1 is uniquely determined and proportional to the quantity of monetary assets.

Proof. Since $\phi_t^* M_t = W_h$ and equation (4) hold, we have that the following equation must hold in equilibrium:

$$\phi_t^* MA_t = \phi_t^* M_t + \phi_t^* F_t^* = W_h + \frac{\rho}{1-\rho}\left(\Gamma^* l^* - W_e\right).$$

Hence, there is an inverse relationship between the total amount of monetary assets and the inverse of the equilibrium price level. ■

It is important to understand that the central bank in this model sets the value of MA_t only; how much of MA_t is dedicated to currency and how much to reserves is endogenously determined. In the stationary equilibrium of Proposition 1, where reserve requirements are binding, there is a one-to-one link between the price level and the quantity of monetary assets supplied by the central bank. This result is in the spirit of the traditional Quantity Theory of Money. We will see later in Proposition 4 that there are situations in which this one-to-one equilibrium relationship between money and prices does not necessarily hold.

Corollary 1.2 (Rates of return). In the equilibrium of Proposition 1 we have that $r_k^* = r_f^* > r_d^* = r_q^* > r_{ior}^*$ and

$$r_f^* = r_{ior}^* + \frac{1}{1-\rho}\left(r_d^* - r_{ior}^*\right).$$

[16] It is easy (but somewhat messy) to provide conditions on $\gamma, \widehat{\tau}_b, \beta, W_e$, and $G(r)$ so that $\Gamma^* l^* > W_e$. This guarantees that deposits are positive, something we assumed from the beginning.

Proof. The proof is straightforward from the first order conditions to the bank's problem and the fact that the multipliers λ_t and μ_{1t} are positive and constant for all t. $\blacksquare$

Even though capital appears to be a more expensive source of funding than deposits (i.e., $r_k^* > r_d^*$), in effect for the bank it is not. Deposits are indirectly "taxed" by the imposition of reserve requirements and the policy of not paying interest on reserves (Lacker, 1987; and Kashyap and Stein, 2012). Once the implicit tax is taken into consideration, as the optimizing bank does, then both forms of funding (capital and deposits) have the same cost.

Binding capital constraints

Suppose now that $\kappa > \overline{\kappa}(\rho)$ but strictly lower than unity, since we are considering equilibria where bank deposits are positive. We are interested in understanding a situation where the capital constraint is binding, while the liquidity constraint is not.

Lemma 2. When $i_{ior} = 0$, if the bank capital constraint is binding in a stationary equilibrium with $d > 0$ and the bank liquidity constraint is not binding, then the reserve requirement constraint must also be binding.

Proof. Suppose not. Then the bank's first order condition with respect to reserves is $r_{ior} - r_d = \kappa\lambda > 0$. But we know that $1 + r_d = 1/\beta$ and $1 + r_{ior} = 1/(1+\gamma) \leq 1 < 1/\beta$ so we reach a contradiction. $\blacksquare$

Given the result in Lemma 2, we only need to consider the case when both the capital and the reserves constraints are binding.[17] This is the situation characterized in the following proposition.

Proposition 2. Let $i_{ior} = 0$. Given a value of $\rho \in (0,1)$ and the threshold values $\overline{\kappa}(\rho)$ and $\overline{\theta}(\rho)$ (as defined in Proposition 1), for all $\kappa \in (\overline{\kappa}(\rho), 1)$ and $\theta < \overline{\theta}(\rho)$ there is a boundary $\overline{\overline{\xi}}(\theta; \rho, \kappa) \in (0,1)$, such that for all $\xi < \overline{\overline{\xi}}(\theta; \rho, \kappa)$ there exists a unique stationary (monetary) equilibrium where the bank capital and the reserve requirement constraints are binding and the bank liquidity constraint is not binding.

Proof. It is still the case that in equilibrium $r_d = r_q$. Since we are considering the case when the capital and reserves constraints are binding, it is immediate from the first order condition of the bank's problem that $s_b = 0$. Now, using the bank's balance sheet condition and the capital and reserves constraints, both holding with equality, we have that bank capital and the real value of reserves are proportional to bank loans in equilibrium. That is:

$$\phi_t f_{bt} = \rho\phi_t d_{bt} = \frac{\rho(1-\kappa)}{1-\rho(1-\kappa)} l_b, \qquad \text{and} \qquad k_b = \frac{\kappa}{1-\rho(1-\kappa)} l_b.$$

Now, for each triple (θ, ρ, κ) let

$$\overline{\overline{\xi}}(\theta; \rho, \kappa) = \left[(1 + r_{ior})\rho - \theta(1 + r_d)\right] \frac{1-\kappa}{1-\rho(1-\kappa)}.$$

Using these expressions to substitute in the liquidity constraint, it is easy to verify that, if $\theta < \overline{\theta}(\rho)$ and $\xi < \overline{\overline{\xi}}(\theta; \rho, \kappa)$, then the bank liquidity constraint is not binding.

To find the rest of the equilibrium values, define the weighted average of rates of return r_b as follows:

$$r_b = \frac{1-\kappa}{1-\rho(1-\kappa)} r_d - \frac{\rho(1-\kappa)}{1-\rho(1-\kappa)} r_{ior} + \frac{\kappa}{1-\rho(1-\kappa)} r_k.$$

[17] The result in Lemma 2 can be generalized. In particular, whenever the capital constraint is binding, the reserves constraint must be binding. Lemma A1 in the appendix shows that when $i_{ior} < i_d$ if both the capital and the liquidity constraints are binding, then the reserves constraint must also be binding.

The objective of the bank can then be written as $(r_l - r_b)\, l - \chi(l) - \widehat{\tau}_b$. Taking first order condition with respect to l and using the zero profit condition, we have that $l^* = l^o(\widehat{\tau}_b)$. Now using the market clearing condition for bank capital and for loans we have that:

$$W_e = \Gamma^* k_b^* = \frac{\kappa}{1-\rho(1-\kappa)} \Gamma^* l^* = \frac{\kappa}{1-\rho(1-\kappa)} \left[1 - G(r_l^*)\right],$$

which can be used to determine the equilibrium level of r_l^* and Γ^*. Since $r_b^* = r_l^* - \chi'(l^*)$ we can use the definition of r_b to determine the equilibrium level of r_k^*. The rest of the equilibrium values can be obtained by straightforward substitution. ■

Corollary 2.1 (Price level determination). Given a sequence for MA_t set by the central bank, the price level in the equilibrium described in Proposition 2 is uniquely determined and proportional to the quantity of monetary assets.

Proof. Since $\phi_t^* M_t = W_h$ and $\phi_t^* F_t^* = [\rho(1-\kappa)/\kappa]\, W_e$, we have that following equation must hold in equilibrium:

$$\phi_t^* MA_t = W_h + \frac{\rho(1-\kappa)}{\kappa} W_e,$$

and hence there is an inverse relation between the total amount of monetary assets and the inverse of the price level ϕ_t, as in the equilibrium of Proposition 1. ■

Corollary 2.2 (Rates of return). In the equilibrium of Proposition 2 we have that $r_k^* > r_f^* > r_d^* = r_q^* > r_{ior}^*$. Furthermore, the equilibrium interest rate on interbank loans is given by:

$$r_f^* = r_{ior}^* + \frac{1}{1-\rho(1-\kappa)} \left(r_d^* - r_{ior}^*\right) + \frac{\rho\kappa}{1-\rho(1-\kappa)} \left(r_k^* - r_{ior}^*\right).$$

A look at some relevant data (1995-2005)

We now discuss some U.S. data for a period when the Fed was not paying interest on reserves and contrast the data with predictions from the model. Even though the Fed started paying interest on reserves in 2008, we stop the data period for this section in 2005 to concentrate on a relatively stable time in financial markets. Figure 3 shows total and required reserves in the banking system. This data suggest that considering the case where reserve requirements are binding is a good first approximation for this time period.

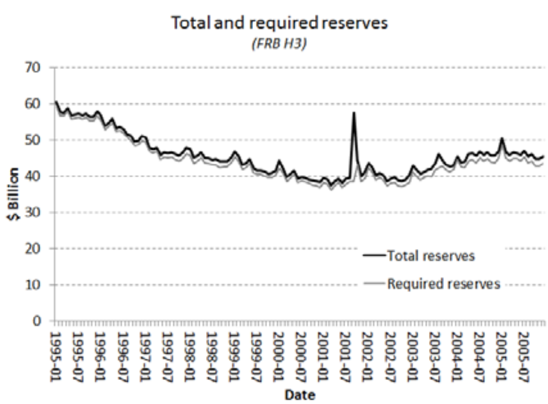

Figure 3

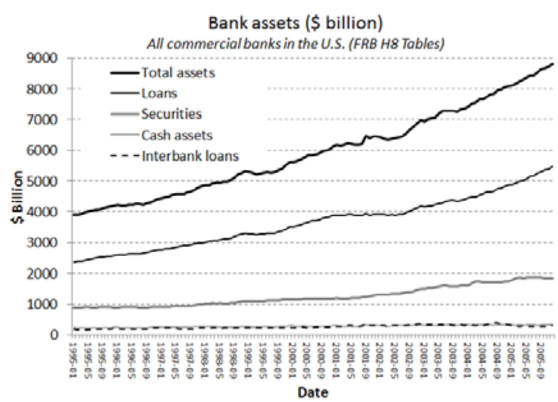

Figure 4

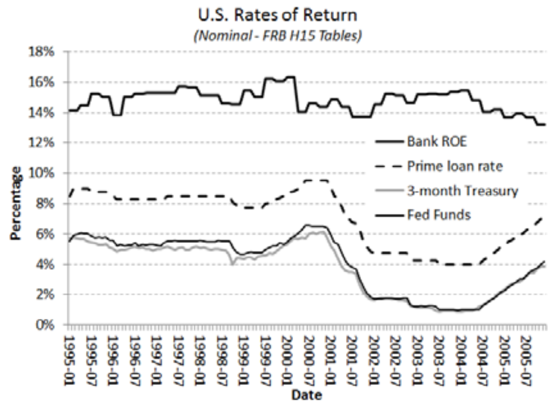

Figure 5

Figure 4 shows that banks held around 20% of their assets in securities during this period of zero interest on reserves. This does not correspond with the predictions of the model in Propositions 1 and 2. The reasons for this gap are clear: Securities and deposits play the same store-of-value role in the consumer's problem; since consumers are pricing the securities in the market and they hold both securities and deposits in equilibrium, both saving instruments must have the same rate of return in equilibrium. A bank, on the other hand, to hold securities needs to hold more deposits *and* more reserves, which makes the benefits from holding securities (r_q) lower than the costs (as reserves pay no interest). There are two ways to fix this shortfall of the model: One way is to lower the cost of deposits for the bank by allowing deposits to provide a liquidity/transaction service to consumers (as in Van den Heuvel, 2008, Stein, 2012, Ireland, 2013, or Begenau, 2013). We pursue this alternative formulation in Section 5. The other way is to have securities provide an extra benefit to the bank, which can happen when the liquidity constraint is binding (see Proposition A1 in Part 1 of the Appendix).

To look at rates of return during this period and compare them with the predictions of the model is complicated. For example, a possible approach to approximate the return on bank capital r_k is to consider the average return on equity for banks. However, banking in the model is riskless and the appropriate adjustment for risk to the observed return on equity is hard to determine. Subject to this caveat, Figure 5 shows some of the relevant rates of return to assess the comparison in corollaries 1.2 and 2.2. We see that the return on equity of banks is the highest of the rates of return involved. Also, the fed funds rate (which stands for r_f in the model) moves very closely with the rate on short term treasuries, the counterpart of r_q in the model.

Empirically approximating the effective interest rate on deposits is difficult. There are many different types of deposits and even within each type, different banks offer to their clients different bundles of services that may include a special pricing for deposits (see also the discussion in Part 5 of the appendix). Based on an arbitrage argument, the model suggests that deposits and short term Treasuries should yield comparable returns ($r_d = r_q$). We conclude from these observations that the ordering of rates of return is most in line with the predictions in Corollary 2.2.

4.2 Interest on reserves

When the central bank pays interest on reserves, the strong result in Lemma 1 is no longer true. That is, there are situations where none of the three constraints in Problem A are binding. This is an interesting case to study to the extent that, as we will see, such a situation is consistent with increases in monetary assets that are unmatched by changes in the price level. But, before turning to that particular case, we will discuss the implications of paying interest on reserves more generally.

