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Abstract

We study new transaction-level data of bank borrowings at the U.S. Federal Reserve discount window from 2010 to 2019. We merged this data with quarterly information about banks' balance sheets and income statements. To aid in the interpretation of our empirical analysis, we also develop a detailed model of the decision of banks to borrow from various sources, including the discount window. The objective is to contribute to a better understanding of the reasons why banks use the discount window during periods of relative calm in financial markets. We find that borrowing from the discount window is tightly linked to basic bank characteristics —such as size and location— and to the composition of banks' balance sheets. Most importantly, banks holding less reserves tend to borrow more often (and more) from the discount window. Similarly, banks with less liquid and riskier asset portfolios, and less market-pledgeable collateral, are also more likely to borrow from the Fed's discount window.

JEL Classification: E52, E58, G28
Keywords: Banking, Federal Reserve, Central Bank, Liquidity

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1 Introduction

The discount window is the most prominent and long-standing liquidity-support program offered by the Federal Reserve, the U.S. central bank. As such, it is a crucial institution of the U.S. financial system. Studying the role of the discount window has generally been constrained by data availability. This has changed recently: there is now detailed and reliable transaction-level data being published by the Fed on a regular basis. We take advantage of this new data and study, empirically and theoretically, the determinants of bank activity at the discount window between 2010 and 2019, a period of relative calm in U.S. financial markets.

The idea behind having the central bank acting as the lender of last resort has been around for more than two hundred years (Humphrey (1989)). While the role of the central bank as a backup source of funding has been particularly prominent during financial crises, it is the case that the U.S. discount window is open *at all times*, during crises and in "normal" times. It is also *regularly used* during those normal times. It is natural then to ask the questions: What is the role of the discount window in normal times? What is the discount window helping with at those times? Which banks borrow from the discount window outside of crises and why? We address these questions below with a combination of multivariate regression analysis and a theoretical framework that is deliberately designed to help us interpret the empirical findings that we uncover.

Having the discount window open at all times has benefits and costs. On the benefits side, the discount window plays a dual role in the interbank market. One role is to induce a ceiling on overnight interest rates, aiding the Fed in maintaining control on short-term rates as part of the process of monetary policy implementation (Ennis and Weinberg (2016)). The second role is to provide short-term liquidity insurance to eligible depository institutions. Both roles are related insofar as interest rates in the interbank market reflect, at times, liquidity events experienced by clusters of banks. A further, yet more practical consideration for having the discount window open at all times is to have it "ready to go" in case a crisis happens.

On the costs side, there are explicit and implicit costs of having the discount window open at all times. Aside from the normal operational costs, the central bank is also often involved in monitoring potential counterparties to be able to discern in a timely manner when illiquidity is (and is not) directly tied to insolvency. In addition to these explicit costs, there are also potential moral hazard costs linked to the availability of central bank funding, as this support changes banks' incentives to manage their liquidity and credit risk (Ennis and Price (2015)).

To assess the balance between costs and benefits of having a discount window open at all times, it is critical to better understand the motivations of those borrowers actually accessing the facility. With that in mind, we search for and identify systematic patterns in the data that describe the type of banks that use the discount window and the financial conditions of those banks at the time of borrowing. Knowing these patterns should allow us to make better judgments related to the essentiality (or lack of thereof) of a permanent discount window. It could also help in the design of alternative or complementary arrangements aiming at addressing the specific issues that we characterize.

There is also an important and long-standing policy debate around the value of having a discount window. On one side are those who question the need to have a discount window for the provision of liquidity to the market (Goodfriend and King (1988), Selgin (2017)) and those who emphasize the risk of the discount window becoming a vehicle for subsidizing poorly operated and possibly insolvent institutions (Friedman (1960), Schwartz (1992)). On the other side of the debate are those who advance the view that the discount window can be an effective source of liquidity for banks, filling gaps produced by transitory liquidity shortfalls (Clouse (2000)) and those who emphasize the role that the discount window can play in promoting stability of the financial system, not only during crisis, but more generally, at all times (Fischer (2016)). Our study is of course also relevant for this more general policy debate.

Market frictions are a key justification behind discount window lending. If frictions are not significant, then open market operations should be sufficient to achieve the objectives of the central bank (Goodfriend and King (1988)). The nature, strength, and implications of such frictions can vary widely depending on general financial market conditions. For example, some of the frictions most important for understanding discount window activity during normal times may be trumped by the general disruptions occurring during crisis. So, while at calmer times banks carefully evaluate the time and resources necessary to secure suitable trading counterparties in the market, those trading links might be globally disrupted during a financial crisis. For these reasons, we have taken an approach here that intends to account and emphasize aspects of the problem at hand that are particularly relevant in a non-crisis period.[1]

A crucial and novel feature of our data is its level of detail. Until recently, public information about activity at the discount window was limited. Traditionally, the Federal Reserve published discount window lending only at an aggregate level and at a weekly frequency. One justification for providing limited information has been the fear that the disclosure of information could impact the effectiveness of the facility (Kleymenova (2016)). Indeed, in March 2020, the Fed made changes to its weekly reporting to reduce the amount of discount window information available at higher frequencies. The view supporting such a change is that banks might become reluctant to access the discount window if they perceive that the information will be made public and subsequently interpreted negatively by potential counterparties. This type of stigma effect is often discussed by policymakers (Bernanke (2008)) and has received attention in the theoretical and empirical literature.[2]

A competing view and a common reaction to the events of the 2008 financial crisis is that transparency is particularly important when it comes to the administration of government lending programs. In response to such demand for extra information, starting in July 2010, the Dodd-Frank Act requires the Fed to make public detailed information about individual loans extended at the discount window. In a compromise that reflects the concerns associated with excessive disclosure of information, the transaction-level data is released with a two-year delay. The availability of this new, more detailed information provides an opportunity to take a closer look at what motivates actual borrowing at the discount window. We take that opportunity in this paper using the first ten years of data.

Aside from the relative calm during our sample period, an important factor is that the banking system was operating in an environment with ample reserves (Carpenter et al. (2015), Ennis and Wolman (2015)). This was unprecedented and a signifi-

[1] The role of the discount window during crises has been more extensively studied in recent years. For example, Berger et al. (2017), Gauthier et al. (2015), Li, Milne, and Qiu (2020), Gilbert et al. (2012), and Gerlach and Beyhaghi (2020) study empirically discount window lending in the U.S. during the financial crisis. Klee (2019) and Armantier et al. (2015) focus more narrowly on discount window stigma, also during the financial crisis. Drechsler et al. (2016) study empirically discount window lending in Europe during the financial and sovereign debt crises. The study by Gerlach and Beyhaghi (2020) includes the period under consideration here, but the focus of that paper is on the signal value of discount window activity about the financial conditions of the bank and, in particular, their probability of failure. Here, instead, we focus on the determinants of discount window activity.

[2] See, for example, Klee (2019), Ennis and Weinberg (2013), Armantier et al. (2015), Gauthier et al. (2015), Ennis (2019), Hu and Zhang (2021), Ennis and Price (2020), and the citations therein.

cant change from pre-crisis conditions (i.e., the previous period of normal times). In principle, abundant reserves would tend to reduce the chances of banks experiencing liquidity shocks that push them to borrow from the discount window. This tendency notwithstanding, we observe non-trivial amounts of borrowing at the discount window during the period under consideration (Ackon and Ennis (2017)).[3]

As a preliminary step, we report in Section 2 some broad correlations between bank characteristics and the use of the discount window. We find that larger banks are more likely to borrow from the discount window —that is, of the group of large banks, the percentage that borrows is relatively high— even though most of the borrowing is done by smaller banks, which are more numerous. Borrowers tend to hold less reserves and more illiquid asset portfolios. On the liability side, borrowers rely more on short term funding (such as repurchase agreements). In this way, based on the composition of their liabilities, borrower banks look more similar to larger banks, while on the asset side similarities are less evident. Borrowers also seem to have more risky assets that tend to lower their risk-based capital ratios. Again, this is something that one can also observe for larger banks. In general, then, discount window borrowers share some characteristics with larger banks, although less than 10 percent of the banks borrowing at the discount window are larger than $10 billion in assets. Accounting for these broad patterns is relevant as a preliminary step, but the confounding of size with other various characteristics quickly makes clear that a multivariate analysis is needed to untangle the origin of such patterns.

Before moving to a more thorough empirical analysis, we present in Section 3 a model of the decision of a bank that, under some circumstances, values having access to the discount window and borrows from it. The model is intended as a framework to guide our thinking when interpreting the various patterns uncovered in the data. The decision of the bank in our model is similar to that studied recently in Ennis (2018) and Afonso, Armenter, and Lester (2019) and is in the tradition of Poole (1968) and the extensive literature that sprang after that seminal paper.[4] Relative to the previous literature, and given our interest in aiding the interpretation of the discount

[3]Large quantities of excess reserves in the system imply that many banks may be close to indifference when it comes to holding the marginal unit of reserves. Hence, holding patterns in the cross section of banks may be harder to identify. However, as our model will illustrate, for those banks that actually borrow from the discount window, their holdings of reserves are likely more tightly linked to other financial decisions.

[4]See, for example, Ennis and Keister (2008) and, more recently, Bianchi and Bigio (2022), Armenter and Lester (2017), Berentsen, Kraenzlin, and Mueller (2018).

window data, our model involves a more complete description of the bank's balance sheet and a more flexible interpretation of the trading possibilities of the bank facing a liquidity or payment shock.

In the model, when the interest rate charged at the discount window is higher than the rate in the interbank market, as is generally the case in the U.S., a bank with access to the interbank market will not use the discount window. However, in some situations, depending on the nature and timing of a shock, the bank my not have ready access to the interbank market. In such a case, the bank will follow a "pecking order" to cover its liquidity need, using first its holdings of reserves, then discount window borrowing and, finally, if the shock is large enough to exhaust the bank's collateral pledged at the discount window, the bank incurs an overdraft in its account at the central bank, which is more expensive. This pecking order, in turn, determines the way the bank will choose ex ante its level of reserves and other components of its balance sheet.

Importantly, the model also illustrates how the structure of the distribution of shocks influences the ex-ante choice of level of reserves and other components of banks' balance sheets. Those decisions, in turn, interact with the shocks and determine the actual probability of borrowing from the discount window. This interaction makes the relationship between balance sheet components and discount window borrowing far from straightforward and the model helps in parsing out the various forces at play.

Section 4 presents the multivariate empirical analysis and our main empirical results. We start by looking at the probability that a bank borrows from the discount window, conditional on having ready access to it. Not all banks have immediate access to the discount window; some due diligence is required from the part of the bank. Unfortunately, the data does not include explicit bank-level access information, but we are able to construct a reasonable proxy based on the information that we have.

We obtain several important empirical results. First, we establish conclusively that banks holding less bank reserves as a proportion of assets, ceteris paribus, are more likely to tap the discount window for funding. This finding survives when we control for the bank's balance sheet composition, size measured by assets, and a number of other bank characteristics that, in principle, could matter. Furthermore, the finding is robust to accounting for possible endogeneity in reserve holdings.