Let us start with those cases when $1 + r_{ior} \in (1/(1+\gamma), 1/\beta)$. For any interest on reserves within this range, if κ, θ, and ξ are small enough, a similar equilibrium to the one described in Proposition 1 exists. To see this, consider the ratio of capital to loans in the banking system, κ_l^*, as defined in the proof of Proposition 1. Clearly, κ_l^* is a function of the interest on reserves because

L^* is a decreasing function of r_l^*, which is an increasing function of r_k^*, which in turn is a decreasing function of r_{ior} (see the proof of Proposition 1). Since $1 + r_{ior} = (1 + i_{ior}) / (1 + \gamma)$, we have that a higher nominal interest on reserves implies a lower r_k^* and r_l^* and a larger banking system (a larger L^*). In summary, then, for a given ρ, we can define the equilibrium capital to loans ratio as a decreasing function of the interest on reserves; that is, $\kappa_l^* (i_{ior}; \rho)$ decreasing in i_{ior}. In line with the proof of Proposition 1, define now the following threshold for the capital requirement κ:

$$\overline{\kappa} (\rho; i_{ior}) = \frac{(1-\rho) \kappa_l^* (i_{ior}; \rho)}{1 - \rho \kappa_l^* (i_{ior}; \rho)}. \tag{5}$$

It is now easy to verify that given a value of $\rho \in (0, 1)$, whenever $\kappa < \overline{\kappa} (\rho; i_{ior})$, $\theta < \overline{\theta} (\rho, i_{ior})$ and $\xi < \overline{\xi} (\xi; \rho, i_{ior})$ there is a unique stationary (monetary) equilibrium where the bank reserves requirement is binding and the bank capital and bank liquidity constraints are not binding.[18]

As in Proposition 1 (see Corollary 1.2), we have that $r_k^* = r_f^* > r_d^* > r_{ior}^*$ and we can express the *nominal* interest rate on interbank loans, the equivalent to the fed funds rate, as follows:

$$i_f^* = i_{ior} + \frac{1}{1-\rho} (i_d^* - i_{ior}) = \frac{1}{1-\rho} i_d^* - \frac{\rho}{1-\rho} i_{ior}. \tag{6}$$

Clearly, the interest on reserves acts as a floor for the interbank rate. However, increases in the interest rate paid on reserves tend to decrease the interbank rate. This result is in sharp contrast with the standard result in the Poole model of demand for reserves, where the interest on reserves and the fed funds rate move (*ceteris paribus*) in the same direction (see, for example, Ennis and Keister, 2008).

The mechanism behind this relationship between equilibrium interest rates is clear. Paying higher interest on reserves makes reserves a better asset, which reduces the cost of funding via deposits (since deposits require holding reserves to satisfy reserve requirements). Unless the interbank rate falls, all banks would want to attract deposits and lend the funds out in the interbank market, which would be inconsistent with the clearing of that market.

To make this logic more evident, we can re-write expression (6) as follows:

$$i_d^* = \rho i_{ior} + (1-\rho) i_f^*.$$

The left-hand side is the cost for the bank of attracting one extra unit of deposits; that is, i_d^*. The right-hand side is the benefits: To hold one extra unit of deposits, the bank has to hold ρ units of reserves, which get a return of i_{ior} and can lend the rest $(1-\rho)$ in the interbank market at rate i_f^*. Since the equilibrium level of i_d^* is independent of i_{ior} in this model, higher values for i_{ior} result in lower equilibrium values of i_f^*.

When the capital constraint is not binding, the return on capital is the marginal cost of funding for the bank, which can be written as:

$$r_k^* = r_d^* + \frac{\rho}{1-\rho} (r_d^* - r_{ior}^*).$$

That is, the marginal cost of funding for the bank equals the cost of deposits plus the "implicit tax" on deposits originated in the fact that the interest on reserve is lower than the interest on deposits. Managing this tax is at the heart of the monetary policy prescriptions advanced, for example, by Kashyap and Stein (2012). Here, total lending is a decreasing function of the spread $r_d^* - r_{ior}^*$ and, hence, by changing this spread the monetary authority can influence banks' lending activities.

If the capital ratio parameter κ is greater than $\overline{\kappa} (\rho; i_{ior})$, then the equilibrium is analogous to that described in Proposition 2. Since $\overline{\kappa} (\rho; i_{ior})$ is decreasing in i_{ior} we have that, for a given value of κ, higher levels of the interest on reserves make more likely that the capital constraint would be

[18] Here, again, we are using $\overline{\theta} (\rho, i_{ior})$ and $\overline{\xi} (\xi; \rho, i_{ior})$ as were defined in the proof of Proposition 1 and we only change notation to make explicit that the thresholds now depend on i_{ior}, which we no longer considered fixed at zero (as we did in Proposition 1).

binding. This is a natural consequence of the fact that when only the reserve requirement is binding, higher levels of interest on reserves are associated with a banking system that has a larger balance sheet.

Interestingly, once the capital constraint becomes binding, this is no longer the case: Instead, higher levels of the interest rate on reserves do not change the size of the banking system but only induce changes in the equilibrium value of the various rates of return. In particular, higher interest on reserves imply higher return on bank capital, r_k (see Proposition 2).

So far, we have considered cases where the interest paid on reserves is the lowest among the prevailing rates in the economy. In such situations, reserve requirements are binding and banks do not hold excess reserves. To think about large holdings of excess reserves in the model, we need to consider values of the interest on reserves such that $r_{ior} \geq r_d$. We turn our attention to those cases next.

Excess reserves

We want to study situations where the banking system is holding plentiful of reserves, over and above their requirements. We will abstract from the liquidity constraint in the analysis but point out that given the large quantity of reserves, the liquidity constraint is relatively slack, and conditions for it to not be binding are easy to establish.

Lemma 3. If $i_{ior} > i_d$ for all t, then in any stationary equilibrium with $d > 0$ the capital constraint is binding.

Proof. This is a direct implication of the first order conditions for Problem A. Given that $\phi_t d_t > 0$, the reserve requirement constraint implies that $\phi_t f_t \geq \rho \phi_t d_t > 0$ and the first order condition for bank reserve holdings is:

$$(r_{ior} - r_d) - \kappa\lambda + (1-\rho)\,\mu_1 = 0,$$

where we are using that $\mu_2 = 0$, since we are considering the case when liquidity constraints are not binding. The only way that the first order condition for bank reserves can hold is if $\lambda > 0$, that is, if the capital constraint is binding. ■

Since we are interested in situations with excess reserves, let us concentrate attention on the case when *only* the capital constraint is binding. Recall that from the consumer's problem, with positive equilibrium deposits, the rates of return on deposits and securities satisfy $1 + r_d = 1 + r_q = 1/\beta$. Given that $\lambda > 0$, we have that $r_q < r_d + \kappa\lambda$, and hence the banking system does not hold securities; that is, $S_b = 0$ in equilibrium. Since the banking system is holding (excess) reserves, it must be the case that $r_{ior} > r_d$. Also from the bank's problem, we have that:

$$r_k = r_d + \frac{1}{\kappa}\,(r_{ior} - r_d) > r_q,$$

so experts do not hold securities in equilibrium and $K_e = W_e$ in equilibrium.

Using the equilibrium value for the rates of return and the fact that the binding capital constraint implies that $\phi_t f_t = (k/\kappa) - l$ we have that the representative bank chooses loans to maximize $(r_l - r_{ior})\,l - \chi\,(l) - \widehat{\tau}_b$. Together with the zero profit condition, this determines that $l = l^o\,(\widehat{\tau}_b)$ and $L^*\,(i_{ior}) = 1 - G\,(r_l^*\,(i_{ior}))$ where $r_l^*\,(i_{ior}) = r_{ior}^* + \chi'\,(l^*)$. Define again $\kappa_l^*\,(i_{ior}) = W_e/L^*\,(i_{ior})$.[19] With these values, we are ready to characterize the situation when banks hold excess reserves even though capital is scarce in the economy.

Proposition 3. Let $i_{ior} > i_d$. Given a value of $\rho \in (0,1)$, there is a threshold value $\overline{\kappa}\,(i_{ior},\rho) > 0$ and, for all $\kappa < \overline{\kappa}\,(i_{ior},\rho)$, a non-empty set $\Omega\,(i_{ior},\rho,\kappa)$ of values of $\theta > 0$ and $\xi > 0$ such that for all $\kappa < \overline{\kappa}\,(i_{ior},\rho)$ and $(\theta,\xi) \in \Omega\,(i_{ior},\rho,\kappa)$ there exists a unique stationary (monetary) equilibrium where the bank capital constraint is binding and banks hold excess liquidity and excess reserves.

[19] Note here that $r_l^*\,(i_{ior})$ is different than in Proposition 1, so the values of κ_l^* and $\overline{\kappa}\,(i_{ior},\rho)$ are also different.

Proof. Define the threshold value $\overline{\kappa}(i_{ior},\rho)$ as:

$$\overline{\kappa}(i_{ior},\rho) = \min\left\{1, \frac{(1-\rho)\,\kappa_l^*(i_{ior},\rho)}{1-\rho\kappa_l^*(i_{ior},\rho)}\right\}$$

and the set $\Omega(i_{ior},\rho,\kappa)$ for $\kappa < \overline{\kappa}(i_{ior},\rho)$ as:

$$\Omega(i_{ior},\rho,\kappa) = \left\{(\theta,\xi) \mid \theta < \overline{\theta}(i_{ior},\rho,\kappa) \text{ and } \xi < \overline{\overline{\xi}}(i_{ior},\rho,\kappa,\theta)\right\},$$

where

$$\overline{\theta}(i_{ior},\rho,\kappa) = \frac{(1+r_{ior})(\kappa_l^*-\kappa)}{(1+r_d)(1-\kappa)\,\kappa_l^*},$$

and

$$\overline{\overline{\xi}}(i_{ior},\rho,\kappa,\theta) = \frac{1}{\kappa}\left[(1+r_{ior})(\kappa_l^*-\kappa) - \theta(1+r_d)(1-\kappa)\,\kappa_l^*\right].$$

Since $\kappa_l^*(i_{ior},\rho) > 0$ we have that $\overline{\kappa}(i_{ior},\rho) > 0$. Similarly, for $\kappa < \overline{\kappa}(i_{ior},\rho)$ we have that $(\kappa_l^*-\kappa)/(1-\kappa)\,\kappa_l^* > \rho > 0$ so $\overline{\theta}(i_{ior},\rho,\kappa) > 0$ and, clearly, for $\theta < \overline{\theta}(i_{ior},\rho,\kappa)$ we have that $\overline{\overline{\xi}}(i_{ior},\rho,\kappa,\theta) > 0$. So, $\Omega(i_{ior},\rho,\kappa)$ is non-empty for $\kappa < \overline{\kappa}(i_{ior},\rho)$. If only the capital constraint is binding, then $\phi_t F_t = (W_e/\kappa) - L^*$ and $\phi_t D_t = ((1-\kappa)/\kappa)\,W_e$. When $\kappa < \overline{\kappa}(i_{ior},\rho)$, we have that $\phi_t F_t > \rho\phi_t D_t$ so the reserve requirements are not binding. Similarly, it is easy to check that when $\kappa < \overline{\kappa}(i_{ior},\rho)$ and $(\theta,\xi) \in \Omega(i_{ior},\rho,\kappa)$ the liquidity constraint is also not binding. The rest of the equilibrium values can be easily obtained by straightforward substitution. ■

The condition that $\kappa \in (0,\overline{\kappa}(\rho;i_{ior}))$ can be translated into a condition over ρ; that is, $\rho \in (0,\overline{\rho}(\kappa;i_{ior}))$ (see Figure 6). When $\rho > \overline{\rho}(\kappa;i_{ior})$ the reserve requirement is too high for the equilibrium in Proposition 3 to exists. Instead, the equilibrium has both the capital and the reserve requirement constraints binding (that is, no excess reserves; see Part 2 of the Appendix for a description of this case).

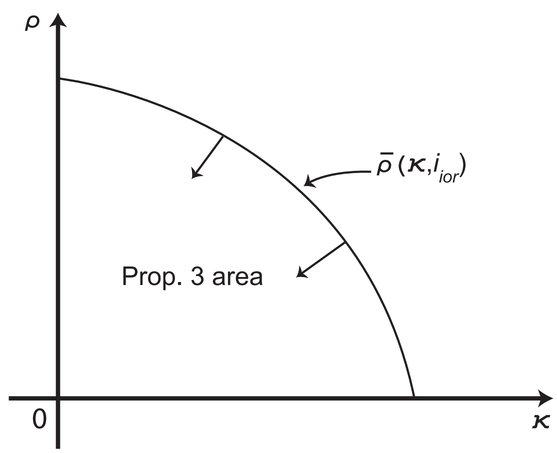

Figure 6

Corollary 3.1 (Price level determination). Given a sequence for MA_t set by the central bank, the price level in the equilibrium described in Proposition 3 is uniquely determined and proportional to the quantity of monetary assets.

Proof. Since $i_d > 0$, we have that $\phi_t^* M_t = W_h$. Using $\phi_t^* F_t^* = (W_e/\kappa) - L^*$ and given that L^* is independent of MA_t and ϕ_t, we have that prices ϕ_t satisfy:

$$\phi_t^* MA_t = W_h + \frac{1}{\kappa}W_e - L^*,$$

and hence there is an inverse relationship between the total amount of monetary assets and the inverse of the price level, as in the equilibrium of Proposition 1 and 2. ■

Corollary 3.2 (Rates of return). In the equilibrium of Proposition 3 we have that $r_k^* > r_{ior}^* > r_d^* = r_f^*$. Furthermore, the equilibrium interest rate on interbank loans is given by:

$$r_f^* = r_{ior}^* - \kappa \left(r_k^* - r_d^* \right).$$

This result is interesting because it tell us that even if the banking system is flushed with reserves, when bank capital is scarce the equilibrium interest rate on interbank loans is lower than the rate of interest on reserves.[20]

Paying interest on reserves at 'market rates'

Suppose now that the central bank pays interest on reserves at a level such that $r_{ior} = r_d$ in equilibrium. Given that the return from holding reserves is the same as the cost of obtaining extra deposits, if bank capital is scarce (and the bank capital constraint is binding) then the reserve requirement constraint will also be binding. Holding reserves requires holding more (expensive) capital and given that the return on reserves is the same as the cost of deposits, the return on reserves is not sufficient to compensate for the higher cost in terms of bank capital. However, if bank capital *is not* scarce, then banks will be indifferent with respect to their holdings of reserves and may increase them by becoming bigger or by holding fewer securities. The next two propositions describe these two situations formally, starting with the latter situation first, in Proposition 4, and then moving on to the situation with scarce capital in Proposition 5.

Proposition 4. Let $i_{ior} = i_d$. Given a value of $\rho \in (0, 1)$ there is a threshold value $\overline{\kappa} (i_{ior}, \rho) > 0$ and, for all $\kappa \in (0, \overline{\kappa} (i_{ior}, \rho))$, a non-empty set $\Omega (i_{ior}, \rho, \kappa)$ of values of $\theta > 0$ and $\xi > 0$ such that for all $\kappa < \overline{\kappa} (i_{ior}, \rho)$ and $(\theta, \xi) \in \Omega (i_{ior}, \rho, \kappa)$ there exists a continuum of equilibria where the bank capital constraint is not binding and banks hold excess liquidity and excess reserves. The equilibrium set induces a continuum of initial (inverse of the) price levels ϕ_0 that belong to the bounded segment $\left[\phi_0^L, \phi_0^U \right]$.

Proof. If none of the constraints in Problem A are binding in equilibrium, then we must have that $r_k^* = r_f^* = r_d^* = r_q^* = r_{ior}^*$. The solution to the bank's problem is given by $l^* = l^o (\widehat{\tau}_b)$ and hence we have that $r_l^* = r_{ior}^* + \chi' (l^*)$. Also, loan market clearing implies that $\Gamma^* = [1 - G (r_l^*)] / l^*$ and $L^* = \Gamma^* l^*$. Assume that $L^* > W_e$ and define the threshold value $\overline{\kappa} (i_{ior}, \rho)$ as follows:

$$\overline{\kappa} (i_{ior}, \rho) = \frac{(1 - \rho) \kappa_l^*}{1 - \rho \kappa_l^*},$$

where $\kappa_l^* = W_e / L^*$.[21]

Given the equality of equilibrium rates of return on different assets, there are many different possible sizes of the banking system's balance sheet that are consistent with equilibrium. The smallest possible banking system (measured by assets) is one where $S_b^* = 0$ and reserve holdings are minimal. This requires that $\phi_t^* F_t^* = \rho \phi_t^* D_t^*$ and that deposits are minimal. From the balance sheet condition of the banking system we have that:

$$\phi_t^* D_t^* = \frac{1}{1 - \rho} \left(L^* - K_e^* \right),$$

[20] Bech and Klee (2011) provide a different (but complementary) argument to explain such a relationship among rates of return. Their explanation is mainly based on limited competition in the interbank market when some participants have no access to the payment of interest on reserves by the monetary authority.

[21] Imposing conditions on parameters so that $L^* > W_e$ is straightforward in this case. Essentially, the equation $1 - G \left[\frac{i_{ior} - \gamma}{1 + \gamma} + \chi' \left(l^o \left(\widehat{\tau}_b \right) \right) \right] > W_e$ must hold. This is simply a condition on parameters and the value of i_{ior} set by the central bank.

and deposits are minimal when $K_e^* = W_e$. When $\kappa < \overline{\kappa}(i_{ior}, \rho)$ we have that

$$\frac{1}{1-\rho}(L^* - W_e) + W_e < \frac{W_e}{\kappa},$$

so the capital constraint is not binding.

The largest possible banking system has the largest possible deposits consistent with a non-binding capital constraint and bank capital less than or equal to W_e so that capital is not scarce in equilibrium. Using the banking system balance sheet and the capital constraint, we have that:

$$\phi_t^* D_t^* + K_e^* \leq \frac{K_e^*}{\kappa},$$

must hold. So, the largest possible deposits are:

$$\phi_t^* D_t^* = \frac{1-\kappa}{\kappa} W_e.$$

When $S_b^* = 0$, this banking system has excess reserves. In fact, this level of reserves achieves the maximum real value of reserves consistent with equilibrium. The level is given by:

$$\phi_t^* F_t^* = \frac{W_e}{\kappa} - L^* > \rho \frac{1-\kappa}{\kappa} W_e,$$

where the last inequality holds whenever $\kappa < \overline{\kappa}(i_{ior}, \rho)$.

Since $r_d^* = 1/\beta$ and $\gamma > 0$, we have that in these equilibria i_d^* is positive. Then, it follows that:

$$\phi_t^* MA_t = \phi_t^* (M_t^* + F_t^*) = W_h + \phi_t^* F_t^*,$$

which, for a given sequence of values of MA_t implies a one-to-one increasing relationship between the inverse of the price level and the real value of reserves. To obtain the limits of the interval of possible values of ϕ_0 we need to consider the cases when the equilibrium level of real reserves is minimal and maximal. Denote by $\phi_t^L = (1+\gamma)^t \phi_0^L$ the inverse of the price level that satisfies:

$$\phi_t^L = \frac{1}{MA_t}\left[W_h + \frac{\rho}{1-\rho}(L^* - W_e^*)\right],$$

and denote by $\phi_t^U = (1+\gamma)^t \phi_0^U$ the inverse of the price level that satisfies:

$$\phi_t^U = \frac{1}{MA_t}\left(W_h + \frac{W_e}{\kappa} - L^*\right).$$

It is easy to verify that when $\kappa < \overline{\kappa}(i_{ior}, \rho)$ we have that $\phi_t^L < \phi_t^U$.

Now define the set $\Omega(i_{ior}, \rho, \kappa_l^*)$ as:

$$\Omega(i_{ior}, \rho, \kappa) = \left\{(\theta, \xi) \mid \theta < \rho \text{ and } \xi < \overline{\overline{\xi}}(i_{ior}, \rho, \kappa_l^*, \theta)\right\},$$

where

$$\overline{\overline{\xi}}(i_{ior}, \rho, \kappa, \theta) = \frac{(1+r_{ior})(\rho-\theta)(1-\kappa_l^*)}{1-\rho}$$

for $\theta < \rho$. When $\kappa < \overline{\kappa}(i_{ior}, \rho)$ and $(\theta, \xi) \in \Omega(i_{ior}, \rho, \kappa_l^*)$, the liquidity constraint for banks is not binding. To see this, note that the liquidity constraint is satisfied in equilibrium if

$$(1+r_{ior})(q^* S_b^* + \phi_t^* F_t^* - \theta \phi_t^* D_t^*) \geq \xi L^*.$$

Also, in equilibrium $q^* S_b^* + \phi_t^* F_t^* \geq \rho \phi_t^* D_t^*$ since reserves holdings satisfy reserve requirements and banks cannot short securities. So, we have that the liquidity constraint is satisfied whenever

$$(1+r_{ior})(\rho-\theta)\phi_t^* D_t^* \geq \xi L^*.$$

This expression also tells us that the liquidity constraint will be satisfied for all equilibria if it is satisfied for the equilibria with minimal level of deposits. This is the case when $\xi \leq \overline{\overline{\xi}} \left(i_{ior}, \rho, \kappa_l^*, \theta\right)$.

Household consumption in all these stationary equilibria is given by:

$$C^* = W_h + S + r_l^* L^* - \Gamma^* \chi \left(l^*\right) - r_{ior} W_e,$$

and consumption of experts is given by $X_2^* = \left(1 + r_{ior}\right) W_e$. ■

As we move from one extreme of the set of possible equilibrium price levels, several adjustments in the banking system could be happening. The equilibrium associated with ϕ_0^L is a situation where the reserve requirements are not restricting the behavior of banks but yet there are no excess reserves in the banking system. This is also a situation where deposits are at its minimum value consistent with equilibrium.

Moving to situations with higher ϕ_0 requires that banks hold more deposits, which results in a larger equilibrium banking system. Once deposits are not minimal, banks can hold either more reserves or more securities. In principle, they could also hold less capital. Figure 7 shows a representation of the equilibrium set in the space of real deposits and real reserves. The equilibrium set is the triangle ABC. Two lines are plotted to delimit the set: one is the minimal reserves for each level of deposits $\phi_t^* F_t^* = \rho \phi_t^* D_t^*$ and the other is the maximal level of reserves for a given level of deposits, which is a representation of the balance sheet condition when securities holdings are set to equal zero. Point A is associated with the price level implied by ϕ^L and point C with the price level implied by ϕ^U. Just as an example, point B is an equilibrium where bank deposits are maximal but the bank is holding securities in the amount represented by the segment BC so that reserves are not maximal.

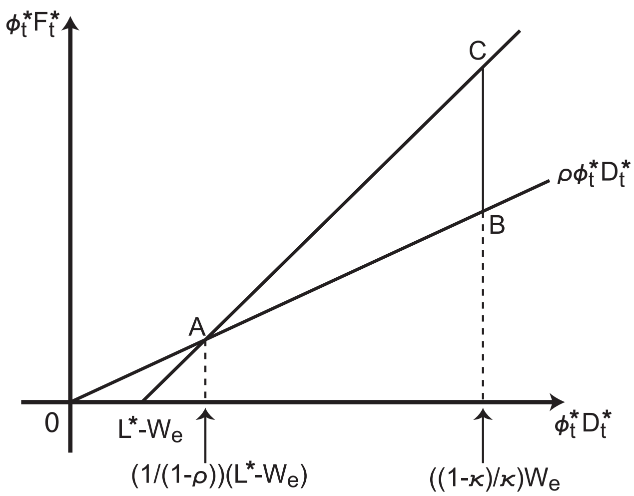

Figure 7

Proposition 4 implies that there is a disconnect between the quantity of monetary assets set by the central bank and the price level in the stationary equilibrium of the model when $r_{ior} = r_d$. There are two ways to look at this disconnect: On one side, as Proposition 4 demonstrates, there are multiple price levels consistent with the same amount of monetary assets. On the other side, for a given price level, there are multiple amounts of monetary assets consistent with an equilibrium with that price level. The next corollary establishes this fact formally.

Corollary 4.1 (Monetary assets/price level disconnect). Given a value for the initial (inverse of the) price level $\widetilde{\phi}_0 \in \left(\phi^L, \phi^U\right)$, there is a continuum of values of MA_0 consistent with the equilibrium of Proposition 4.

Proof. Take $MA_0 = x$. Then, by Proposition 4 there exists $\phi^L(x)$ and $\phi^U(x)$ such that $\phi^L(x) < \phi^U(x)$ and

$$\phi^L(x) = \frac{1}{x}\left[W_h + \frac{\rho}{1-\rho}(L^* - W_e)\right], \quad \text{and} \quad \phi^U(x) = \frac{1}{x}\left(W_h + \frac{W_e}{\kappa} - L^*\right).$$

These two equations define the functions $\phi^L(x)$ and $\phi^U(x)$ for all $x > 0$. Figure 8 plots this two functions.

Consider now a value $\widetilde{\phi}_0 \in \left(\phi^L(x), \phi^U(x)\right)$. We know that there is a stationary equilibrium with $MA_0 = x$ and price level $1/\widetilde{\phi}_0$. Now define two threshold values:

$$x^L\left(\widetilde{\phi}_0\right) \equiv \frac{1}{\widetilde{\phi}_0}\left(W_h + \frac{\rho}{1-\rho}(L^* - W_e)\right), \quad \text{and} \quad x^H\left(\widetilde{\phi}_0\right) \equiv \frac{1}{\widetilde{\phi}_0}\left(W_h + \frac{W_e}{\kappa} - L^*\right).$$

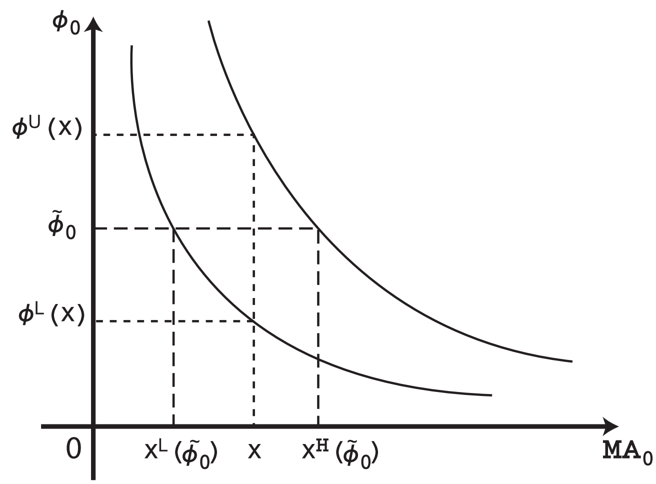

Figure 8

By Proposition 4, for any $y \in \left(x^L\left(\widetilde{\phi}_0\right), x^H\left(\widetilde{\phi}_0\right)\right)$ there are thresholds $\phi^L(y)$ and $\phi^U(y)$ such that for all $\phi_0 \in \left(\phi^L(y), \phi^U(y)\right)$ there is a stationary equilibrium with $MA_0 = y$ and price level $1/\phi_0$. But, since

$$\phi^L(y)\, y = \widetilde{\phi}_0 x^L\left(\widetilde{\phi}_0\right) < \widetilde{\phi}_0 y < \widetilde{\phi}_0 x^H\left(\widetilde{\phi}_0\right) = \phi^U(y)\, y,$$

then $\widetilde{\phi}_0 \in \left(\phi^L(y), \phi^U(y)\right)$. Hence for each $y \in \left(x^L\left(\widetilde{\phi}_0\right), x^H\left(\widetilde{\phi}_0\right)\right)$ there is an stationary equilibrium with $MA_0 = y$ and price level $1/\widetilde{\phi}_0$. ■

To illustrate the policy implications of these results, consider the case of a central bank operating in a stationary equilibrium like the one described in Proposition 4. Suppose, now, that the central bank can make an unanticipated change to the size of its open market operations at time $t_1 > 0$. In particular, suppose that:

$$MA_{t_1} = (1+\gamma)\, MA_{t_1 - 1} + \triangle_{MA},$$

and that:

$$\triangle_{MA} = \frac{q \triangle_{S_c}}{\phi^*_{t_1}} > 0,$$

where $\triangle_{S_c} = S_{c,t_1} - S_{c,t_1-1}$. With this notation, we are ready to state the next corollary.