We also confirm that larger banks are more likely to borrow even after controlling for various observable bank characteristics. One possibility that may explain this finding is that larger banks have a more sophisticated liquidity-management process that integrates the discount window more effectively. In line with this idea, we find that size matters most when we restrict the sample to banks below $10 billion in total assets. That sample presumably includes highly heterogeneous banks, with very different levels of sophistication, which asset size is helping to capture.

In terms of balance sheet composition, banks with more illiquid and riskier asset portfolios are more likely to borrow from the discount window, even after controlling for bank size. Similarly, those banks relying more heavily on less stable funding sources also find themselves more often needing to borrow from the discount window. Interestingly, even after controlling for size and other main bank characteristics, banks in certain Federal Reserve districts remain more likely to use the discount window than those in other districts. This could reflect some unobserved (un-controlled) heterogeneity in the composition of banks across Fed districts, or it may indicate a certain degree of (informal) differential treatment with respect to discount window usage across those districts.

Taken together, our empirical results suggest that banks' decisions to borrow from the discount window in "normal" times is the consequence of a deliberate liquidity-management decision. Banks are intentional with respect to their use of the discount window and use the discount window when it becomes the economically relevant marginal source of funding for the bank.

We close the paper with a short Section 5, where we briefly discuss how we read and understand our findings more broadly and, then, conclude.

2 Background

2.1 Data

Our dataset comes from a combination of various regulatory and central bank data-collecting efforts. The primary source is detailed information on daily borrowings at the discount window, available on the Federal Reserve Board's public website.[5] The data include information on the name of the borrower, the size and duration of the discount window loan, the type of loan (primary credit, secondary credit, or seasonal

[5]Refer to https://www.federalreserve.gov/regreform/discount-window.htm.

credit), and the borrower's Federal Reserve district. Also available is information on the types and amounts of collateral the borrower has posted at the discount window for borrowing purposes. The universe is all discount window loans extended in each quarter from July 2010 to September 2019.

We pair the discount window data with information on bank balance sheets. We use quarterly Call Report filings for all depository institutions eligible to borrow at the discount window, which includes commercial banks (form FFIEC 031/041), foreign banking organizations (FFIEC 002), and credit unions (NCUA 5300). These reports provide information on balance sheet items, including various assets and liabilities, as well as some off-balance sheet items, such as unused loan commitments.

2.2 Overview

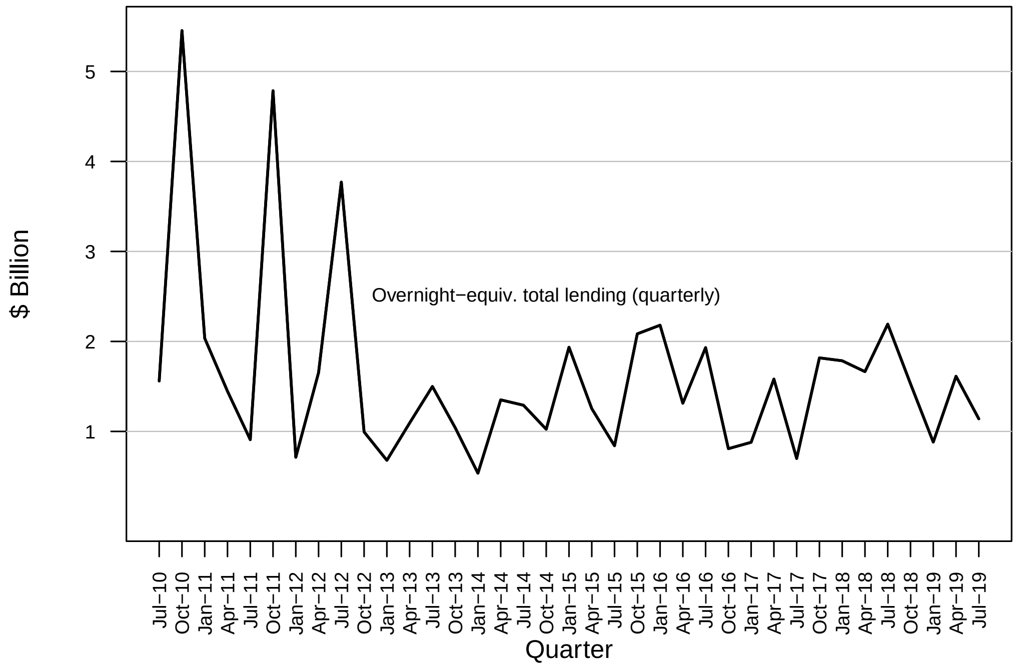
There are three discount window programs: primary credit, secondary credit, and seasonal credit. Primary and secondary credit are the main programs through which the Federal Reserve provides back-up, short-term funding to depository institutions. These two programs conform closely with the traditional lender-of-last-resort view of central bank liquidity provision. By contrast, the seasonal credit program is aimed at satisfying seasonal fluctuations in the funding needs of particular institutions.

The primary credit program is a standing facility (no questions asked) available to depository institutions in sound financial condition. Primary credit loans carry a penalty interest rate, which was 50 basis points higher than the upper bound of the policy target range for the federal funds rate during our sample period. Secondary credit is available (subject to the discretion of each Reserve Bank) to depository institutions not eligible to borrow from the primary credit program. The penalty rate in the secondary credit program is 50 basis points higher than the rate paid for primary credit. Most lending in the primary and secondary credit programs is overnight.[6]

We focus mainly on primary credit, as it is the program most often considered and used. Figure 1 plots the total amount of lending done in the primary credit program each quarter between July 2010 and September 2019, our period of normal times.

[6]By contrast, seasonal credit is not provided at a penalty interest rate and instead is offered at a floating market rate based on the average of the federal funds rate and the rate on three-month CDs. The interest rate is reset every two weeks and applies to all outstanding seasonal credit loans. Moreover, seasonal credit is generally longer term than overnight. Refer to https://www.frbdiscountwindow.org/ for more details.

Figure 1: Total lending – Primary credit (quarterly)



Because some loans have longer maturities than overnight, we calculate overnight-equivalent amounts that are then summed to a quarterly frequency.[7] While discount window lending is an order of magnitude smaller during normal times than in crisis periods, it is still a meaningful amount – in many quarters during the sample period more than $1 billion dollars in loans were made in the primary credit program. In contrast to the financial crisis period of 2007 to 2009, primary credit borrowing did not meaningfully increase with broader market events, including the European debt crisis in the summer of 2011. On the other hand, early on in the sample period, there appears to be some seasonality in the total lending series, with credit picking up in the last quarter of each year. This will motivate our use of time-based dummies (quarterly and yearly) to assess robustness of results. For a more detailed discussion of possible seasonality, see Ackon and Ennis (2017) (including their Figure 4 and Table 7).

In terms of the number of loans, between July 2010 and September 2019, there were 22,352 discount window loans made in the primary credit program and 792 loans made in the secondary credit program (Table 1). Many of those loans were for

[7] In these calculations, a loan of $100 million for two days is equivalent to two overnight loans of $100 million each. This transformation is necessary to account for varying maturities across loans in the computation of aggregates.

relatively small amounts and were initiated by the borrowing institutions in order to *test* the processes and systems involved in executing a transaction.

Table 1: Primary and secondary credit, July 2010 to Sept. 2019

	All loans	Loans greater than or equal to $1 million						
			Amount in $ millions					
	N	N	Mean	Median	Min	Max	Std. dev
Primary credit	22,352	6,133	9.88	3	1	2,700	50.56
Secondary credit	792	30	6.01	4.25	1	51	9.16

Note: Reflects overnight-equivalent borrowing amounts. If restated in raw amounts, summary statistics for primary and secondary credit loans of $1 million or more are somewhat lower, with a maximum primary credit loan of $1.017 billion. Source: Federal Reserve Board, available at https://www.federalreserve.gov/regreform/discount-window.htm.

The data do not include information on which loans are tests and which ones are not. To account for this fact and study banks' lending behavior uncontaminated from other administrative decisions, we focus on loans greater than or equal to $1 million. Even though this subsample of 6,133 loans is much smaller than the full sample, we think that only loans of relatively larger size would reflect a deliberate economic decision on the part of the borrower.[8]

We use the information on smaller loans and testing to construct our proxy for "access" to the discount window. To have immediate access, institutions need to take some basic steps, including internal approvals, a lending agreement, and the pledging and evaluation of collateral at their respective Reserve Bank. These steps can take time; sometimes days or even weeks. Since we do not have information on which institutions have the needed arrangements in place to borrow at the discount window, we use a proxy, described in detail in section 3.5 below. The behavior of banks that have access to the discount window can be expected to differ from the behavior of banks without access; we address this issue theoretically and empirically below.

The discount window data include borrower information that allows us to split the sample into institutions of different types, such as domestic banks, credit unions, and foreign banking organizations (FBOs). As shown in Table 2, while most of the

[8]Smaller loans are not likely to receive much managerial scrutiny, either because the loan is just a test loan or because it is too small to warrant much evaluation —just as an example, a $1 million overnight loan at an interest rate 100 basis points higher than the alternative best rate produces a total extra cost of approximately $30, a very small amount.

borrowing (defined as loans greater than or equal to $1 million) was done by smaller domestic banks (defined as those with less than $10 billion in assets), the percentage of institutions of each type that borrowed from the discount window is the highest for larger domestic banks (38 percent), followed by larger credit unions (20 percent) and larger FBOs (14 percent). Interestingly, smaller credit unions are the least likely to borrow from the discount window.

Table 2: Discount window borrowing by type and size, July 2010-Sept. 2019

	Number of banks	Borrow at least once		Borrow at least five times	
		Number	Percent of total	Number	Percent of total
Smaller banks					
Domestic banks	7,177	669	9	185	3
Credit unions	7,509	215	3	59	1
Foreign banks	191	37	19	9	2
Larger banks					
Domestic banks	108	41	38	9	2
Credit unions	5	1	20	0	0
Foreign banks	49	14	47	7	2

Note: "Borrowers" are defined as domestic banks that file Call Reports, and that take out a discount window loan at least once for $1 million or more over the sample period. "Larger banks" are defined as banks with at least $10 billion in assets in the fourth quarter of 2014, roughly the midpoint of the sample.

To get a sense of intensity of use, we also compute the number of banks that borrowed at least five times during our sample period. Of the roughly 110 larger domestic banks in our sample, 9 borrowed at least five times during the period, or 2 percent of the total. This figure is 3 percent for smaller domestic banks. Taken together, these statistics suggest that both larger and smaller domestic banks rarely borrow repeatedly from the discount window.

An important caveat here is that we have defined the group of large banks as those that are $10 billion or larger. As is well-known, the size distribution of banks in the U.S. is very skewed, with only a few very large banks. As it turns out, none of those very large banks borrowed from the discount window during our sample period. Even when considering those banks with $100 billion or more in assets, only a few borrowed — and when they did, it was very infrequently and for a very small amounts relative to their size. For this reason, when we discuss the borrowing behavior of larger banks, it should be kept in mind that this is not a reference to the very largest ones that are so prominent in the U.S. system.