Corollary 4.2 (An unanticipated open market operation). Consider an economy that, for all $t < t_1$, is in a stationary equilibrium as described in Proposition 4. Given a value for the initial (inverse of the) price level $\phi_0^* \in \left(\phi^L, \phi^U\right)$, there is a maximum size of an unanticipated open market operation at time $t_1 > 0$ that is consistent with maintaining a constant inflation equal to γ in all periods (including t_1).

Proof. Take $MA_0 = x$. We have that $\phi_0^* \in \left(\phi^L(x), \phi^U(x)\right)$ and since $\phi_t^* MA_t = \phi_0^* x$ for all $t < t_1$ (see Proposition 4) we have that:

$$\phi^L(x)\,x = W_h + \frac{\rho}{1-\rho}(L^* - W_e) < \phi_t^* MA_t < W_h + \frac{W_e}{\kappa} - L^* = \phi^U(x)\,x,$$

for all $t < t_1$.

To simplify notation, denote by $\widetilde{\phi}$ the price level for period t_1 consistent with a constant inflation rate γ, that is $\widetilde{\phi} = (1+\gamma)^{-t_1}\phi_0^*$; and denote by y the level of monetary assets at time t_1 consistent with a constant growth rate equal to γ, that is $y = (1+\gamma)^{t_1} x$. If $\widetilde{\phi} \in \left(\phi^L(y + \triangle_{MA}), \phi^U(y + \triangle_{MA})\right)$, then $\widetilde{\phi}$ is consistent with equilibrium and the constant inflation rate γ for all t (including t_1) is also consistent with equilibrium. However, if $\widetilde{\phi}$ is greater than $\phi^U(y + \triangle_{MA})$, then the inverse of the price level has to change so that $\phi_{t_1} \in \left(\phi^L(y + \triangle_{MA}), \phi^U(y + \triangle_{MA})\right)$, which immediately implies that $\phi_{t_1} < \widetilde{\phi}$ and hence inflation at time t_1 is greater than γ.[22]

To determine the values of $\triangle_{MA}$ that are consistent with a constant inflation rate γ for all t (including t_1) note that for $\widetilde{\phi} \in \left(\phi^L(y + \triangle_{MA}), \phi^U(y + \triangle_{MA})\right)$ we need that:

$$\phi^L(y)\,y = \phi^L(y + \triangle_{MA})(y + \triangle_{MA}) < \widetilde{\phi} y + \widetilde{\phi}\triangle_{MA} < \phi^U(y + \triangle_{MA})(y + \triangle_{MA}) = \phi^U(y)\,y,$$

and hence:

$$\left(\frac{\phi^L(y)}{\widetilde{\phi}} - 1\right) y < \triangle_{MA} < \left(\frac{\phi^U(y)}{\widetilde{\phi}} - 1\right) y.$$

Since $\phi^U(y)\,y = \phi^U(x)\,x$, we have that $\phi^U(y) = \phi^U(x)/(1+\gamma)^{t_1}$ and $\phi^U(y)/\widetilde{\phi} = \phi^U(x)/\phi_0^*$. Then, we have that the maximum size of the open market operation at time t_1 consistent with a constant inflation rate γ is the one that involves a percentage increase in monetary assets at time t_1 equal to $\overline{\delta}$ given by:

$$\overline{\delta} = \frac{(W_h + \frac{W_e}{\kappa} - L^*) - \phi_0^* x}{\phi_0^* x}(1+\gamma).$$

Whenever $\triangle_{MA} > \overline{\delta} MA_{t_1-1}$ inflation has to increase in period t_1 when the ('larger') open market operation occurs. ■

We have analyzed in Proposition 4 equilibria where banks do not face any binding capital, reserve, or liquidity constraint and the price level is indeterminate. Such situations can only happen if the central bank pays interest on reserves at rate $i_{ior} = i_d$. However, even when the central bank pays interest on reserves at rate $i_{ior} = i_d$, bank capital can be scarce in equilibrium, and in that case the stationary equilibrium is unique, sharing similar properties to the equilibrium described in Proposition 2. We state this result formally in the next proposition.

Proposition 5. Let $i_{ior} = i_d$. Given a value of $\rho \in (0,1)$ and the threshold value $\overline{\kappa}(i_{ior}, \rho) > 0$ defined in Proposition 4, for all $\kappa \in (\overline{\kappa}(i_{ior}, \rho), 1)$ there is a non-empty set $\Omega(i_{ior}, \rho, \kappa)$ of values of $\theta > 0$ and $\xi > 0$ such that for all $\kappa > \overline{\kappa}(i_{ior}, \rho)$ and $(\theta, \xi) \in \Omega(i_{ior}, \rho, \kappa)$ there exists a unique stationary equilibrium where the bank capital and the reserve requirement constraints are binding and the bank liquidity constraint is not binding.

[22] Using the budget constraint for the central bank, it can be shown that if $\phi_{t_1} = \widetilde{\phi}$, then the real value of taxes $\phi_t T_t$ does not change at time t_1. However, if $\phi_{t_1} < \widetilde{\phi}$ then the real value of taxes can actually be smaller, due to the extra seigniorage obtained as a result of the increase in inflation.

Proof. Let $l^* = l^o(\widehat{\tau}_b)$, $\Gamma^* = \frac{1-\rho(1-\kappa)}{\kappa}\frac{W_e}{l^*}$, and a loan rate r_l^* that solves:

$$1 - G(r_l^*) = \frac{1-\rho(1-\kappa)}{\kappa} W_e = L^*.$$

Since $\kappa > \overline{\kappa}(i_{ior}, \rho)$ we have that $r_l^* > r_{ior}^* + \chi'(l^*)$. Now let $r_b^* = r_l^* - \chi'(l^*) > r_{ior}^*$ where:

$$r_b^* = \left(1 - \frac{\kappa}{1-\rho(1-\kappa)}\right) r_d^* + \frac{\kappa}{1-\rho(1-\kappa)} r_k^*.$$

This implies that $r_k^* > r_d^* = r_{ior}^*$ and these rates imply that the capital and reserve constraints are binding in the bank's problem. Now define the set $\Omega(i_{ior}, \rho, \kappa)$ as:

$$\Omega(i_{ior}, \rho, \kappa) = \left\{(\theta, \xi) \mid \theta < \overline{\theta}(i_{ior}, \rho) \text{ and } \xi < \overline{\overline{\xi}}(i_{ior}, \rho, \kappa, \theta)\right\},$$

where

$$\overline{\theta}(i_{ior}, \rho) = \rho,$$

and

$$\overline{\overline{\xi}}(i_{ior}, \rho, \kappa, \theta) = \frac{(1-\kappa)(\rho-\theta)(1+r_{ior})}{1-\rho(1-\kappa)},$$

which are essentially the same thresholds for θ and ξ as those used in Proposition 2 but when $i_{ior} = i_d$. If $(\theta, \xi) \in \Omega(i_{ior}, \rho, \kappa)$, the liquidity constraint is not binding. Given that the capital and the reserve constraints are binding, deposits are given by $\phi_t^* D_t^* = ((1-\kappa)/\kappa) W_e$ and reserves are $\phi_t^* F_t^* = \rho((1-\kappa)/\kappa) W_e = (W_e/\kappa) - L^*$, which confirms the values of L^* and Γ^* postulated at the beginning of the proof. The rest of the equilibrium values can be obtained by simple substitution. ■

Corollary 5.1 (Price level determination). Given a sequence $\{MA_t\}_{t=0}^{\infty}$ set by the central bank, the price level in the equilibrium described in Proposition 5 is uniquely determined and proportional to the quantity of monetary assets.

Proof. Since $1 + r_d^* = 1 + r_q^* = 1/\beta$, we have that $i_d^* > 0$ and $\phi_t^* M_t = W_h$. Using $\phi_t^* F_t^* = \rho((1-\kappa)/\kappa) W_e$ we have that prices ϕ_t^* satisfy:

$$\phi_t^* MA_t = W_h + \rho \frac{1-\kappa}{\kappa} W_e,$$

and hence there is an inverse relationship between the total amount of monetary assets and the inverse of the equilibrium price level. ■

Corollary 5.2 (Rates of return). In the equilibrium of Proposition 5 we have that $r_k^* > r_f^* > r_{ior}^* = r_d^*$. Furthermore, the equilibrium interest rate on interbank loans is given by:

$$r_f^* = r_{ior}^* + \frac{\rho\kappa}{1-\rho(1-\kappa)}(r_k^* - r_{ior}^*).$$

A look at some relevant data (2005-2013)

In October 2008 the Federal Reserve started paying interest on reserves. At the same time, the Fed engaged in a series of programs (lending facilities and asset purchases) that resulted in a significant increase in the level of bank (excess) reserves, as illustrated in Figure 9 (see also Figures 1 and 2 in the Introduction). Keister and McAndrews (2009) provide a good discussion of the process of buildup in bank reserves during the financial crisis and its potential implications.

Figure 9 also plots the currency component of the money stock measure provided by the Federal Reserve in its H6 data release. This measure of currency is basically currency in circulation minus bank vault cash (which is accounted for in bank reserves) and can be taken as the empirical counterpart of the variable M_t in the model. At the same time that total monetary assets in the economy were growing substantially, the price level grew only moderately at an average rate that was below 2% (see Figure A5). This suggests that a situation like the one described in Proposition 4 may be relevant for understanding this period.

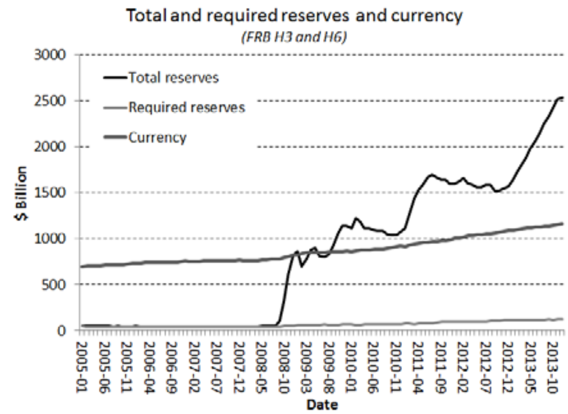

Figure 9

Since 2009, the fed funds rate has been persistently below the interest rate on reserves (see Figure 10). In principle, this is consistent with binding capital requirements, as indicated by Corollary 3.2. Also consistent with that corollary is the fact that the short-term rate on securities (3-month Treasury bills) has been below interest on reserves.

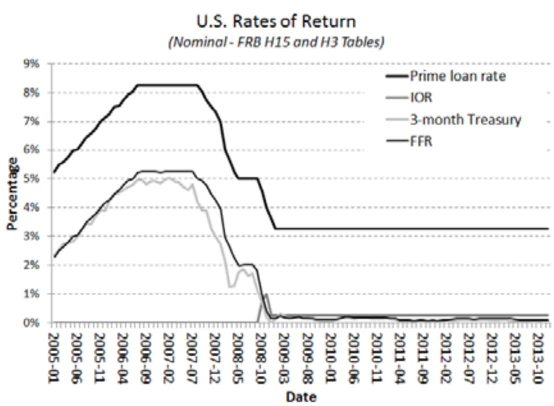

Figure 10

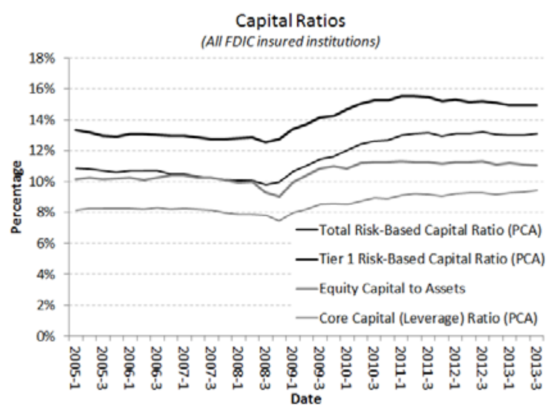

Figure 11

In the model of this paper, the real rate of interest on deposits is pinned down by the real side of the economy, and in steady state it is given by $1/\beta$. Also, inflation is determined by the (long run) growth rate of monetary assets. In such situation, it is completely up to the central bank to set the interest on reserves such that it is equal, higher, or lower than the rate on deposits.[23]

The fact that the rate on 3-months Treasury bills is below the interest on reserves in Figure 10 suggests a situation such as that described in Proposition 3. At the same time, the disconnect between the growth in monetary assets and prices seems consistent with Proposition 4. A crucial distinction between the two propositions is the state of the bank capital constraint.