Table 3: Balance sheet ratios–Domestic banks

	Non-borrowers		Borrowers		Large banks	
	Mean	Median	Mean	Median	Mean	Median
Share of assets						
Reserve balances	0.046	0.032	0.035	0.021	0.055	0.028
Treasury securities	0.007	0	0.005	0	0.017	0.001
C & I loans	0.076	0.063	0.098	0.079	0.126	0.113
Residential mortgages	0.132	0.091	0.127	0.089	0.116	0.076
CRE loans	0.206	0.181	0.268	0.268	0.158	0.139
Short loans to total loans	0.279	0.262	0.304	0.280	0.470	0.479
Share of liabilities						
Transaction deposits	0.286	0.291	0.200	0.142	0.095	0.085
Fed funds borrowed	0.002	0	0.003	0	0.005	0
Repurchase agreements	0.007	0	0.014	0	0.020	0.006
FHLB advances	0.033	0.006	0.049	0.030	0.046	0.021
Balance sheet and capital						
Log(total assets)	12.144	12.012	13.557	13.367	17.542	17.217
Unused commitments to assets	0.112	0.079	0.153	0.122	0.478	0.238
Tier 1 ratio	0.177	0.153	0.152	0.138	0.142	0.128
Annual ROA	0.034	0.034	0.035	0.034	0.033	0.029
Number of banks	6,575		710		108	

Note: This table provides summary statistics on balance sheet items for domestic banks. "Borrowers" are defined as domestic banks that file Call Reports, and that take out a discount window loan at least once for $1 million or more over the sample period. "Non-borrowers" are defined as banks that file Call Reports and do not take out a discount window loan over the sample period. "Large banks" are defined as banks with at least $10 billion in assets at the end of 2014; these can be either borrowers or non-borrowers. Summary statistics are calculated as means and medians of bank averages over the sample period. We eliminate banks with missing or negative assets, zero loans, negative cash-to-asset ratios, loan-to-asset ratios greater than 1, deposit-to-liability ratios greater than 1, negative federal funds borrowings, negative capital, and reported posted discount window collateral greater than total assets. Eliminating these banks leads to a smaller sample size than in Table 2.

Due to regulatory reporting rules, more information is available for domestic banks. For this reason, in general, we will narrow our attention to this important group. As we discussed, Table 2 suggests that larger domestic banks are more likely to borrow. At the same time, larger banks tend to have business models that can produce a differential approach to discount window borrowing. In Table 3, we report the average balance-sheet composition of domestic banks that borrowed at least one time from the discount window (middle column). We use the table to compare them with non-borrowing banks in the sample (left column) and with all large domestic banks in the sample (with more than $10 billion in assets, right column), regardless of whether they were borrowers or non-borrowers.

On the asset side, discount window borrowers tend to have less liquid assets than non-borrowers. Overall, borrowers have lower average shares of assets in reserves, lower shares of Treasury securities, and higher shares of C&I and CRE loans. Some of these patterns are also observed for larger banks and could be (partly) behind the higher incidence of borrowing on the part of larger banks. Still, the median borrower seems to hold less reserves than both non-borrowers and large banks, which suggests that size is not the primary determining factor.

On the liability side, borrowers tend to have less liquid liabilities than non-borrowers, as well. Borrowing banks hold lower shares of transaction deposits, although not quite as low as the largest banks. Fed funds and repo borrowings tend to be small in general, and the preponderance of larger banks probably boosts these shares among borrowing banks. Banks that borrow at the discount window also tend to borrow at FHLBs; differences in shares are particularly notable when looking at the difference in medians for borrowers and non-borrowers. Based on these patterns, we cannot rule out that the differences between borrowers and non-borrowers on the liability side of the balance sheet is driven by differences in size.

Borrowing from FHLBs deserves further discussion. FHLBs are active providers of backup liquidity to member banks, and sometimes are considered to be the "lenders of next-to-last resort" (Gissler and Narajabad (2017)). It is particularly important, then, to understand the interaction between discount window activity and this alternative source of funding. FHLB advances are collateralized loans, with terms that are usually longer than the regular overnight discount window loan. Furthermore, the collateral acceptable at the discount window tends to be broader than that required by FHLBs —which is, mainly, real estate related assets and government securities (with some minor exceptions). Along the way in the paper, we will make a deliberate effort to account for the role of FHLB advances within the context of our study.[9]

Turning now to other balance sheet measures, on average, borrowers are somewhat larger in size than non-borrowers (in line with Table 2). Still, the median borrower is

[9]See, also, Ashcraft, Bech, and Frame (2010) for a detailed discussion of FHLB lending to banks during the 2008 financial crisis.

much smaller than the median large bank, consistent with the fact that there is little to no borrowing among the very largest banks. In addition, borrowers have more unused commitments compared with non-borrowers. This pattern may be driven by the relative predominance of larger banks among borrowers: Unused commitments at larger banks are higher than those at both borrowers and non-borrowers, and given the difference between mean and median, this pattern may be driven by a few banks with a very high proportion of unused commitments relative to assets.

Also in the bottom panel of Table 3, we see that there are no significant differences in return on assets across the different subsamples. Borrowers do appear to have lower tier-1 capital ratios relative to non-borrowers. Larger banks also have lower capital ratios, so the difference could be driven mainly by size; the median large bank has a tier-1 capital ratio that is 1 percentage point below that of the median non-borrower.

Table 4 further investigates issues related to assets and capital. One of the main findings in Drechsler et al. (2016) is that banks that borrowed from the discount window in Europe during the sovereign debt crisis in 2011-12 held less capital and more risky assets. In principle, this could be a pattern that arises mainly during crises. However, Table 4 suggests that U.S. domestic banks present a similar pattern in our sample period of "normal" times in the financial system. As in Table 3, we see that borrower banks tend to have lower tier-1 capital ratios (15 percent) relative to non-borrowers (18 percent) but a bit higher than those for the largest banks (14 percent). The lower ratios may be explained by higher risk-weighted assets (RWA) for a given level of total assets (i.e., more risky assets) or lower capital levels.

Table 4: Domestic banks–capital ratios, DW borrowers and non-borrowers

	Non-borrowers		Borrowers		Large banks	
	Mean	Median	Mean	Median	Mean	Median
Tier-1 capital ratio	0.177	0.154	0.152	0.138	0.141	0.127
Risk-weighted over total assets	0.654	0.664	0.705	0.715	0.712	0.732
Tier-1 capital to total assets	0.108	0.101	0.103	0.097	0.095	0.091
Number of banks	6,575		710		104	

The bottom two rows of Table 4 suggest risky assets drive the difference. Borrowers tend to have more risky assets that translate into higher levels of risk-weighted assets relative to un-weighted assets–this ratio hovers around 71 percent for borrowers and large banks but is 6 percentage points lower for the mean non-borrower. By contrast, tier-1 capital to total assets is similar across all three categories of banks.

In general, it seems fair to say that in terms of the composition of assets and liabilities, capital ratios and profitability, borrowers are different from non-borrowers, and also, they do not just replicate larger banks. In summary, multiple factors simultaneously influence discount window borrowing. The multivariate approach we take in Section 4 addresses this complexity in detail.

2.3 Interest rates

Before we move to the theoretical framework, it is helpful to discuss briefly the configuration of interest rates most relevant for understanding this period in the U.S. financial system. As we will see, portfolio decisions depend on the relative level of the various interest rates confronted by banks. During our sample period, interest rates exhibit a pattern that will allow us to narrow the discussion of the theoretical and empirical possibilities.

The interest rate on overnight overdrafts at the Federal Reserve was set at a penalty rate equal to the interest rate at the discount window primary credit program plus 4 percentage points (annual rate). There was also a minimum fee, and the rate was adjusted upward after multiple days of running an overdraft.

The interest rate at the discount window primary credit program was 50 basis points higher than the top of the target range for the policy interest rate (i.e., the effective federal funds rate). During our sample period, the interest rate on reserves was set equal to the top of the target range from 2010 through June 2018 and hence 50 basis points lower than the primary credit rate. Thereafter, the interest rate on reserves was lowered to below the top of the target range in a series of steps, and so the spread between the primary credit rate and the rate paid on reserve balances widened to as much as 70 basis points.

Interest rates in the interbank market were generally below the interest on reserves, as market rates traded within the range set by policy. Part of the interbank transactions are, of course, between a commercial bank and the FHLBs. While the comparison between FHLB posted interest rates and other market and administered rates is complicated by the additional requirements involved in taking advances from FHLBs (Ashcraft, Bech, and Frame (2010)), as a general matter, the all-in cost of borrowing from FHLBs was below the discount window rate during the full span of our sample period.

3 A theoretical framework

In this section, we introduce a framework to help with the interpretation of our empirical strategy and results. The framework describes the decisions of a bank that is exposed to shocks and needs to make adjustments to its balance sheet in response to those shocks. Under some conditions, but not always, the optimal response of the bank is to borrow from the discount window. The framework is intended to illustrate the mechanisms that generate the patterns observed in the data.

3.1 The model

Consider the problem of a bank that can make loans (l), hold liquid and illiquid securities (s^L and s^I, respectively) and reserves (f) and is able to fund those assets by attracting deposits (d) and engaging in other borrowing (b), and by holding equity capital (k). The bank also has an administrative resource cost $\chi(l)$ from managing a portfolio of loans of size l.

After choosing the initial allocation of assets and the structure of its liabilities, the bank is exposed to various shocks that can alter certain components of its balance sheet.[10] For example, the bank may experience an outflow of borrowed money (b), or a valuable client may choose to draw down a line of credit with the bank that changes total bank lending (l). To confront the funding needs that result from those shocks, the bank may use its reserves, liquidate some of its holdings of securities, or borrow from the interbank market (b^{FF}) or the central bank (via a discount window loan b^{DW} or with an overnight overdraft o). From the perspective of the model, FHLB advances can be seen as part of the bank's activity in the interbank market.

The framework is general in the sense that it allows us to interpret shocks as potentially reflecting access (or lack thereof) to different markets that the bank can use to adjust its balance sheet in response to those shocks. In particular, the bank may be able to trade in the securities market, or in the fed funds market, or in no market at all, depending on the timing of the shock and the time-sensitivity of the required adjustment.

For example, if a source of borrowed funds disappears late in the day, a bank's only

[10] The bank's problem is similar to the one presented in Ennis (2018), but modified to consider a situation where the bank experiences liquidity shocks that need to be accommodated with reserves, other holdings of liquid assets, or short-term borrowing from the interbank market or central bank.

alternative may be to use owned reserves or to borrow from the central bank (through the discount window or with an overnight overdraft) to cover certain payments needs (as in Poole (1968)). Some shocks, however, may give the bank more time to adjust, in which case the bank may be able to liquidate short-term securities or borrow in the interbank market.