Capital regulations in the United States are complex and undergoing a process of reform. For these reasons, it is hard to evaluate the extent to which bank capital constraints may have been binding in the period 2009-2013. Expectations about future adjustments to the required ratios may have played a critical role. After a long period of gestation, a new rule was introduced in October

[23] Sargent and Wallace (1985) study a model where central bank policy can change the equilibrium real rate of interest. In that case, a policy of setting the interest on reserves equal to the real rate does not pin down the equilibrium value of the real rate, and some policy induced indeterminacy arises. See the appendix for a detailed discussion of the connection between this and Sargent and Wallace's papers.

2013. The rule established a minimum common equity tier 1 capital to risk adjusted assets ratio of 4.5% with an added 2.5% conservation buffer. For tier 1 capital the required ratio is 6% plus the conservation buffer, and for total capital the required ratio is 8%. The leverage ratio was set at 4% with a supplement for off-balance sheet assets that may take it to as high as 5%. Figure 11 plots the ratio of capital to assets for different categories of capital and for the U.S. banking system as a whole. The figure suggests that bank capital holdings are not uniformly close to the regulatory constraint. Of course, it could be important to look at the distribution of these ratios across banks. The information provided in Ennis and Wolman (2014) on this respect supports a similar conclusion.

If capital constraints are not binding, then the situation described in Proposition 4 appears as the most relevant. Since rates of return are equalized in such an equilibrium, explaining why the fed funds rate and other short-term rates have been below the level of interest on reserves may require an alternative explanation (such as that advanced in Bech and Klee, 2011, for example).

5. Convenience yield on deposits

As we saw from Figure 4, between 1995 and 2005, while the Fed was not paying interest on reserves, banks held around 20% of their assets in securities. In contrast, the equilibrium of the model in the previous sections when $i_{ior} = 0$ and the liquidity constraint is not binding has banks holding no securities. The logic behind this outcome is clear: Since households price the securities and are holding deposits, then the interest rate on securities and deposits is equalized. Banks, however, would only hold securities if the return on them would be enough to compensate for the cost of attracting deposits and the cost of holding more reserves and/or more capital.

In the appendix, I study the case where liquidity constraints are binding. Securities can be a source of liquidity for banks, and when the liquidity constraints are binding, banks may hold securities in equilibrium even if the reserves and the capital constraints are binding.

As an alternative, in this section, I provide a minimal modification of the model presented in Section 2 that results in the possibility of banks holding securities in equilibrium even if liquidity constraints are not binding. The basic idea is to capture the fact that deposits provide a liquidity/transaction service to depositors (as in Van den Heuvel, 2008; Stein, 2012; Ireland, 2013; or Begenau, 2013). When this is the case, depositors will be willing to hold deposits even if the return on deposits is lower than the return on securities. The spread between these two rates of return, then, can compensate banks for the full balance sheet cost of holding securities.

Suppose that there are two goods in the economy, the consumption good introduced in Section 2, and a '*big-ticket item*' that can only be paid with deposits. We call these goods 1 and 2, respectively. Every period t each household receives an endowment $\left(w_h^1, w_h^2\right)$ of the goods, and s units of the one-period securities. As before, households do not consume their own endowment. Suppose that each household has three members. At the beginning of the period, two members of the household leave to sell the endowment of the goods in the market. The remaining member of the household trades securities, makes deposits, pays taxes, and buys good 1 from other households. Once banks and the securities market close, the household purchases good 2 and pays those goods with deposits.

The problem of the household is:

$$\max \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t^1\right) + v\left(c_t^2\right)\right],$$

subject to

$$\begin{aligned}
c_t^1 + \phi_t \widetilde{d}_t + \phi_t n_t + q_t s_{ht} &= \phi_t \left(1 + i_{dt-1}\right) d_{t-1} + s_{ht-1} + q_t s_t + \phi_t m_{t-1} - \phi_t \tau_{ht}, \\
\phi_{2t} c_t^2 &= \phi_t \left(\widetilde{d}_t - x_t\right) \\
\phi_t m_t &= w_{ht}^1 + \phi_t n_t, \\
\phi_t d_t &= \phi_{2t} w_{ht}^2 + \phi_t x_t \\
c_t^1, c_t^2, n_t, x_t &\geq 0,
\end{aligned}$$

where ϕ_{2t} is the relative price of good 2 in terms of good 1 and $\widetilde{d}_t$ is the amount of deposits held by the household when banks close in period t. Let us define the nominal interest rate i_t by the following equation:

$$1 + i_t = \frac{\phi_t}{\phi_{t+1}} (1 + r_{qt}).$$

Using the first order conditions for this problem, it is easy to show that whenever $i_t > 0$ we have that $n_t = 0$. Also, in similar fashion, if $r_{dt} < r_{qt}$ then $x_t = 0$. Finally, the optimal values of c_t^1 and c_t^2 satisfy the following two conditions:

$$\begin{aligned} -u'\left(c_t^1\right) + \frac{1}{\phi_{2t}} v'\left(c_t^2\right) &= 0, \\ -q_t u'\left(c_t^1\right) + \beta u'\left(c_{t+1}^1\right) &= 0. \end{aligned}$$

The market clearing condition for good 2 is $c_t^2 = w_{ht}^2$. The rest of the model is unchanged from that presented in Section 2 and analyzed in Sections 3 and 4.

Note that now even if the return on deposits is lower than the return on securities, we have that $d_t = \phi_{2t} w_{ht}^2 / \phi_t > 0$; that is, the household wants to hold deposits to be able to purchase the big-ticket item, good 2. This is in sharp contrast with the equilibrium of the model in the previous sections. There, for the banking system to attract deposits, the rate of return on deposits has to be as high as the rate of return on securities. The modification introduced in this section allows the rate of return on deposits to differ from the rate of return on securities. As a result, banks may hold both securities and deposits in equilibrium even when the interest on reserves is relatively low (or zero).

To illustrate how the modified model works, we characterize a stationary equilibrium when the central bank is not paying interest on reserves and only the reserve requirements are binding. This equilibrium parallels that in Proposition 1 of Section 4. As in that case, then, we adopt basic stationarity assumptions: Endowments are constant, and monetary assets grow at rate γ. To simplify the analysis, assume that θ and ξ are equal to zero, so that the liquidity constraint is always satisfied. Generalizing the result to positive values for θ and ξ is straightforward using the same logic as in the proof of Proposition 1.

Before stating the proposition, we make one extra technical assumption and introduce one extra piece of notation. Assume the utility function $v(c)$ has relative risk aversion strictly lower than one for all c and $v'(c)\,c \to 0$ as $c \to 0$. Finally, using the definition of $l^o(\widehat{\tau}_b)$ introduced in Section 4, define $L^o = 1 - G\left(\frac{1-\beta}{\beta} + \chi'\left(l^o\left(\widehat{\tau}_b\right)\right)\right)$.

Proposition 6. Fix $W_e < L^o$ and assume that $i_{ior} = 0$. There is an interval Ω_{W^2} of values of W_h^2 and a threshold value $\overline{\kappa} \in (0, 1)$ such that for all $\kappa < \overline{\kappa}$ and $W_h^2 \in \Omega_{W^2}$ there exists a stationary equilibrium where banks hold securities and the bank reserve requirement constraint is binding, while the bank capital and bank liquidity constraints are not binding.

Proof. Using the household's problem, we have that $q^* = \beta$ and $r_q^* = (1-\beta)/\beta$. Also, we have that $\phi_2^* = v'\left(W_h^2\right)/u'\left(C_1^*\right)$ where we have used the market clearing condition for good 2 and denote by C_1^* total equilibrium consumption of good 1.

As in Section 4, stationarity implies that $\phi_{t+1}^*/\phi_t^* = 1/(1+\gamma)$ and since $i_{ior} = 0$ we have that $r_{ior}^* = -\gamma/(1+\gamma)$. These rates of return imply that $n^* = 0$ in the household's problem and $\phi_t^* M_t^* = W_h^1$.

Assume now that $S_b^* > 0$ and that the bank capital constraint is not binding. We will verify that this is the case later in the proof. Now, since $r_q^* > r_{ior}^*$ the reserve requirements must be binding and $r_k^* = r_f^* = r_q^*$. Furthermore, we have that $r_l^* = r_q^* + \chi'(l^*)$ and:

$$r_d^* = (1-\rho)\, r_q^* + \rho r_{ior}^*,$$

which implies that $r_d^* < r_q^*$ because r_{ior}^* is negative. From the household's problem, these rates of return imply that $x^* = 0$ and hence:

$$\phi_t^* D_t^* = \phi_2^* W_h^2.$$

Given that reserve requirements are binding, $\phi_t^* F_t^* = \rho \phi_t^* D_t^* = \rho \phi_2^* W_h^2$ and hence

$$\phi_t^* MA_t = \phi_t^* M_t + \phi_t^* F_t^* = W_h^1 + \rho \phi_2^* W_h^2,$$

which uniquely determines a sequence of values for ϕ_t^* given values for MA_t for all t. Using the bank's zero profit condition we have that $l^* = l^o(\widehat{\tau}_b)$, and from the market clearing condition for loans we have $\Gamma^* = (1 - G(r_l^*))/l^*$, which implies $L^* = L^o$.

Since $r_k^* = r_q^*$, experts are indifferent between holding securities and supplying bank capital. For concreteness, assume that $K_e^* = W_e$ and define $\overline{\kappa} = W_e / \left(\phi_2^* W_h^2 + W_e\right) \in (0,1)$ and Ω_{W^2} as follows:

$$\Omega_{W^2} = \left\{ W_h^2 \mid u'(C_1^*) \frac{L^o - W_e}{1-\rho} < v'\left(W_h^2\right) W_h^2 < u'(C_1^*) \frac{\beta(S - S_c) + L^o - W_e}{1-\rho} \right\},$$

where it is important to note that C_1^* is independent of W_h^2. Given the properties of the function $v(c)$, the set Ω_{W^2} is non-empty. Aggregating across banks, the balance sheet condition for the banking system is:

$$L^* + q^* S_b^* = (1-\rho)\phi_2^* W_h^2 + W_e,$$

and when $W_h^2 \in \Omega_{W^2}$ we have that $q^* S_b^*$. Finally, when $\kappa < \overline{\kappa}$ the bank capital requirement is not binding in equilibrium. This completes the proof. ■

Note that in this equilibrium of the modified model, the interest rate on deposits gets determined in the solution to the banks problem and the quantity of deposits, in the solution to the household problem. The opposite was true in the model of Section 2. Aside from this fundamental difference, the analysis of equilibrium is very similar in both models. The point here was to show that a minimal modification based on an idea that is common in the literature (i.e., the convenience yield on deposits) moves the model closer to the data in terms of having banks hold securities in their balance sheets.

As for the situation characterized in Proposition 4, note that once the central bank can pay interest on reserves at market rates and none of the constraints in the bank's problem are binding, then $r_d^* = r_q^*$ and the household choice of x^* can be positive. Households become indifferent between holding deposits or securities in the margin, and so do banks. For this reason, the basic logic behind Proposition 4 still applies to this case.

6. Conclusions

We have constructed a relatively simple general equilibrium model of a macro economy with a banking system and non-trivial bank balance sheets. The model is consistent with traditional price level determination (Lucas, 1990) when the central bank pays no (or low) interest on reserves. However, when the central bank pays interest on reserves at 'market' rates, banks hold excess reserves and the price level can be indeterminate. In other words, there is a disconnect between the total amount of monetary assets issued by the central bank and the value of the equilibrium price level: A higher quantity of monetary assets does not necessarily imply a higher price level.

However, the *range* of values that the price level may take is linked to the quantity of monetary assets. Hence, not all price levels are consistent with equilibrium for a given quantity of monetary assets. As higher levels of monetary assets are considered, eventually the price level needs to increase to remain in the interval that makes it consistent with equilibrium. These results suggest that it may be possible for a central bank to increase monetary assets for some time without impacting the price level, but that eventually, as monetary assets continue to increase, the price level would need to increase to accommodate the nominal liabilities issued by the central bank that are used for transaction purposes and as reserves in banks' balance sheets.

The two key elements of the model that drive these results are an endogenous split of total monetary assets between bank reserves and currency in circulation, and the modeling of the explicit potential costs faced by banks from expanding the size of their balance sheets given that reserves can only be held by banks. The model was kept as simple as possible while incorporating these two dimensions that seem essential for understanding the role that bank excess reserves play in the process of price level determination. The analysis in this paper was limited to the non-stochastic stationary equilibrium, for simplicity. However, an extension considering a stochastic version of the model, where central bank policy can be studied in a more systematic way, seems a promising next step on the research agenda pursued in this paper.