Denote by ϵ the vector of shocks that a bank experiences. Initially, the bank chooses loans, securities, reserves, deposits, other borrowing, and capital, subject to the balance sheet constraint:

$$l + s^L + s^I + f = d + b + k, \tag{1}$$

with all variables restricted to be positive. After these decisions are made, the bank is exposed to the shocks ϵ which (possibly) impact the values of l, d, and/or b. We denote by $l(\epsilon)$, $d(\epsilon)$, and $b(\epsilon)$ the value of these variables, respectively, after the shocks. In response to a shock, the bank may be able to adjust its reserves and securities holdings. We denote by $f(\epsilon)$, $s^L(\epsilon)$, and $s^I(\epsilon)$ the ex-post value (after the adjustment) of these variables. Furthermore, the bank may borrow in the interbank market ($b^{FF}(\epsilon)$), at the discount window ($b^{DW}(\epsilon)$), or may run an overnight overdraft on its account at the central bank ($o(\epsilon)$). All these decisions together must satisfy the following "flow" constraint:

$$\begin{aligned}(l(\epsilon) - l) + (d - d(\epsilon)) + (b - b(\epsilon)) &= \\ &(f - f(\epsilon)) + (s^L - s^L(\epsilon)) + \omega(s^I - s^I(\epsilon)) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + o(\epsilon), \end{aligned} \tag{2}$$

where the parameter ω is the liquidation value per unit of illiquid securities.

If a variable is not affected by the shocks or is not adjusted (potentially due to the presence of market frictions and the timing of the shocks), then its ex-post value equals its ex-ante value. For example, if total loans are not affected by the shock and cannot be adjusted in a timely manner in response to the shock (because, say, they are longer-term commitments), then $l(\epsilon) = l$. Similarly, if the shock ϵ is such that, due to its timing, it does not allow the bank to adjust its securities holdings, then $s^L(\epsilon) = s^L$ and $s^I(\epsilon) = s^I$. That is, the bank's securities holdings after the shock are the same as before the shock. One way to interpret such a situation is that the shock is realized after securities markets are closed for the day, or activity in these markets is so reduced that no significant trading can be executed effectively.

Discount window lending in the U.S. is collateralized. For simplicity, we assume that only securities can be used as collateral.[11] Discount window borrowing, then, has to satisfy the following collateral constraint:

$$b^{DW}(\epsilon) \leq s^L(\epsilon) + \theta s^I(\epsilon), \tag{3}$$

where θ is the haircut applied on illiquid securities, while liquid securities have no haircut. When ω is expected to be less than one, the value of θ will likely also be set at a level lower than one to appropriately reflect the liquidation risk. We also assume that, after the shock, the bank cannot sell more than the amount of securities it is holding (no shorting of securities is allowed).

Finally, there are natural non-negativity constraints on reserves, discount window borrowing, and overnight overdrafts:

$$f(\epsilon) \geq 0, \quad b^{DW}(\epsilon) \geq 0, \quad o(\epsilon) \geq 0. \tag{4}$$

The bank takes as given the interest rates paid on deposits, r_L, interbank loans, r_{FF}, and other borrowings, r_B, the rates earned on loans, r_L, securities, r_{SL} and r_{SI}, and the cost of capital, r_K. Also, the bank takes as given the interest rates fixed by the central bank: the interest on reserves rate, r_{IOR}, the discount window rate, r_{DW}, and the rate charged on overnight overdrafts, r_o.

Given all those rates, the bank chooses the initial values of l, s^L, s^I, f, d, b, and k. The bank also chooses the functions $f(\epsilon)$, $s^L(\epsilon)$, and $s^I(\epsilon)$ subject to the feasibility constraints imposed by the timing of trade and the possibility that some markets are no longer available at the time that particular shocks are realized. Finally, the bank also chooses $b^{FF}(\epsilon)$, $b^{DW}(\epsilon)$, and $o(\epsilon)$. The objective of the bank is to maximize the following profit function:

$$\begin{aligned} E[(1 + r_L)l(\epsilon) + (1 + r_{SL})s^L(\epsilon) + (1 + r_{SI})s^I(\epsilon) + (1 + r_{IOR})f(\epsilon) \\ -(1 + r_D)d(\epsilon) - (1 + r_B)b(\epsilon) - (1 + r_K)k - \chi(l) \\ -(1 + r_{FF})b^{FF}(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_o)o(\epsilon)], \end{aligned} \tag{5}$$

[11] Banks can pledge many other types of assets as collateral at the discount window. Extending the model to accommodate broader classes of collateral is not difficult but notationally cumbersome.

subject to constraints (1), (2), (3), and (4). Note that in general $b(\epsilon)$ will be a function of the initial value of other borrowings chosen by the bank, b, modified by the impact of the shock. In principle, this is the case for loans and deposits as well.

To understand the decisions of the bank, we start with the ex-post adjustment that the bank optimally makes in response to a shock. Then, we study the ex-ante decisions on reserves holdings and other variables given the optimal ex-post response previously analyzed.

3.2 Ex-post response to shocks

Consider a bank that has chosen the level of loans (l), deposits (d), securities holdings (liquid and illiquid, s^L and s^I), and capital (k). After the shock ϵ, the bank liquidity needs are $\Delta(\epsilon)$ given by

$$\Delta(\epsilon) \equiv (l(\epsilon) - l) + (d - d(\epsilon)) + (b - b(\epsilon)).$$

To simplify the exposition, assume that the timing of the shock is such that the bank is not able to adjust securities after the shock. Then, using equation (2), we have

$$\Delta(\epsilon) = f - f(\epsilon) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + o(\epsilon), \tag{6}$$

which tells us the bank will use reserves, borrowings (from the interbank market or the discount window), and/or overnight overdrafts to cover its ex-post liquidity needs.

The relevant portion of the payoff function (5) for the bank in the ex-post decision-making process is given by

$$(1 + r_{IOR})f(\epsilon) - (1 + r_{FF})b^{FF}(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_o)o(\epsilon), \tag{7}$$

with the bank still subject to constraints (3) and (4), the collateral and non-negativity constraints, respectively. The bank needs to choose $b^{FF}(\epsilon)$, $b^{DW}(\epsilon)$, and $o(\epsilon)$ to maximize objective (7) given that $f(\epsilon)$ satisfies (6).

In terms of the relevant configurations of interest rates to consider, as discussed in Section 2.3, it is standard to have that $r_{IOR} < r_{DW} < r_o$. That is, the central bank's lending rate is higher than the deposit rate, and overnight overdrafts carry a penalty over borrowing at the discount window. With respect to the interbank market, given the simplified nature of the model, it makes sense to restrict attention to $r_{IOR} \leq r_{FF} < r_{DW}$. If $r_{FF} < r_{IOR}$, it would be profitable for any bank (facing no other balance sheet costs, as assumed here) to borrow in the interbank market to hold reserves and earn interest on reserves paid by the central bank. Since all banks would want to do the same, such configuration of interest rates would be inconsistent with the clearing of the interbank market.[12]

3.2.1 Active interbank market

The ex-post funding decisions of the bank will depend crucially on the funding alternatives open at the time of receiving the liquidity shock. In particular, if the bank still has access to the interbank market when the shock occurs, then the discount window will not be used, as the following proposition demonstrates.[13]

Proposition 1 (Active interbank market) *If the timing of the shock ϵ is such that the bank can trade in the interbank market when the shock is realized, and $r_{IOR} \leq r_{FF} < r_{DW} < r_o$, then $b^{DW}(\epsilon) = 0$ and $o(\epsilon) = 0$. Furthermore, if $r_{IOR} < r_{FF}$, then $b^{FF}(\epsilon) = \Delta(\epsilon) - f$.*

For the relevant configurations of interest rates, if the bank can trade in the interbank market, then it does not borrow from the central bank. Note that $b^{FF}(\epsilon)$ may be positive or negative, depending on the relative size of $\Delta(\epsilon)$ compared with the ex-ante level of reserves held by the bank, f. When $r_{IOR} < r_{FF}$, the bank will borrow or lend (respectively) in the interbank market the reserves that it needs to end the period with no holdings of reserves (i.e., so as to have $f(\epsilon) = 0$). If instead $r_{IOR} = r_{FF}$, then whenever $\Delta(\epsilon) < f$, the bank may choose to finish the period with a positive level of (*excess*) reserves (i.e., so as to have $f(\epsilon) > 0$).

The case when $r_{FF} = r_{DW}$ is less relevant in practice and hence not discussed in the proposition. This would be a situation where the system as a whole is systematically "short" on reserves and some banks have to borrow at the discount window to balance aggregate supply with aggregate demand. While there have been times when this situation was within the realm of possibilities (see, for example, Kasriel and Morris (1982)), the recent history in the U.S. is inconsistent with such general configuration of interest rates.

[12]For a recent paper where balance sheet costs are explicitly modeled, and hence the interbank rate can be below the interest on reserves, see Afonso, Armenter, and Lester (2019).

[13]Formal proofs are provided in the appendix.

It is, of course, still possible for certain banks at certain times to face interest rates in the interbank market that are higher than the discount window rate. This could happen, for example, if segmentation in the market created market power on the lending side (Bech and Klee (2011)). The situation in that case is equivalent to the case when the bank does not have access to the interbank market altogether, which we study next.

3.2.2 No active interbank market

Proposition 1 describes the case when the bank can (effectively) use the interbank market as its marginal source of funding to accommodate a given shock. Other shocks may happen at a time when the interbank market is not immediately accessible to the bank, either because the shock occurs late in the day, when the interbank market is thin or no longer active, or because the bank's usual counterparties are not able to accommodate its liquidity demand and the bank is not able to find other suitable trading partners on short notice. In that case, some discount window borrowing may be optimal as the next proposition demonstrates.

Evidently, when $s^L + \theta s^I = 0$, the bank has no available collateral to borrow at the discount window and, as a consequence, $b^{DW}(\epsilon) = 0$ regardless of the shock. In this case, whenever $\Delta(\epsilon) > f$, the bank incurs an overnight overdraft $o(\epsilon) = \Delta(\epsilon) - f$. To focus on the more interesting case when discount window borrowing can happen, assume that $s^L + \theta s^I > 0$.

Proposition 2 (No active interbank market. The pecking order) *Assume that $s^L + \theta s^I > 0$. If the timing of the shock ϵ is such that the bank cannot trade in the interbank market when the shock is realized, and $r_{IOR} < r_{DW} < r_o$, then:*

- *when $f \geq \Delta(\epsilon)$ we have that $b^{DW}(\epsilon) = 0$ and $o(\epsilon) = 0$;*
- *when $f < \Delta(\epsilon)$ we have that $b^{DW}(\epsilon) > 0$ and $f(\epsilon) = 0$. Furthermore,*
    - *if $\Delta(\epsilon) - f \leq s^L + \theta s^I$ then $o(\epsilon) = 0$,*
    - *if $\Delta(\epsilon) - f > s^L + \theta s^I$ then $o(\epsilon) > 0$.*

The bank follows *a pecking order* for funding the liquidity shock. If the shock is relatively small, the bank uses its holdings of reserves to cover the shock. For larger shocks, when the ex-ante stock of reserves held by the bank is not enough, the bank borrows from the discount window. In such case, the bank may or may not need to also incur an overnight overdraft, depending on whether the collateral pledged at the central bank is sufficient to back the required discount window loan.

Figure 2: The pecking order

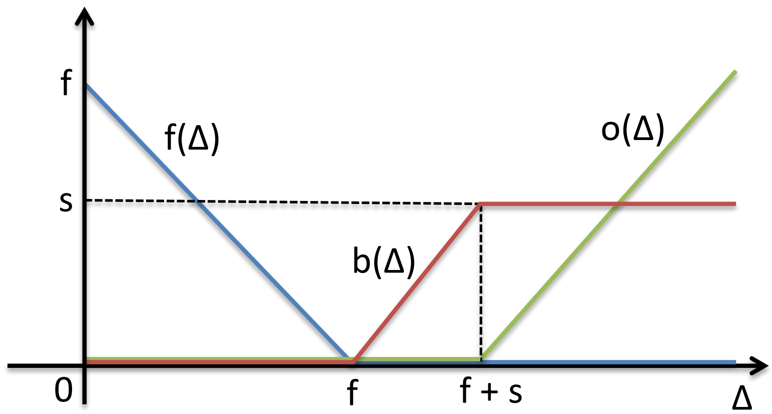


Figure (2) illustrates this pecking order. To simplify notation, we denote with s the total value of available collateral for the bank (i.e., $s \equiv s^L + \theta s^I$). On the horizontal axis, we measure the size of the shock Δ. When Δ is smaller than ex-ante reserves f, the bank adjusts its reserves holdings down to accommodate the shock. No central-bank credit is used in this case, and ex-post reserves are given by $f(\Delta) = f - \Delta$. When Δ is greater than the level of ex-ante reserves f, discount window borrowing $b(\Delta)$ is positive and ex-post reserves are zero. Finally, if the shock is greater than ex-ante reserves plus the discount window borrowing capacity of the bank, given by its available collateral s, then the bank incurs a positive overnight overdraft $o(\Delta)$ (i.e., a negative balance in its account at the central bank).

It is important to note that some of the structural parameters (such as, for example, the variance of the distribution of possible shocks) that influence the likelihood of observing a bank borrowing from the discount window also determine the bank's ex-ante choice of the level of reserves (and other components of its balance sheet). We turn to this issue next.

3.3 Ex-ante balance sheet decisions

When the bank is choosing the composition of its balance sheet, it anticipates that it will be exposed to shocks. Depending on the size and timing of those shocks, the bank may have different alternatives (including discount window borrowing) for addressing the resulting liquidity needs. In this section, we study the ex-ante portfolio decision of banks. To simplify the analysis, consider a situation when a bank has already decided the amount of loans, other borrowed money, and capital in its portfolio and now has to decide the amount of reserves, securities, and deposits to hold. For concreteness, we also assume that the shocks affect only other borrowed money. We discuss the more general case in Section 3.4.

At such point in the decision process, the problem of the bank is to choose reserves (f), securities (s^L and s^I) and deposits (d) to maximize

$$\widehat{V} \equiv E_\epsilon[(1+r_{SL})s^L + (1+r_{SI})s^I + (1+r_{IOR})f(\epsilon) - (1+r_D)d + (1+r_B)\Delta(\epsilon) \\ -(1+r_{FF})b^{FF}(\epsilon) - (1+r_{DW})b^{DW}(\epsilon) - (1+r_o)o(\epsilon)], \quad (8)$$

subject to (1), (2), (3), (4), and $f > 0$. Using (2) we have that

$$f(\epsilon) = f - \Delta(\epsilon) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + o(\epsilon),$$

which can be substituted into the objective function (8). Here, the functions $b^{FF}(\epsilon)$, $b^{DW}(\epsilon)$, and $o(\epsilon)$ are ex post optimal (as described in the previous section).

Particularly relevant for the interpretation of our empirical analysis, the objective here is to show how the choice of reserves (and securities) depends on the distribution of shocks and the ability of banks to use different sources of funding to accommodate those shocks. To this end, we use a simple example that clearly captures the tradeoffs involved. In this example, the structure of the shock-distribution is:

$$\Delta(\epsilon) = \begin{cases} \Delta_0 = 0 & \text{with prob. } 1 - p_1 - p_2, \\ \Delta_1 & \text{with prob. } p_1, \\ \Delta_2 & \text{with prob. } p_2, \end{cases} \quad (9)$$

with $0 < \Delta_1 < \Delta_2$.

Furthermore, after the shock happens, with probability q the bank is able to trade in the interbank market and with probability $1-q$ the bank can only cover a liquidity shortfall via the central bank (in the form of a discount window loan or an overnight overdraft). We denote with the subscript A the value of a variable when the bank can find a trade in the interbank market, and with the subscript N when it cannot.

Given our maintained assumptions on interest rates ($r_{FF} < r_{DW} < r_o$), Proposition 1 tells us that when the bank can trade in the interbank market after the realization of the shock, it will neither borrow at the discount window nor run an overnight overdraft. That is, $b_A^{DW} = 0$ and $o_A = 0$ regardless of the size of the shock. This is the case because it is cheaper for the bank to borrow in the interbank market at rate r_{FF} than to borrow from the central bank at rates r_{DW} or r_o.

When the bank cannot find a trade in the interbank market (with probability $1-q$), as in Proposition 2, whether or not it borrows from the central bank depends on its chosen level of securities and reserves. This choice, of course, depends on the cost of funding (in this simple case, the interest rate on deposits).

Denote by r_{CB} the relevant interest rate paid by the bank at the central bank. This rate could be equal to r_{DW}, r_o, or a combination of the two. There are three relevant thresholds for the level of the interest rate on deposits (i.e., the bank's funding cost) that we need to consider:

$$\begin{aligned}
r^{T1} &= qr_{FF} + (1-q)r_{IOR} \\
r^{T2} &= qr_{FF} + (1-q)[(1-p_2)r_{IOR} + p_2 r_{CB}] \\
r^{T3} &= qr_{FF} + (1-q)[(1-p_1-p_2)r_{IOR} + (p_1+p_2)r_{CB}].
\end{aligned}$$

A way to think about these thresholds is that they represent the value for the bank of holding an extra unit of reserves, depending on whether or not the bank needs to borrow from the central bank in response to the different realizations of the liquidity shock Δ. The bank will compare such value with the cost of obtaining an extra unit of reserves ex ante, which is given by r_D here.[14]

So, for example, if the bank is holding reserves sufficient to cover all possible realizations of the liquidity shock, then with probability q the bank will be able to lend out leftover reserves in the interbank market. With probability $1-q$, however, the bank will not be able to trade in the interbank market and will keep those leftover reserves, remunerated at the level of the interest on reserves. This situation generates the interest rate r^{T1}, and if the interest rate on deposits is higher than this threshold rate, then the bank would have no incentives to hold such a high level of reserves.

[14] In principle, the cost of funding an extra unit of reserves is the cost of the marginal liabilities created by the bank to obtain those reserves. Here, we have simplified the timing so that deposits always represent the marginal liability for the bank. Importantly, the assumption here is that the bank does not need to increase capital as its balance sheet grows – the capital constraint is not binding. If it were binding, then the marginal cost of funding would include a capital charge (as in Ennis (2018)).

Notice that if the interest rate on deposits r_D is below the threshold rate r^{T1}, then the bank would benefit from increasing deposits and reserves indefinitely. This situation would not be compatible with equilibrium, so we proceed by considering only situations where $r_D \geq r^{T1}$. Interestingly, if $r_D = r_{FF} = r_{IOR}$, then r_D equals r^{T1} and the bank will choose to hold sufficient reserves to cover all possible shocks and possibly a significant level of *excess reserves*. While this is a situation that appears relevant for many banks in the U.S. during the past several years, it is inconsistent with active discount window lending in the model. For this reason, we proceed by studying the model under the assumption that $r_D > r^{T1}$.

If the rates of return on securities (r_{SL} and r_{SI}) are equal to the rate of interest on deposits, then the bank will hold enough securities to avoid overnight overdrafts. In that case, $r_{CB} = r_{DW}$. We consider this case first and later explain how things change when the rate of return on securities is lower than the deposit rate and the collateral constraint at the discount window is occasionally binding.

Denote by b_{iN}^{DW} the amount borrowed at the discount window when the shock equals Δ_i with $i = 0, 1, 2$. Depending on the level of the interest rate on deposits, the bank will decide to hold reserves to cover, partially or fully, the different possible realizations of the liquidity shock. The following proposition describes such decision.

Proposition 3 (<u>Ex-ante decisions. No overdrafts.</u>) *When $r_{IOR} \leq r_{FF} < r_{DW} < r_o$ and $r_{SI} = r_{SL} = r_D$, we have that:*

- ∘ *if $r^{T1} < r_D < r^{T2}$ then $f = \Delta_2$, and if $r_D = r^{T2}$ then $\Delta_1 \leq f < \Delta_2$,*
- ∘ *if $r^{T2} < r_D < r^{T3}$ then $f = \Delta_1$, and if $r_D = r^{T3}$ then $0 \leq f < \Delta_1$,*
- ∘ *if $r^{T3} < r_D$ then $f = 0$.*

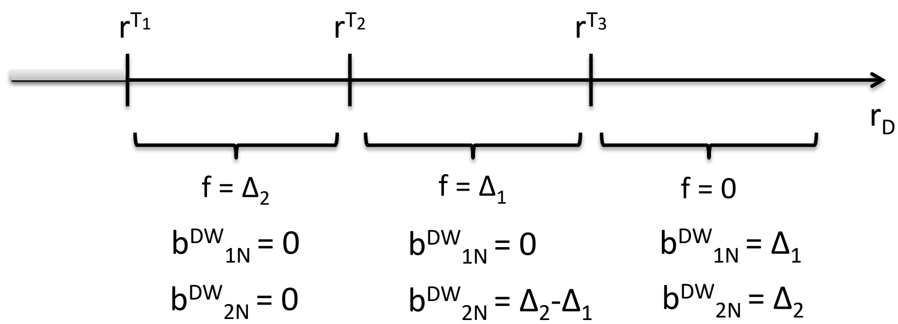
Furthermore,

- *when the bank can access the interbank market, $b_{iA}^{DW} = 0$ and $o_{iA} = 0$ for $i = 0, 1, 2$; and*
- *when the bank cannot access the interbank market, $b_{iN}^{DW} = \max \{0, \Delta_i - f\}$ and $o_{iN} = 0$ for $i = 1, 2$.*

Finally, $s_L + \theta s_I \geq \max \{b_{iN}^{DW}\}_{i=1,2}$.

Figure (3) summarizes the results from the proposition. The most interesting situation occurs when $r_D \in (r^{T2}, r^{T3}]$ because then, if the shock is large (equal to Δ_2) and the bank has no access to the interbank market, it borrows from the discount window even when holding a positive amount of reserves ex ante. For other interest rate values, either the bank never borrows from the discount window or it chooses to hold no reserves and hence borrows from the discount window whenever it receives a liquidity shock and has no access to the interbank market.

Figure 3: Interest rate thresholds



The proposition suggests a negative association between the level of reserves and discount window borrowing, given that $b_{iN}^{DW} = \max\{0, \Delta_i - f\}$. In other words, for a given shock process, higher levels of reserves holdings tend to be associated with lower discount window lending. However, this negative association may weaken when looking at a cross section of banks facing different shock processes. We illustrate this situation in the following corollary, where the structure of the shock-distribution is as in Proposition 3 (see expression (9)).