Appendix

The appendix has five parts. In the first part, we discuss equilibrium when liquidity constraints are binding. In Part 2, as complement to Proposition 3, we consider equilibrium when both capital and reserves constraints on banks are binding even for relatively high interest on reserves. In Part 3, we discuss in detail some of the related literature. In Part 4, we briefly discuss the case when banks fund lending only with capital (and no deposits). Finally, in Part 5 we provide additional discussion of U.S. data that is useful to evaluate the empirical relevance of the results in the model.

Part 1: Binding bank liquidity constraints

In this part of the appendix, we study situations in which the banks' liquidity constraints are binding. Again, we restrict attention to stationary equilibrium and maintain the same general assumptions as in Section 4, including the assumptions that guarantee that bank deposits are positive in equilibrium. We consider first the case when the central bank is not paying interest on reserves.

Lemma A1. If $i_{ior} = 0$ and the liquidity constraint in Problem A is binding, then the reserves requirement constraint is also binding.

Proof. Suppose the reserves constraint is not binding. If the capital constraint is also not binding, then from the first order conditions of the bank problem, we have that:

$$r_q - r_d + \left[(1 + r_q) - \theta (1 + r_d)\right] \mu_2 \leq 0$$

must hold. However, since from the household's problem we have that $r_q = r_d$ in equilibrium, this implies that $\mu_2 < 0$ which is a contradiction.

Suppose, instead, that the capital constraint is binding. From the first order condition of the bank's problem we have that:

$$\begin{aligned} -\kappa\lambda + (1 - \theta)(1 + r_d)\mu_2 &\leq 0, \text{ and} &\text{(A1)}\\ -\kappa\lambda + \left[(1 + r_{ior}) - \theta (1 + r_d)\right]\mu_2 &= r_d - r_{ior}. &\text{(A2)}\end{aligned}$$

Condition (A2) can be rewritten as:

$$-\kappa\lambda + (1 - \theta)(1 + r_d)\mu_2 = (1 + \mu_2)(r_d - r_{ior}).$$

Since the right side of this expression is positive, this contradicts condition (A1). ∎

The logic behind this lemma is clear: Given that $r_d = r_q$, the bank can always increase its holdings of securities financed with an increase in deposits or with a decrease in reserves and in that way relax the liquidity constraint without changing profits. Lemma A1 implies that we only need to consider two cases: one when the liquidity and reserves constraint are binding, and one when all three constraints (liquidity, reserves, and capital) are binding.

To simplify the presentation, assume that $\xi = 0$ so that loans do not require liquidity. Considering the general case of $\xi \geq 0$ is a straightforward extension. Also, for concreteness, assume that $1 - \rho > \theta > \rho$. This condition guarantees that $\theta > \overline{\theta}(\rho)$, as defined in Proposition 1.

Proposition A1. Let $i_{ior} = 0$. Given a value of $\rho \in (0, 1)$, there is a threshold $\overline{\kappa}^A(\rho) \in (0, 1)$ such that for all $\kappa < \overline{\kappa}^A(\rho)$ there exists a unique stationary equilibrium where the bank reserve requirement and liquidity constraints are binding and the bank capital constraint is not binding.

Proof. In equilibrium, the reserves and liquidity constraints imply:

$$\begin{aligned} \phi_t f_t &= \rho \phi_t d_t, \text{ and} \\ (1+r_q)\, q s_b + (1+r_{ior})\, \phi_t f_t &= \theta\,(1+r_d)\, \phi_t d_t. \end{aligned}$$

Substituting the first into the second constraint and rearranging terms, we get:

$$(1+r_q)\, q s_b = [\theta\,(1+r_d) - \rho\,(1+r_{ior})]\, \phi_t d_t > 0,$$

which implies that the bank is holding securities in its balance sheet. In fact, both reserves and securities holdings by banks are proportional to deposits. To simplify notation, define the following variable

$$\varsigma \equiv \frac{1+r_d}{(1-\rho-\theta)\,(1+r_d) + \rho\,(1+r_{ior})}$$

Now, using the reserves and liquidity constraints and the balance sheet identity for the bank, we can write the bank's objective as $(r_l - r_k^*)\, l - \chi(l) - \widehat{\tau}_b$, with r_k^* satisfying:

$$r_k^* = r_d + (r_d - r_{ior})\, \rho\varsigma.$$

Then, the level of lending for each bank satisfies $r_l^* - r_k^* = \chi'(l^*)$ and from the zero profit condition we have that $l^* = l^o(\widehat{\tau}_b)$. Loan market clear implies that $\Gamma^* = [1 - G(r_l^*)]\,/ l^*$. Since $1-\rho > \theta$, we have that $r_k^* > r_d^*$ and hence experts dedicate all their resources to bank capital. Aggregating across banks, we obtain the banking system balance sheet identity:

$$L^* + qS_b^* + \phi_t^* F_t^* = \phi_t^* D_t^* + W_e.$$

Since equilibrium reserves and securities holdings are proportional to deposits, using the banking system balance sheet we get that total bank deposits are given by $\phi_t^* D_t^* = \varsigma\,(\Gamma^* l^* - W_e)$. Since we are interested in the case of positive deposits, we required that $L^* = \Gamma^* l^* > W_e$. It is easy to give conditions on parameters so that this condition is satisfied. The threshold $\overline{\kappa}^A(\rho)$ is given by:

$$\overline{\kappa}^A(\rho) = \frac{\kappa_l^*}{1 + (\vartheta + \rho)\,\varsigma\,(1-\kappa_l^*)},$$

where $\kappa_l^* = W_e / L^*$ and $\vartheta \equiv \frac{\theta(1+r_d) - \rho(1+r_{ior})}{1+r_q}$. The real quantity of total reserves in the banking system is $\phi_t^* F_t^* = \rho\varsigma\,(\Gamma^* l^* - W_e)$. Since the implied nominal interest rate on deposits is positive, we have that $\phi_t^* M_t = W_h$ and, in equilibrium:

$$\phi_t^* MA_t = \phi_t^* M_t + \phi_t^* F_t^* = W_h + \rho\varsigma\,(\Gamma^* l^* - W_e),$$

which tells us that there is a one to one relationship between the total amount of monetary assets and the equilibrium price level (the inverse of ϕ_t). For a given sequence of MA_t, there is a unique sequence of ϕ_t^* consistent with a stationary equilibrium. Finally, note that in equilibrium we have $r_k^* > r_f^* = r_d^* > r_{ior}^*$, where the equality between r_f^* and r_d^* can be verified using the first order conditions for the bank's problem. ■

Proposition A1 deals with the case when only the reserve and liquidity constraints are binding. The next proposition considers the case when all three constraints (reserves, liquidity, and capital) are binding.

Proposition A2. Let $i_{ior} = 0$. Given a value of $\rho \in (0,1)$, there is a threshold $\overline{\kappa}^A(\rho) \in (0,1)$ such that for all $\kappa > \overline{\kappa}^A(\rho)$ there exists a unique stationary equilibrium where the banks' reserve requirements, the bank liquidity constraints, and the bank capital constraints are binding.

Proof. When the capital constraint is binding, we can write banks deposits and holdings of securities and reserves as a function of bank capital. Using the constraints in Problem A, we obtain:

$$\begin{aligned}\phi_t^* f_t^* &= \rho\phi_t^* d_t^* = \rho\frac{1-\kappa}{\kappa}k_b^*, \text{ and} \\ qs_b^* &= \frac{\theta(1+r_d)-\rho(1+r_{ior})}{(1+r_q)}\frac{1-\kappa}{\kappa}k_b^*.\end{aligned}$$

Now using the balance sheet condition we can express loans as a function of bank capital:

$$l^* = \frac{[(1-\rho)-\theta](1+r_d)(1-\kappa)+\kappa[(1+r_d)-\rho(1+r_{ior})]+\rho(1+r_{ior})}{(1+r_d)\kappa}k_b^* \equiv \omega k_b^*.$$

Using this expression, we can show that the bank's objective function can be express as $\left(r_l - r_b^A\right)l - \chi(l) - \widehat{\tau}_b$ where r_b^A is given by:

$$r_b^A = r_d + \frac{(1-\kappa)\rho(r_d - r_{ior}) + \kappa(r_k^* - r_d)}{\kappa\omega},$$

and using the bank's zero profit condition, we conclude that $l^* = l^o(\widehat{\tau}_b)$. Since, from the first order conditions of the bank's problem, it is evident that $r_k^* > r_d^*$ then we have that the equilibrium rate of return on loans r_l^*, the size of the banking system Γ^*, and the level of bank capital per bank k_b^* satisfy the following equation:

$$\Gamma^* l^* = \omega\Gamma^* k_b^* = \omega W_e = 1 - G(r_l^*).$$

These derivations determine the real allocation of resources in the economy.[24]

As before, the real quantity of total reserves in the banking system does not depend on any nominal quantity and is given by $\phi_t^* F_t^* = \rho\frac{1-\kappa}{\kappa}W_e$. Furthermore, as the implied nominal interest rate on deposits is positive, we have that $\phi_t^* M_t = W_h$ and, in equilibrium:

$$\phi_t^* MA_t = \phi_t^* M_t + \phi_t^* F_t^* = W_h + \rho\frac{1-\kappa}{\kappa}W_e,$$

so that for any given sequence of MA_t there is a unique sequence of ϕ_t^* consistent with a stationary equilibrium. ■

So far in the appendix we have considered the situation when $i_{ior} = 0$. It is relatively straightforward to prove similar results when $i_{ior} < i_d$. The more interesting cases arise when $i_{ior} = i_d$. Proposition 4 considers the situation when $i_{ior} = i_d$ and none of the bank constraints are binding. It is easy to show that when $i_{ior} = i_d$ the liquidity condition cannot be the only binding constraint on the banks' problem. It is interesting to note that it is possible to have the capital constraint and the liquidity constraint binding while the banks are still holding excess reserves. This is in contrast with the result in Proposition 5, which suggested that banks would not hold excess reserves if the capital constraint is binding (and the liquidity constraint is not). The important logic here is that, from the perspective of a bank dealing with a binding liquidity constraint, reserves and securities are equivalent assets when the central bank pays interest on reserves at the market rate (so that $r_{ior} = r_d = r_q$). Most importantly, this asset indifference can act as a source of price level indeterminacy even when bank capital is binding in the model. See the discussion of Hornstein (2010) in Part 3 of this appendix.

[24] Note that ω is a function of κ. It is easy to show that, as should be expected, $\omega(\kappa) \to 1/\kappa_l^*$ as $\kappa \to \overline{\kappa}^A(\rho)$ where both κ_l^* and $\overline{\kappa}^A(\rho)$ are defined as in Proposition A1. In other words, the equilibrium with a binding capital constraint converges to the equilibrium with a non-binding capital constraint as the coefficient κ converges to the threshold $\overline{\kappa}^A(\rho)$ from above.

Part 2: Binding capital and reserve requirements with high interest on reserves

Consider the case when the central bank is paying $i_{ior} > i_d$ but $\kappa > \overline{\kappa}(i_{ior}, \rho)$, where the threshold value $\overline{\kappa}(i_{ior}, \rho)$ was defined in Proposition 3. Suppose the liquidity constraint is not binding. Then, the equilibrium in this case is similar to that described in Proposition 2. First note that we still have $1 + r_q^* = 1 + r_d^* = 1/\beta$ and, since by Lemma 3 the capital constraint is binding, from the bank's and the experts' problems we have that $S_b^* = S_e^* = 0$. Hence, $K_e^* = W_e$. Also using the bank balance sheet constraint and the binding capital and reserves constraints, we have that $\phi_t^* F_t^* = \rho \phi_t^* D_t^* = \rho ((1-\kappa)/\kappa) W_e$ and $L^* = ((1 - \rho(1-\kappa))/\kappa) W_e$. Now, as in Proposition 2, define the weighted average of rates of return r_b as follows:

$$r_b = \frac{1-\kappa}{1-\rho(1-\kappa)} r_d - \frac{\rho(1-\kappa)}{1-\rho(1-\kappa)} r_{ior} + \frac{\kappa}{1-\rho(1-\kappa)} r_k.$$

The objective of the bank can then be written as $(r_l - r_b) l - \chi(l) - \widehat{\tau}_b$. Taking first order condition with respect to l and using the zero profit condition, we have that the optimal value of $l = l^0(\widehat{\tau}_b)$ as defined at the beginning of Section 4. Using the market clearing condition for bank capital and for loans, we have that:

$$W_e = \frac{\kappa}{1-\rho(1-\kappa)} \Gamma^* l^* = \frac{\kappa}{1-\rho(1-\kappa)} [1 - G(r_l^*)],$$

which can be used to determine the equilibrium level of r_l^* and Γ^*. Since $r_b^* = r_l^* - \chi'(l^*)$, we can use the definition of r_b to determine the equilibrium level of r_k^*. By definition, the capital and reserves constraints hold with equality. Following the same logic as in the proof of Proposition 2, we can determine threshold values for θ and ξ so that the liquidity constraint is indeed not binding. The rest of the equilibrium variables are easily derived from simple substitution.