Corollary 3.1 (Ex-ante effects of heterogeneous shock-distributions.) *Consider two banks, 1 and 2, facing two different shock processes, with $\Delta^2(\epsilon) = \rho\Delta^1(\epsilon)$ and $\rho > 1$ so that bank 2 experiences larger shocks than bank 1. When the conditions in Proposition (3) are satisfied and $r_D \in (r^{T2}, r^{T3})$, bank 2 will hold (ex ante) higher levels of reserves and borrow more (ex post) from the discount window.*

As we saw in Proposition 3, when $r_D \in (r^{T2}, r^{T3})$, bank i will hold reserves $f_i = \Delta_1^i$ and will borrow at the discount window $b_i^{DW} = \Delta_2^i - \Delta_1^i$ when the shock $\Delta^i(\epsilon)$ equals Δ_2^i. As a result, bank 2 will hold higher reserves, since $\Delta_1^2 > \Delta_1^1$, and will borrow more from the discount window since $\Delta_2^2 - \Delta_1^2 > \Delta_2^1 - \Delta_1^1$.

The proportionality factor ρ is, of course, not necessary for the result. It is assumed here for convenience.[15] The corollary highlights the importance of recognizing the endogeneity of reserves holdings. Conditional on a shock process, higher reserves imply that a bank is able to accommodate more of those shocks without tapping the discount window. However, banks exposed to larger liquidity shocks may choose to hold higher levels of reserves and, at the same time, may need to borrow more (and more often) from the discount window. While the first logic indicates a negative relationship between reserves and discount window borrowing, the second can generate a positive relationship (as the corollary illustrates).

3.3.1 The impact of discount window access

The ex-ante level of reserves (and other components of the balance sheet) also depends on the ability of banks to access the discount window. If the bank is not able to access the discount window (because it has not made the necessary arrangements, for example), then $r_{CB} = r_o$ and the values for the thresholds of the interest rate on deposits change. The change reflects the fact that the bank will need to incur an overnight overdraft when it is short of reserves, since it has no access to the discount window. Given these new thresholds, denoted with a prime below, the bank's ex-ante choice of reserves is given by the following proposition.

Proposition 4 (Ex-ante decisions. No discount window access.) *When $r_{IOR} \leq r_{FF} < r_{DW} < r_o$ and the bank has no access to the discount window, we have that:*

- *if $r^{T1'} < r_D < r^{T2'}$ then $f = \Delta_2$, and if $r_D = r^{T2'}$ then $\Delta_1 \leq f < \Delta_2$,*
- *if $r^{T2'} < r_D < r^{T3'}$ then $f = \Delta_1$, and if $r_D = r^{T3'}$ then $0 \leq f < \Delta_1$,*
- *if $r^{T3'} < r_D$ then $f = 0$.*

Furthermore, $o_{iA} = 0$ for $i = 0, 1, 2$ $o_{iN} = max\ \{0, \Delta_i - f\}$ and for $i = 1, 2$.

The parallels between propositions (3) and (4) highlight the fact that during "normal" times, the discount window operates, in part, as an alternative central-bank source of funding that is less costly than overnight overdrafts.[16] It is also the case that for certain combinations of rates of return and funding costs, a bank with no

[15] As long as Bank 2 faces a shock process such that Δ_1 and $\Delta_2 - \Delta_1$ are both larger quantities than for Bank 1, then it will hold higher reserves and borrow more from the discount window.

[16] In fact, a common discussion in policy circles is the possibility of automatically transforming any shortfall in a bank's account at the central bank into a discount window loan, as long as the bank has the appropriate amount of collateral pledged (see, for example, Nelson (2019)).

access to the discount window will tend to hold higher levels of reserves than a similar bank that has access to the discount window, as the following corollary demonstrates.

Corollary 4.1 (Ex-ante effects of discount window access.) *Consider two banks, one with access to the discount window and one without it. Both banks face the same funding cost r_D. When the conditions in propositions (3) and (4) are satisfied and $r^{T2} < r_D < r^{T2'}$ or $r^{T3} < r_D < r^{T3'}$, the bank without access to the discount window holds more reserves than the bank with access the discount window.*

The basic logic behind this result is simple. When a bank has no access to the discount window, if the shock exhausts its reserves, then it has to incur an overdraft with the central bank, which is more expensive than a discount window loan ($r_{DW} < r_o$). For this reason, given the interest rates, the bank with no access to the discount window has more incentive to hold reserves.

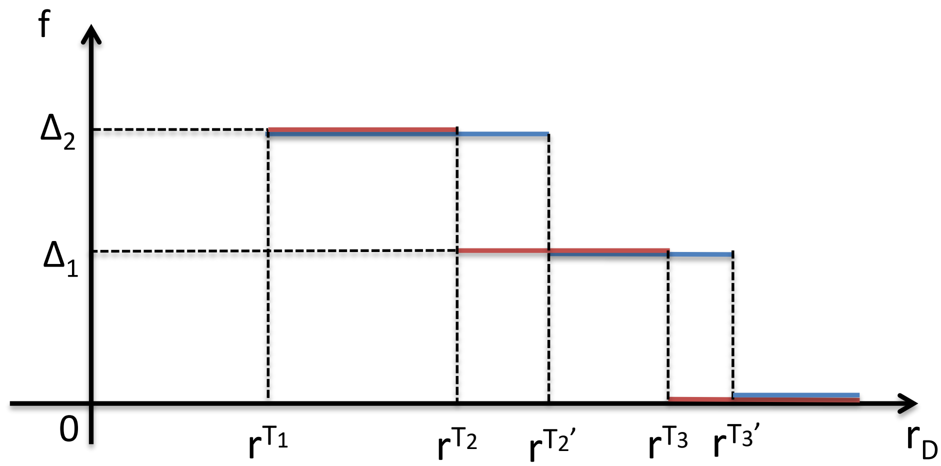
More specifically, in the context of the model, this logic is captured by the fact that the relevant interest rate thresholds for a bank with access to the discount window are lower than the thresholds for the bank without access. As we see in Figure 4, when funding costs are between the two values of a given threshold, the bank with access to the discount window (red lines) chooses a lower level of reserves than the bank without access (blue lines). For example, when $r^{T2} < r_D < r^{T2'}$, the bank with access to the discount window will choose reserves equal to Δ_1, and the bank without access to the discount window will choose reserves equal to Δ_2.

Of course, in principle, economic reasons may be driving a bank to make the necessary arrangements to be able to access the discount window. In that sense, "access" could be partly determined by, for example, the distributions of shocks faced by the bank, as was the case with the level of reserves. For this reason, the relationship between reserves and access in a cross section of heterogeneous banks can be difficult to disentangle, as we discuss further in the Appendix.

3.3.2 Binding collateral

The ability of a bank to use the discount window also depends on the amount of eligible collateral it has available. In this model, we have simplified collateral to be represented only by securities holdings. The ex-ante decision by the bank to hold securities, then, determines the amount of collateral held by the bank and its ability to use the discount window. When the return on securities (r_{SL} and/or r_{SI}) is equal

Figure 4: Endogenous reserves with and without discount window access



to the funding cost (r_D), the bank will hold enough securities so that the collateral constraint is never binding. This is the case we have discussed so far (see Proposition 3).

When the collateral constraint is binding for some realizations of the shock, the value of holding an extra unit of securities would be equal to the rate of return on that security plus the shadow value of relaxing the collateral constraint, which we denote by $\lambda_{CC} \geq 0$. For the bank to hold both types of securities in its portfolio, the following two conditions must hold:

$$1 + r_{SL} + \lambda_{CC} = 1 + r_D$$
$$1 + r_{SI} + \theta\lambda_{CC} = 1 + r_D.$$

Since we are assuming that illiquid securities are subject to a haircut in the collateral pool ($\theta < 1$), the bank will only hold both kinds of securities if the illiquid securities have a higher rate of return than the liquid ones.[17]

Additionally, the shadow value of relaxing the collateral constraint depends on the level of reserves chosen by the bank. For example, if the funding rate is low enough so that the bank is choosing $f = \Delta_2$, then $\lambda_{CC} = 0$, and the bank will hold no collateral

[17] When the structure of shocks is such that for some realizations of the shocks the bank can liquidate securities to obtain necessary funding, the decision to hold securities is also driven by these considerations.

whenever the return on securities is below the funding cost.

Furthermore, if the return on securities is low enough, the bank will hold no securities regardless of its level of reserves, and the choice of reserves is equivalent to the case when the bank has no access to the discount window (as in Proposition 4). For intermediate values of the rate of return on securities, the bank simultaneously chooses reserves and securities to minimize the costs associated with funding the liquidity needs originated in the $\Delta(\epsilon)$ shocks. The general direction of this relationship is that, for a given shock process, a bank with higher reserves can afford to hold less collateral. But, as with Corollary 3.1, when banks differ in their exposure to shocks, the cross-sectional heterogeneity may attenuate these basic patterns.

3.4 Generalized implications

In the general version of the model, the shock can affect the amount of loans and deposits, in addition to the change in borrowed money discussed above. Also, depending on the timing of the shock, the bank may be able to liquidate securities to accommodate a shock. So, the bank can use reserves, central bank borrowing, and/or sales of securities to respond to changes in its liquidity needs. And, the distribution of shocks can have a more complex structure than the example studied here, including continuum support and mean and variance heterogeneity.

More generally, we can posit that bank i at time t is facing general liquidity risk, which is proxied by a variable ψ_{it} and is, in turn, a function of the bank's size, balance sheet composition, and other factors. That is, we have

$$\psi_{it} = \psi(A_{it}, \mathbf{p}_{it}, ...),$$

where A_{it} is total assets of bank i at time t (a measure of size) and $\mathbf{p}_{it}$ is a vector of portfolio ratios capturing the bank's exposure to liquidity risks and access to funding.

As it was clear from the model, reserves holdings are in turn also a function of the bank's liquidity risk and whether the bank has access to the discount window (denoted with the indicator variable I^{DW}). That is

$$R_{it} = R(\psi_{it}, ...; I_i^{DW})$$

Discount window activity for bank i at time t, then, is a function of its liquidity risk, its holdings of reserves R_{it}, and other factors, such as discount window access.

So, we have

$$DW_{it} = DW(\psi_{it}, R_{it}, ...; I_i^{DW}).$$

This is the representation of the generalized framework that we will use to approach our empirical investigations in the next section.

3.5 Conditioning on discount window access

Our data do not include information about the universe of banks that have discount window access. If a bank does not have immediate access to the discount window when a liquidity need materializes, then the bank would have to accommodate the shock some other way. Gaining access to the discount window involves approval processes that may take days or weeks, so observing discount window borrowing is contingent on the bank having previously established access to the facility.

Furthermore, as we explained using the model, a bank will choose its holdings of reserves taking into account its access to discount window credit. Table 5 confirms this logic: Looking across all domestic commercial banks in our sample, banks with access to the discount window hold on average a lower proportion of reserves in their asset portfolios. However, the amount of variation in the reserves to assets ratio explained by discount window access and balance sheet factors is small, suggesting that multiple factors drive such decisions.

In our empirical strategy, we restrict attention to banks that are most likely to have immediate access to the discount window. In this way, we aim to control for common factors determining reserve and access decisions. Banks that take the necessary steps to gain discount window access are likely to share unobserved characteristics and controlling for those will reduce endogeneity issues.

Because we cannot directly identify the universe of banks with discount window access in our data, we use as a proxy whether the bank in question has taken a loan (test or otherwise) at any time during our sample period. The largest banks are also very likely to have taken the steps necessary to access the discount window. For this reason, we assume that banks with $50 billion or more in assets also have immediate access, even if we do not observe these banks borrowing.

This assumption is surely only an approximation. There may be banks that have access to the discount window but have not borrowed or tested during our sample

Table 5: Reserves and discount window access

	Dependent variable: Reserves to assets				
	(1)	(2)	(3)	(4)	(5)
Discount window access	-0.0698***	-0.0827***	-0.0656***	-0.0748***	-0.0834***
	(0.00537)	(0.00713)	(0.00489)	(0.00625)	(0.00734)
Other asset controls	N	Y	N	N	Y
Liability controls	N	N	Y	N	Y
Other balance sheet controls	N	N	N	Y	Y
R^2					
Over time (within)	0.085	0.141	0.101	0.140	0.180
Across banks (between)	0.0043	0.0054	0.0147	0.0171	0.0182
Overall	0.0052	0.0096	0.0155	0.0201	0.0244

Clustered standard errors at bank level in parentheses.
Number of observations: 223,750. Number of banks: 7,276.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: This table provides estimates from a fixed effects panel regression model of the impact of discount window access (and other selected bank characteristics) on the share of assets held in reserve balances. Sample includes all domestic banks with positive assets, and eliminates some outlier observations. Regression includes quarter fixed effects and various sets of controls (see Table 3).

period. Likewise, some banks may have gained access late in our sample period, but we consider them as having access for the entire period. While these are important considerations to keep in mind, we think that, overall, the proxy is a good way to address selection issues linked to discount window access in our empirical strategy.

4 Empirical analysis

Our empirical analysis is motivated by the ideas brought to the fore by the model presented in Section 3. At the same time, we intend to be more general with our approach, in order to accommodate as much as possible the multiple dimensions of the complex problem behind a bank's decision to borrow from the discount window.

We restrict our attention to discount window loans at the primary credit facility.[18] For the most part, we focus on loans to domestic banks, although in some cases we use data for foreign-related institutions and credit unions to broaden the perspective.

4.1 Incidence of borrowing, conditional on access

Our first specification relates the probability of borrowing at the discount window to a bank's holdings of reserves and other balance sheet characteristics. As discussed

[18] Secondary credit loans are not common and often involve very special circumstances. For a discussion of the specifics behind some recent secondary credit loans, see Ennis, Ho, and Tobin (2019).

above, to limit the effect of endogeneity, we restrict our sample to banks that are observed or, in the case of the very largest banks, assumed to have discount window access. As suggested by our model, because discount window borrowing and reserves may be co-determined, there is some risk of unobserved heterogeneity or endogeneity that could affect our parameter estimates. On the other hand, banks must obtain discount window access well in advance of any borrowing decision. By limiting our sample to banks that have access, we are conditioning on some of this unobserved heterogeneity, which minimizes the effect on our parameter estimates and permits a cleaner interpretation of our results.

We estimate the following equation:

$$DW_{it} = \alpha_{dw} + \gamma_{dw} R_{it} + \beta_{dw} \psi_{it} + \zeta_{it}, \tag{10}$$

where DW_{it} equals 1 if bank i borrows from the discount window in quarter t, R_{it} is reserves as a share of bank assets, and ψ_{it} refers to the set of bank characteristics and control variables that proxy for general liquidity risk.[19]

We account for two subsets of covariates. The first subset includes portfolio characteristics of the bank. For assets, we explore the effects on discount window borrowing of liquidity, risk, and duration of bank assets and include balance sheet measures of Treasury securities holdings, C&I and CRE loan holdings, and short versus longer-term loans. For liabilities, we gauge the effect of core funding, money market activity, and other borrowings, and incorporate measures of transactions deposits, federal funds and repo borrowing, and FHLB advances. For balance sheet size and capital, we examine proxies for business models and include (the log of) total assets, unused loan commitments to assets, the tier-1 capital ratio, and return on assets.

The second set of covariates includes factors related to the broader financial and macroeconomic environment. We include indicators of the Federal Reserve district corresponding to each bank in the sample. The idea is to capture systematic differences in discount window behavior beyond those associated with bank business models and sizes. In addition, we consider changes in the macroeconomic environment as proxied by the quarter-over-quarter change in real GDP. We also control for

[19] We eliminate data outliers, including banks observed with missing or negative assets, negative reserves-to-asset ratios, loan-to-asset ratios greater than 1, deposit-to-liability ratios greater than 1, negative federal funds borrowings, negative capital, and reported discount window collateral greater than total assets. We also eliminate trust companies, even though they file call reports, as their balance sheet structure is notably different from that of a typical bank.

short-term shifts in Federal Reserve balance sheet composition as represented by the quarterly standard deviation in the level of balances held in the Treasury General Account (σ_{TGA}).

We assume that ζ_{it} is normally distributed and evaluate the model using an unbalanced panel probit estimator. We use standard clustering procedures by bank to control for potential heteroskedasticity and correlation of errors across observations.

Table 6 reports the results of estimating expression (10).[20] We have six main results. First, conditional on discount window access, banks are more likely to borrow from the discount window if they hold fewer reserves. The first row of column (1) presents the marginal effect of reserves to assets on borrowing from the discount window. Across all specifications, the estimated coefficients on reserves to assets imply that, at the mean of the distribution of the reserves-to-assets ratio, for a one standard deviation decrease in reserves to assets (roughy 4.5 percentage points), the probability of borrowing at the discount window increases by 1 percentage point.[21] As the unconditional probability of borrowing at the discount window for this subsample of banks is 3 percent, this shift in the propensity to borrow is notable.

Second, and consistent with the first result, banks with illiquid balance sheets are more likely to borrow from the discount window. Column (2) presents results from including asset measures in the specification, and column (3) from including liability measures. While measures of Treasury securities holdings appear to have little effect on dicount window borrowing, banks that hold more illiquid, riskier loans, such as C&I loans, are more likely to borrow from the discount window.[22] Analogously, banks that use more intensively wholesale funding, such as repo funding or FHLB advances, are also more likely to borrow from the discount window. The magnitude of these effects is economically meaningful: a one standard deviation increase in the share of C&I loans to assets or the share of FHLB advances to liabilities each boosts the probability of borrowing at the discount window by roughly 50 basis points.

Third, and consistent with our summary statistics reported earlier, larger banks

[20]Banks taking test loans and other smaller loans (of less than \$1 million) are coded as having discount window access, but not as borrowing at the discount window.

[21]The standard deviation of the reserves-to-assets ratio is 0.0463 (4.63 percentage points). This figure, multiplied by -0.219, the coefficient reported on the reserves-to-assets ratio in column (1), is -1.01 percentage points. All subsequent economic impact estimates are calculated analogously.

[22]We focus on Treasury securities as a measure of liquid assets as they are the most comparable to reserve balances in liquidity regulations. Agency MBS, for example, has limits for inclusion in liquidity ratios and counts as less liquid collateral than Treasury securities.

Table 6: Borrowing, conditional on access

Dependent variable	Borrowed ($DW_{it} = 1$)				
	(1)	(2)	(3)	(4)	(5)
Asset composition					
Reserves to assets	-0.219***	-0.213***	-0.183***	-0.200***	-0.176***
	(0.0344)	(0.0350)	(0.0327)	(0.0334)	(0.0328)
Treasuries to assets		-0.00970			-0.0105
		(0.0529)			(0.0559)
C & I to assets		0.0653***			0.0578**
		(0.0171)			(0.0182)
CRE to assets		0.0157			0.0145
		(0.00899)			(0.00978)
Short loans to total loans		0.0123			0.00440
		(0.00847)			(0.00893)
Liability composition					
Transaction deposits to total deposits			-0.0342***		-0.00743
			(0.0101)		(0.0104)
Fed funds borrowed to liabilities			0.115		0.115
			(0.0710)		(0.0704)
Repos to liabilities			0.131**		0.127**
			(0.0426)		(0.0450)
FHLB advances to liabilities			0.0931***		0.0987***
			(0.0205)		(0.0204)
Balance sheet size and capital					
Log(assets)				0.00874***	0.00704***
				(0.00109)	(0.00118)
Tier 1 capital ratio				-0.0230	0.00883
				(0.0253)	(0.0258)
Unused commitments to total assets				-0.00654	-0.00442
				(0.00515)	(0.00514)
ROA				0.186	0.158
				(0.129)	(0.135)
Other controls					
District 2	0.0228*	0.0188*	0.0207*	0.0119	0.0124
	(0.0103)	(0.00844)	(0.00849)	(0.00848)	(0.00859)
District 6	0.0121	0.00937	0.0188**	0.0136*	0.0164*
	(0.00654)	(0.00668)	(0.00681)	(0.00668)	(0.00684)
District 9	0.0136	0.00997	0.0208**	0.0198**	0.0216**
	(0.00793)	(0.00741)	(0.00783)	(0.00745)	(0.00755)
Real GDP growth	-0.000562	-0.000594	-0.000537	-0.000660	-0.000672
	(0.000420)	(0.000419)	(0.000419)	(0.000423)	(0.000423)
TGA	-0.000158***	-0.000166***	-0.000139***	-0.000184***	-0.000175***
	(0.0000347)	(0.0000345)	(0.0000338)	(0.0000356)	(0.0000350)
VIX	0.000147	0.000185	0.0000159	0.000254	0.000193
	(0.000155)	(0.000153)	(0.000154)	(0.000153)	(0.000151)
Number of observations	67,806	67,806	67,806	67,806	67,806
Number of banks	2,035	2,035	2,035	2,035	2,035
Pseudo R^2	0.0098	0.0121	0.0149	0.016	0.0207

Cluster-robust standard errors in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window. Sample is restricted to those banks that executed at least a test loan. The dependent variable is an indicator that equals one if a bank borrowed at the discount window in a quarter. An observation is a bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model.

are more likely to borrow at the discount window. As reported in column (4), conditional on access and at the mean of the distribution of asset holdings, for every one standard deviation increase in assets (from about \$509 million in assets to \$2.4 billion in assets), the probability of borrowing at the discount window climbs by roughly 1.4 percentage points. This result potentially reflects unobserved effects of business models on the probability of borrowing at the discount window. Over our sample period, although banks that have riskier and more illiquid portfolios were more likely to borrow at the discount window, larger banks were also more likely to do so, even though these banks tend to have better access to other funding sources.

Fourth, there are differences in borrowing behavior across Federal Reserve districts. As shown by the statistically significant and economically meaningful coefficients, borrowing banks are more likely to be in districts 6 or 9.[23] There are differences in bank characteristics across Federal Reserve districts; for example, some districts have a higher concentration of smaller banks than other districts, and some have a higher concentration of foreign branches. That said, some of our balance sheet controls are intended to account for these differences, although perhaps imperfectly.