Part 3: A more detailed discussion of some related literature

Sargent and Wallace (1985). The authors use a two-period-lived overlapping generations model to study the effects of paying interest on reserves on equilibrium outcomes. They consolidate the private sector and ignore the currency component of the monetary base, so all base money earns interest. As a consequence, they do not discuss the endogenous split between currency and reserves, which is an important element in my analysis. The Sargent-Wallace result has a similar flavor as the result in Proposition 4 in this paper in the sense that they find that when the monetary authority pays interest on reserves at the market rate, there is a continuum of stationary equilibria, each with a different price level. An important difference, however, is that in the Sargent and Wallace model the real rate of interest is not pinned down by the parameters of the model and it is different across the multiple equilibria. In Proposition 4, the (gross) real rate is given by $1/\beta$ in all the possible equilibria.

It is important to realize that in Sargent and Wallace's model the main role of money is as a store of value (from one period to the next). In the model of this paper, money plays a role in transactions, instead. The idea behind the Sargent-Wallace result can be explained using a very stripped-down description of the model. Basically, the model is a standard overlapping generations endowment economy with money. Agents solve a two-period intertemporal optimization problem by deciding an amount of real savings $s(r, \tau)$ as a function of the real interest rate and endowments net of taxes τ. Call i_t the nominal interest rate paid on cash (Sargent and Wallace's interest on reserves), ϕ_t the inverse of the price level at time t, and MA the amount of monetary assets in the economy (constant over time). We have that in equilibrium the following equations must hold:

$$\begin{aligned} s(r_t, \tau_t) &= \phi_t MA \\ 1 + r_t &= \left(\phi_{t+1}/\phi_t\right)(1 + i_t), \text{ and} \\ \phi_{t+1} i_t MA &= \tau_t, \end{aligned}$$

for all t, where the last equation is the budget constraint of the monetary authority and the first equation is the standard money market clearing equation in monetary overlapping generations models. The second equation is an arbitrage equation between real and nominal returns that would be familiar to the reader since it was repeatedly used in the main body of this paper. In a stationary equilibrium, this system simplifies to:

$$\begin{aligned} s(r,\tau) &= \phi_0 MA, \\ r &= i, \text{ and} \\ \phi_0 iMA &= \tau, \end{aligned}$$

which, after substitutions, can be reduced to $rs(r,\tau) = \tau$. This is one equation in two unknowns and under standard assumptions Sargent and Wallace show that there is a continuum of solutions (r,τ), In turn, for each (r,τ) there is a different equilibrium value of ϕ_0.

When the monetary authority does not pay interests on reserves $i = 0$ and $\tau = 0$. Furthermore, by arbitrage, we also have that $r = 0$. In such a situation, under standard assumptions, there is a unique stationary monetary equilibrium with a constant price level ϕ_0 that satisfies $s(0,0) = \phi_0 MA$. This is a well-known result in monetary theory. In particular, what is required for the existence of a monetary equilibrium is that endowments are such that $s(0,0) > 0$.

Sargent and Wallace's result is a departure from this traditional price level determination mechanism in the monetary overlapping generations model. A way to understand their result is to think about the policy setting process implicit in their formulation. Note that if the monetary authority chooses a value of the interest on reserves, then the stationary equilibrium conditional on that value of the interest rate, would be unique. However, the idea behind the multiplicity is that the monetary authority chooses the nominal interest rate on reserves to make it equal to whatever rate is observed in the market. So, if the rate in the market is r_1, then the monetary authority would set $i = r_1$; and if the rate in the market is $r_2 \neq r_1$, then the monetary authority would choose a different interest on reserves ($i = r_2$), which would validate the different real rate of interest r_2. In a sense, the indeterminacy is the result of the endogeneity of policy. The monetary authority is "chasing" the market, making policy endogenous and the equilibrium indeterminate. This is not the same source of multiplicity as in Proposition 4 of this paper. The nominal and the real interest rates are constant across all the equilibria in Proposition 4.

An important aspect of the Sargent and Wallace analysis is that by changing the nominal interest rate on reserves the monetary authority can validate any given equilibrium real rate of interest (in a continuum). In a monetary equilibrium of the Sargent and Wallace model, money cannot be dominated in rate of return and by changing the return on money the monetary authority changes the real return on savings. By contrast, in the model in this paper, the real rate faced by consumers is independent of policy.

Note that if the monetary authority in the Sargent and Wallace's model were to exogenously set a fix value of τ, then, again, the stationary equilibrium would be unique. Taxes are endogenous in their model. This is a point of contact with the model in this paper, where taxes also are assumed to accommodate endogenously to finance the payment of interest on reserves required by the different equilibria. Perhaps this aspect of the model suggests that my results and those of Sargent and Wallace are not too far apart in this fundamental level.

Hornstein (2010). Following Canzoneri et al. (2008), Hornstein (2010) studies a macroeconomic model with a banking system that may hold securities and reserves in the process of fulfilling its function. Banks take deposits from households and give loans to households that need them to buy the so-called "credit goods." Banks need to hold liquidity to service their deposits and both government bonds and reserves are perfect substitutes as a source of liquidity. Using the notation in this paper, Hornstein's liquidity constraint can be re-stated as follows: each bank must satisfy a technological constraint $s_t + \phi_{t+1} f_t = \theta \phi_{t+1} d_t$ with $\theta < 1$. Banks are also subject to a reserve requirement constraint, $f_t \geq \rho d_t$ with $\rho < \theta$. There is no bank capital constraint in Hornstein's model and, as a result, bank capital plays no role.

Hornstein's model can be interpreted as a particular case of the model in this paper when $\kappa = 0$ and $\xi = 0$. Note, however, that the liquidity constraint is further simplified relative to the one being used in this paper. We think that our more complex specification is more amenable to the interpretation given it here but this is not a fundamental distinction between the models of the two papers.

The model in Hornstein's paper has many features in common with the one studied in this paper. However, the focus of his analysis is different. Hornstein studies the dynamic stability properties of the steady state under the kind of monetary and fiscal policy rules analyzed by Leeper (1991). In that sense, Hornstein's framework is in the Neo-Wicksellian tradition, which is interested in macroeconomic behavior when the monetary authority is following an interest rate rule. Determinacy in Hornstein's analysis refers to the path of inflation and nominal interest rates, but there is no discussion of the level of equilibrium prices.

As Hornstein shows, it is still the case in his model that when the interest on reserves is equal to the return on government bonds, the equilibrium quantity of reserves is indeterminate in a range that depends on the parameters θ and ρ. More formally, in the notation of this paper, when $i_{ior,t} = i_{q,t}$, Hornstein shows that $f_t \in [\rho d_t, \theta d_t]$. It is also the case in Hornstein's model that households have a well-defined demand for cash. In particular, Hornstein uses a money-in-the-utility-function specification and the household demand for cash is given by:

$$\phi_t m_t = \gamma_m \frac{i_t}{i_t - 1} c_t,$$

where γ_m is a parameter in the utility function. Since there is a representative bank and a representative household, we have that $f_t = F_t$ and $m_t = M_t$. Just as in the model of this paper, $M_t + F_t = MA_t$ in equilibrium. Note then that for a given value of MA_t, changes in F_t within the interval $[\rho D_t, \theta D_t]$ will be associated with changes in M_t that would be reflected in different values of ϕ_t to the extent that aggregate consumption and the nominal interest rate remain invariant to changes in the composition of the monetary aggregate.

From this logic I conclude that for a given level of monetary aggregates there is, in fact, a continuum of steady state equilibria in Hornstein's model that can be indexed by the price level. It is true, however, that the set of variables that are the focus of study in Hornstein's model are invariant across these multiple steady states and the stability properties described in Hornstein's paper are the relevant ones when considering the behavior of the real economy under a policy regime that is committed to following Leeper-like interest rate and fiscal rules.[25]

The discussion about the limits on the ability of the central bank to change the monetary aggregate without a necessary adjustment to the price level does not arise in the context of policy based on interest rate rules. In such a setting the monetary aggregates adjust to whatever values are consistent with maintaining the targeted levels of the interest rate and long-run inflation. In consequence, the fact that more than one value of the monetary aggregate is consistent with the policy target is not consequential for the Neo-Wicksellian approach to monetary policy.

Ireland (2012). Ireland studies a dynamic stochastic general equilibrium model with a banking system where currency and bank reserves serve distinct functions and the central bank is able to pay interest on reserves. Households combine currency and bank deposits to economize on shopping time. Banks combine reserves and labor to "create" (produce) deposits. Both the shopping time technology used by households and the deposit production technology used by banks are such that there is imperfect substitutability between cash and deposits in the first case, and reserves and labor in the second case.[26] These assumptions imply that there is a well-defined demand for deposits (and currency) and a well-defined demand for reserves even when reserves earn the "market" interest rate

[25] Toward the end of his article, Hornstein discusses the problems that arise when the demand for government bonds is indeterminate, given that the fiscal policy rule is specified as taxes contingent on government bond demand. He proposes some simple solutions to this problem and hints at some of the issues discussed here (see, for example, footnote 17 in Hornstein, 2010).

[26] Ireland assumes a CES technology for the production of deposits, with reserves and labor as inputs. As a result, when the opportunity cost of holding reserves goes to zero, the demand for reserves grows without bound. To obtain

paid by other securities such as bonds. For given values of the various rates of return, the model pins down the *real* value of all quantities (reserves, deposits, wages, loans, labor allocation, etc.). In a steady state, any change in the quantity of monetary assets results in a proportional change in the price level and leaves all real quantities unchanged. While there are no reserve requirements in Ireland's model, reserves (not liquidity more generally) play a unique role in the production of deposits, and it is never the case that banks are indifferent between holding different levels of reserves or bonds (securities). For this reason, the real quantity of reserves is uniquely pinned down in equilibrium, and a result like the one presented in Proposition 4 of this paper cannot be proved for Ireland's model. Furthermore, Ireland does not study the role of bank capital, an important element in the logic behind Corollary 4.2 in this paper.

Ireland uses his model to study the dynamic response of the economy to various shocks when the central bank follows a Taylor-type rule for setting the market interest rate and firms face a quadratic cost of adjusting prices. The central bank also follows a rule for setting the interest on reserves. Since the market rate enters the interest on reserves rule, effectively, the central bank can be viewed as setting the spread between the market rate and the interest on reserves. Ireland uses the implied behavior of monetary aggregates to calibrate parameters of the technology for production of monetary services. The technological relationship between reserves and labor in the production of deposits is harder to calibrate, and Ireland conjectures parameter values and later performs sensitivity analysis. Ireland concludes that paying interest on reserves has important effects on the levels of reserves and other monetary assets, but not on output and inflation. Paying interest on reserves in Ireland's model allows the central bank to influence the real quantity of reserves, but it is always the case that any monetary policy action intended to decrease the price level in the long run would require a proportional reduction in the nominal supply of reserves. Breaking this tight relationship between the price level and the nominal value of monetary aggregates (present in Ireland's model and most of the existing alternatives) is the main objective of this paper (see Proposition 4).

Martin, McAndrews, Skeie (2013). The authors study a two-period model of an economy with many similarities to the one presented here but with some important differences. There is a set of households, a set of firms, and a set of banks. Banks take deposits from households, make loans to firms, and do not hold bank capital. There is also a central bank that pays interest on reserves, yet banks do not face a reserve requirement. Banks have a (convex) cost of issuing deposits, instead of a cost of providing loans as is assumed in this paper (this difference does not seem critical, though).

Martin, McAndrews, and Skeie normalize the initial price level to be equal to unity, so they are not able to consider the type of price level indeterminacy that is the main subject of this paper (Proposition 4). Furthermore, in Martin, McAndrews, and Skeie's model, there is no endogenous split between reserves and currency. The central bank can choose the initial quantity of reserves in the banking system. Notice that since the price level is equal to unity in the initial period this implies that by choosing the nominal quantity of reserves, the central bank also chooses the real quantity of reserves. As a consequence, Martin, McAndrews, and Skeie need to limit the size of the initial nominal reserves set by the central bank to be consistent with feasibility (see their assumption A2). In contrast, in the model of this paper, since the price level is fully endogenous, we do not need to place any restriction on nominal quantities.

In Martin, McAndrews, and Skeie's model, the equilibrium return on bank loans is always equal to the interest paid on reserves. In the case when the cost for banks of holding deposits is set to zero (the frictionless model), total bank lending is exogenous (given the amount of government bonds in the economy) and the inflation rate adjusts to equalize the nominal return on loans with the nominal interest rate paid on reserves. For this reason, inflation moves one to one with changes in the interest rate on reserves and there is no direct link between monetary assets and prices (or inflation). Furthermore, the rate of return on deposits is also equal to the rate of return on reserves.

The main result from the Martin-McAndrews-Skeie model is that the total amount of reserves

a bounded demand for reserves, Ireland introduces a small labor cost of managing reserves, which puts a positive lower bound on the opportunity cost of holding reserves.

provided by the central bank does not influence equilibrium bank lending. In other words, the quantity of bank lending is independent of the quantity of bank reserves in equilibrium. This is a strong result that cannot be confirmed using the model of this paper: While it is the case that the central bank cannot influence steady-state lending by changing the amount of monetary assets that it issues, in some situations there is an equilibrium correlation between total reserves and total loans (see, for example, Proposition 1). Recall, however, that in the model of this paper both lending and reserves are endogenously and jointly determined, so there is no obvious way in which one can be said to be causing the other.

When the cost of holding deposits is positive in the Martin-McAndrews-Skeie model, the interest rate on deposits can be different from (and lower than) the rate of return on reserves and loans. However, in this situation banks do not hold bonds. This is because households hold both deposits and bonds in equilibrium and the rates of return on these two investments need to be equalized in equilibrium, and they do at a lower level than the return on other bank assets. In this way, the model with costly deposits works very differently from the model of convenience yield on deposits studied in Section 5.

With a moderate cost of holding deposits and moderate levels of reserves, households in Martin, McAndrews, and Skeie's model absorb the changes in the cost of holding deposits induced by the changes in total deposits that result from changes in total reserves. As a consequence, bank lending remains unchanged. However, when the cost of holding deposits is high, an increase in total reserves can result in lower equilibrium levels of bank lending. This is because households have access to a storage technology that puts a lower bound in the equilibrium return on deposits. Increases in reserves do not result in one-to-one increases in deposits and hence, loans have to fall to maintain consistency with the balance sheet condition of banks.

The way reserves influence outcomes in the Martin-McAndrews-Skeie model is, hence, very different from the way reserves interact with other variables in the model studied in this paper. In all the propositions in this paper, with the exception of Proposition 4, the real value of total reserves is endogenously determined and cannot be freely manipulated by the central bank. In Proposition 4, the central bank could in principle induce a change in the real quantity of reserves outstanding, but such a change would be fully accommodated by a change in total deposits and/or holdings of securities by banks, without changing total bank lending. This is very different from what happens in Martin, McAndrews, and Skeie's model, where the central bank has full control over the real quantity of reserves and by manipulating this quantity can induce changes in total bank lending.

Part 4: All-capital banks

Consider the case when ξ and θ are equal to zero. Then, the liquidity constraint is not binding in equilibrium. As it will become clear later, if ξ and θ are positive, then having banks fully financed with capital requires even higher values of the endowment of bank experts. Recall that the balance sheet identity for the banking system implies that $L^* + q^* S_b^* + \phi_t^* F_t^* = \phi_t^* D_t^* + K_e^*$, and the reserves constraint requires $\phi_t^* F_t^* \geq \rho \phi_t^* D_t^*$. Using these expressions, we can show that if $W_e < L^*$, then banks will need deposits to fund their operations. To see this, suppose not. Suppose banks have no deposits. Then, we have:

$$L^* + q^* S_b^* + \phi_t^* F_t^* \geq L^* > W_e \geq K_e^*,$$

which contradicts the balance sheet condition for banks with no deposits. Then, to have an all-capital bank, we need $W_e \geq L^*$. Note that if banks are fully funded with capital, then the capital constraint will not be binding. Furthermore, since deposits are equal to zero, banks can choose to hold no reserves without violating the reserve requirement. Similarly, since the liquidity constraint is not binding, banks can hold no securities. In such a situation, total bank assets equal total loans L^* and as long as $W_e \geq L^*$ it is feasible to have a banking system financed only with bank capital.

Define the following reference interest rate:

$$\widetilde{r} \equiv \frac{1-\beta}{\beta} + \chi' \left(l^o \left(\widehat{\tau}_b \right) \right).$$

Now we can show that when the central bank pays no interest on reserves and $W_e \geq 1 - G(\widetilde{r})$, the equilibrium has banks fully funded with bank capital.

Proposition A3. Let $\xi = \theta = 0$ and $W_e \geq 1 - G(\widetilde{r})$ and $i_{ior} = 0$. Given a value of $\rho \in (0, 1)$ there exists a stationary equilibrium where banks hold no deposits or reserves and loans are fully funded with bank capital.

Proof. In a stationary equilibrium, if the central bank pays no interest on reserve, we have that $1 + r^*_{ior} = 1/(1+\gamma) \leq 1$. Furthermore, we have that if deposits were positive, then $1 + r^*_d = 1 + r^*_q = 1/\beta$ (from the households' problem). From the bank experts' problem, we have that $1 + r^*_k \geq 1/\beta$.

Assume now that the capital constraint is not binding. We will confirm this later. Then, we have that $1 + r^*_k = 1/\beta$. Going back to the bank's problem, when the capital constraint is not binding, the bank solves the following problem:

$$\max\ (r^*_l - r^*_k)\, l_j + \left(r^*_q - r^*_k\right) q s_j + (r^*_{ior} - r^*_k)\, \phi_t f_t - (r^*_d - r^*_k)\, \phi_t d_t$$
$$- \left(r^*_f - r^*_d\right) \phi_t b^f_{jt} - \chi(l_j) - \widehat{\tau}_b,$$

subject to

$$\phi_t f_t \geq \rho \phi_t d_t \geq 0.$$

Given the equilibrium rates of return postulated above, a direct implication of the bank's first order condition with respect to f_t is that the reserve requirement must be binding. The first order condition with respect to deposits is:

$$(r^*_{ior} - r^*_k)\, \rho - (r^*_d - r^*_k) \leq 0,$$

which implies that deposits must be equal to zero (if deposits were not equal to zero, the first order condition would have to hold with equality and from the household problem we have to have that $1 + r^*_d = 1/\beta$, which produces a contradiction). Since $r^*_q = r^*_k$, the banks are indifferent regarding their holdings of securities. The banking system balance sheet condition implies that $L^* + q^* S^*_b = K^*_e$. The bank's first order condition with respect to loans, combined with the zero profit condition for banks, gives us that $l^* = l^o(\widehat{\tau}_b)$ and $r^*_l = \widetilde{r}$. The loan market equilibrium implies that $L^* = 1 - G(\widetilde{r}) \leq W_e$. Hence, for all values of S^*_b such that $K^*_e \leq W_e$ we have that the market for bank capital clears at the rate r^*_k and the bank capital constraint is not binding since $L^* + q^* S^*_b = K^*_e > \kappa(L^* + q^* S^*_b)$. ■

While this proposition indicates that there is nothing in the model inconsistent with having banks funding their operations only with bank capital, it is also clear from the conditions involved that the case of all-capital banks is unlikely to be empirically relevant. In particular, it has to be the case that expertise on banking issues is so abundant in the economy that banks can fund all their lending using the resources directly available to this specialized group of agents.

Part 5: Some extra data

The value of ρ. Calculating the equivalent of the reserve ratio ρ in the data is delicate. In the United States only transaction accounts are subject to reserve requirements. Reserve requirements are also non-linear with a minimum amount not subject to requirements and two segments: For the amount in transaction accounts between \$13.3 million and \$89.0 million, the requirement is 3 percent; and for amounts larger than \$89.0 million, the requirement is 10 percent. In the model, there is only one type of deposit accounts. We could think that "producing" deposits requires a technologically given composition of some transaction accounts and some non-transaction accounts. In the data, the composition of deposits has been shifting significantly in the last couple of decades away from transaction accounts and toward non-transaction accounts (see Figure A1). In the proposed interpretation, we can say that the technology for producing deposits has been shifting over time. Reserve requirements, on the other hand, have not changed significantly. This is reflected in the ratio of required reserves over transaction account deposits in Figure A2. If we consider ρ to be the amount of reserves needed to produce deposits, then the value of ρ has been trending down, due to the technological shift in the production technology for deposits (see Figure A3).

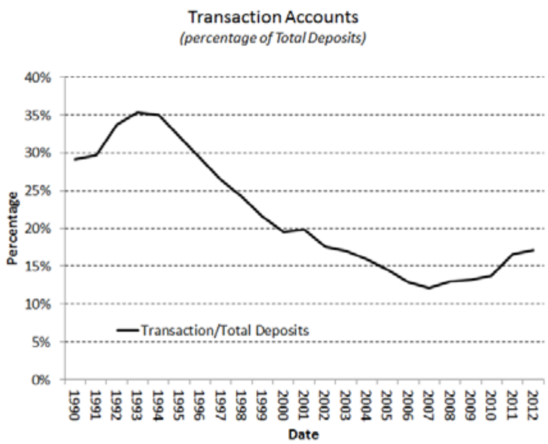

Figure A1

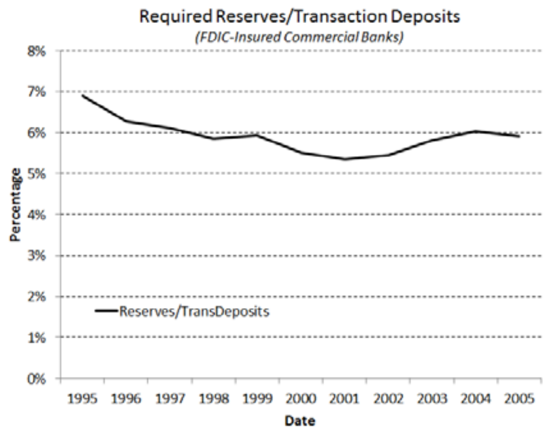

Figure A2

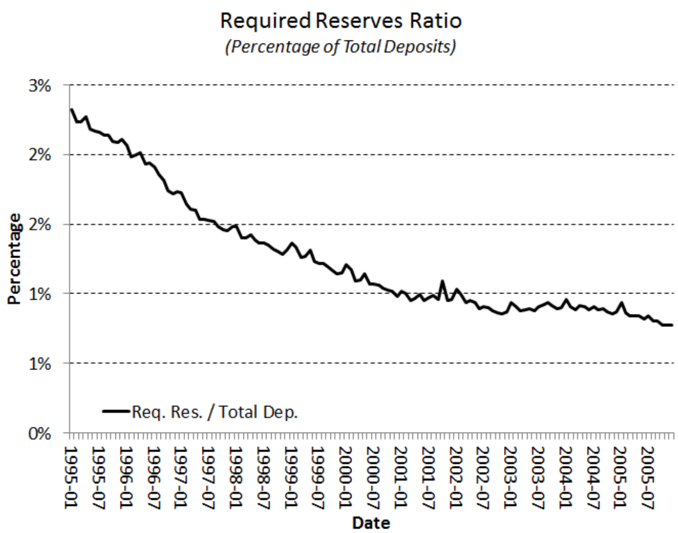

Figure A3

Banks liquid assets. A large proportion of the securities held by banks are Treasury and Agency securities (see Figure A4). Going back to Figure 2 in the Introduction, we see that total securities as a proportion of assets remained fairly stable during the period when reserves increased significantly (starting in 2008). At the same time, the proportion of Treasury and Agency securities in total securities was also relatively stable during that period. From this we conclude that there is not strong evidence of reserves substituting for other liquid assets in banks' balance sheets. See also re-enforcing evidence in Ennis and Wolman (2014).

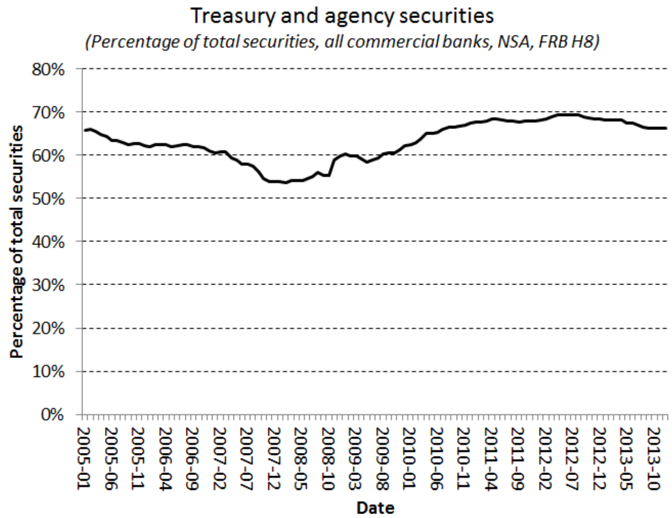

Figure A4

U.S. Inflation. We see in Figure A5 that after the 2008 recession, inflation was relatively stable even though there were periods between 2010 and 2013 when reserves were growing at a significant speed (see Figure 9).

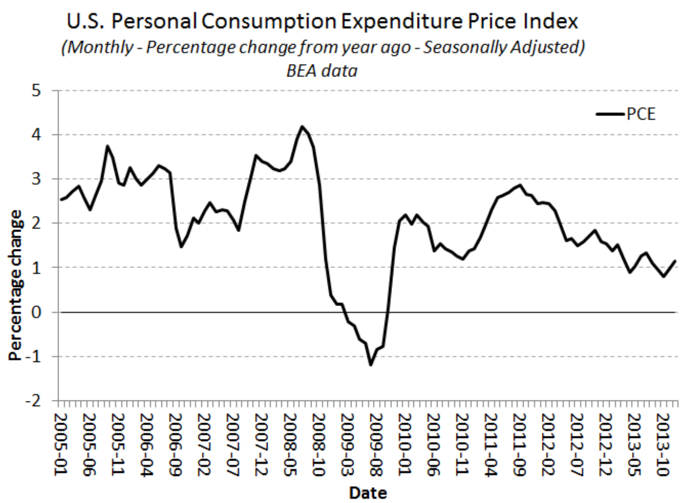

Figure A5

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