Fifth, during our sample period of "normal" times, changes in general macroeconomic conditions do not appear to affect the decision to borrow at the discount window. Specifically, quarterly changes in real GDP have no significant effect on the likelihood of discount window borrowing. In addition, in results not reported, there does not appear to be significant quarter-to-quarter seasonality in discount window borrowing at the bank-quarter observation level. Despite some signs of possible seasonality in figure 1, we cannot reject the hypothesis that the estimated coefficients on a set of quarterly indicator variables are equal to each other. Moreover, the inclusion of these indicator variables leave the coefficient estimates on our balance sheet measures little changed. That said, we do observe (and account for) some weak evidence of annual differences, with more borrowing earlier in the sample period (2010 and 2011) and less later on.

Finally, changes in the Federal Reserve's balance sheet do appear to affect the probability of borrowing at the discount window. Our coefficient estimates suggest that a more volatile TGA over a quarter implies a lower likelihood of borrowing. The volatility of the TGA increased notably as balances in this account climbed over

[23]District 6 is Atlanta and district 9 is Minneapolis. We actually evaluate individual coefficients for each and all the districts. Most of these coefficients are statistically or economically insignificant; we include in the table only those that are significant.

our sample period, together with the overall size of the Federal Reserve's balance sheet. Of note, balances held in the TGA are offset one-for-one by reserve balances held by banks or in other Federal Reserve liabilities. Taken together, our estimated coefficients support the view that more reserve balances outstanding tend to decrease discount window borrowing.

The final row of the table provides goodness-of-fit statistics. Overall, the variation in discount window borrowing explained by our specification is not large. The pseudo-R^2 statistic ranges from 1 to 2 percent, depending on the specification. Still, we reject the hypothesis that all coefficients are jointly equal to zero.

4.2 Additional insights and robustness

The broad brush of the specification and sample in Table 6 provides results that agree with our model. However, a closer investigation of our overall results could provide additional insights, and some robustness checks are warranted.

4.2.1 Heterogeneity on the extensive margin

The decision to borrow at the discount window may vary across broad classes of banks. For example, for any given portfolio allocation, smaller banks may respond differently to funding needs and the tapping of the discount window, as they may have limited access to alternative funding sources relative to larger banks. In addition, foreign branches may have a different approach to discount window borrowing relative to domestic banks, perhaps relying instead on the parent company for funding.

Against this backdrop, we next examine whether factors that determine borrowing vary according to bank size and origin (foreign or domestic). The results, from separate estimates of expression (10) for each group of banks, are presented in the first three columns of Table 7.

Three items stand out. First, our previous result that borrowing banks hold lower shares of reserves remains robust in the size-based subsamples. That said, the response in the probability of borrowing with respect to reserves holdings is greater for larger banks than for smaller ones. Our estimated coefficients imply that a one standard deviation increase in the share of reserves to assets leads to a 1.3 percentage point decline in the probability of borrowing from the discount window for larger banks but only 80 basis points of decline for smaller ones.[24]

Table 7: Borrowing—Robustness, conditional on access

	Heterogeneity			Collateral	
Dependent variable:	$DW_{it} = 1$			$DW_{it} = 1$	Amount borrowed to total liabilities
	Larger banks	Smaller banks	Foreign branches	Incl. total collateral	Borrowing amount
	(1)	(2)	(3)	(4)	(5)
Reserves to assets	-0.180*	-0.186***	0.0193	-0.179***	0.169
	(0.0799)	(0.0343)	(0.0331)	(0.0329)	(0.243)
FHLB advances to tot. liab.	0.0126	0.0994***		0.113***	0.0560
	(0.108)	(0.0206)		(0.0264)	(0.0550)
Log(assets)	-0.0212**	0.0117***	0.00132	0.0115***	-0.00978**
	(0.00766)	(0.00149)	(0.00571)	(0.00204)	(0.00313)
ROA	-1.005	0.241		0.303	-0.494*
	(0.829)	(0.126)		(0.232)	(0.208)
District 2	0.362***	0.0116		0.0118	-0.00225
	(0.0678)	(0.00890)		(0.00854)	(0.00882)
District 6	0.300***	0.0171*		0.0164*	-0.00632
	(0.0576)	(0.00673)		(0.00683)	(0.00855)
District 9		0.0220**		0.0223**	0.0166
		(0.00728)		(0.00750)	(0.0158)
Total collateral to assets				0.0444*	0.280***
				(0.0183)	(0.0749)
Balance sheet controls?	Y	Y	Y	Y	Y
Other controls?	Y	Y	Y	Y	Y
Number of observations	3,200	64,569	3,065	67,437	2,086
Number of banks	91	1,944	90	2,022	710
Pseudo/adj. R^2	0.036	0.025	0.011	0.023	0.161

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window (first four columns) and from a random effects panel regression model of the effects of selected bank characteristics on the amount of borrowing at the discount window. Large banks are those with greater than \$10 billion in assets. All samples are restricted to those banks that executed at least a test loan. In all cases, an observation is a bank-quarter. Columns (1), (2) and (3) evaluate the probit model on large bank, small bank, and foreign branch subsamples, respectively. Column (4) evaluates the probit model on all commercial banks. The dependent variable is an indicator that equals one if a bank borrowed at the discount window in a quarter. Column (5) evaluates the regression model on all commercial banks that have nonzero borrowing in the bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model. Standard errors (shown in parentheses) are clustered at the bank level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Second, turning to the other variables in Table 7, larger and smaller banks appear to have some differential behavior based on balance sheet composition for discount window borrowing. For example, smaller banks that borrow from FHLBs are also more likely to borrow from the discount window. In addition, as total assets increase, larger banks are less likely to borrow, but smaller banks are more likely. There

[24]For the large banks, the standard deviation of the share of reserve balances is roughly 0.071, and for smaller banks it is 0.045. Multiplying these standard deviations by the corresponding coefficients in columns (1) and (2) gives the reported effects.

is some differentiation in borrowing across Reserve Districts as well. Of note, our smaller-banks sample is defined as those banks with \$10 billion or less in assets, and banks of this size are considered community banks for the purpose of supervision and regulation.[25] This suggests smaller banks likely share more in common with each other than with the larger, more complex banks included in our overall sample.

Third, foreign branch decisions to borrow at the discount window do not appear to be driven by the same factors as those for domestic banks. Although the sample size is relatively small (fewer than 100 institutions), neither the share of reserves nor other balance sheet characteristics significantly predict foreign branch discount window borrowing. These observations support our choice to concentrate on domestic banks for the majority of our analysis.

4.2.2 Collateral

The Federal Reserve requires that all discount window loans be fully collateralized, and collateral usually needs to be posted at the Federal Reserve well in advance of obtaining a discount window loan. Consequently, all banks in our sample that borrow have collateral posted at the Federal Reserve. There is significant heterogeneity across banks in their posted collateral, both as an amount relative to assets and across types of collateral. Here, we explore whether the amount of posted collateral significantly predicts borrowing at the discount window.

We observe our variable of interest, bank-level collateral-to-assets, when a bank executes a discount window loan. This information structure generates at least two empirical limitations. First, we do not observe collateral posted for any bank that does not execute a loan over our sample period; this is particularly limiting in the case of the largest banks for which we can only assume have discount window access. We cannot overcome this issue.

Second, we only observe collateral when the bank executes the loan, and not at other times. To deal with this limitation, we leverage a key attribute of bank behavior at the discount window. Discount window collateral is sticky – banks infrequently change the amount and composition of collateral posted. As a result, even though we observe collateral only when banks borrow, the observations we have may well be suf-

[25] Similarly, those with total assets between \$10 and \$100 billion are called regional banking organizations. See, for more details, https://www.federalreserve.gov/supervisionreg/community-and-regional-financial-institutions.htm

ficient. Concretely, for quarterly observations when a bank borrows from the discount window, we use the reported collateral values associated with the loan (and scale that amount by reported bank assets in the corresponding quarter). For quarterly observations when a bank does not borrow, we proxy collateral by using an average of recently reported collateral, weighting observed values more heavily if they are closer in time to the quarter with a missing observation.[26] We then scale this amount of collateral by reported bank assets in the corresponding quarter. Even though this method captures empirical regularities associated with collateral, a degree of caution is warranted in the interpretation of our coefficient estimates, as some bias could result from the imperfect observation of collateral.

The results in the fourth column of Table 7 suggest that banks with more collateral posted relative to assets are more likely to borrow from the discount window. Looking across columns, we see that our previous results are reasonably robust to the inclusion of the collateral control. Against that backdrop, our estimated coefficient on collateral posted as a share of assets indicates that for every one standard deviation decrease in collateral as a share of assets, the probability of borrowing at the discount window climbs by roughly 30 basis points, a modest amount. Banks that pledge a relatively large amount of collateral-to-assets may be more likely to borrow because they have limited access to alternative secured funding sources or do not have access to them altogether. As a result, they post the collateral on hand at the discount window and use it relatively more intensively.

4.2.3 Intensive margin of borrowing

So far, our results have focused on the extensive margin of borrowing. We now turn to the intensive margin by looking at the share of the balance sheet funded with discount window borrowing. We do so by exploring a similar specification to the other columns in the table, but with the dependent variable constructed as the quarterly sum of the amount borrowed on an overnight-equivalent basis divided by total liabilities.

More specifically, we estimate the following representation:

$$DW/L_{it} = \alpha_{dw} + \gamma_{dw} R_{it} + \beta_{dw} \psi_{it} + \epsilon_{it}, \tag{11}$$

[26] As a specific example, if there is no collateral observed in the current quarter, but there are observed collateral values one year prior and six months hence, we weight the one year prior observation by 1/3 and the six months hence observation by 2/3.

where DW/L_{it} is the amount bank i borrows in a quarter divided by total liabilities.[27] Other control variables are defined similarly as above; we also include our collateral controls as the intensity of borrowing is constrained by the amount of collateral posted. We limit our investigation to banks that meaningfully borrow from the discount window at some point in the sample, rather than simply conducting a test loan. We use standard clustering techniques at the bank level to control for potential heteroskedasticity and correlation across observations.

Column 5 of Table 7 displays our results. Smaller and less profitable banks tend to fund a greater share of their balance sheet with discount window loans. Turning first to size considerations, the effects are economically meaningful. For every one standard deviation decline in the (log) of assets, the discount window funding share increases by about 1.5 percentage points. To put this in perspective, this one standard deviation in log assets would push the share of the balance sheet funded by the discount window from the median to the 80th percentile. Turning next to profitability, a one standard deviation drop in profitability implies close to a one percentage point increase in the discount window share, again an economically meaningful amount.

Our results in column (5) also point to a positive correlation between the share of banks' balance sheet funded by the discount window and the amount of collateral pledged there. For banks that borrow at some point in the sample period, a one standard deviation in the share of total pledge collateral to assets is associated with a 1.9 percentage point increase in the borrowing-to-liabilities ratio.

Even with these economically and statistically significant results, some caution regarding broader applicability seems warranted, for two reasons. First, our results are on a small sample of banks, and so patterns uncovered in this sample may not apply to other banks. Second, there is a significant right tail in the distribution of the share of the balance sheet funded with discount window lending, with 75 percent of the sample displaying a share that is less than 2 percent. Again, this suggests that the intensity of use may only be relevant for a small number of banks. Taken together, while the amount borrowed as a share of liabilities does appear to be significantly correlated with certain balance sheet measures, it seems more practical and economically relevant to explore factors that determine *the probability of borrowing* instead of focusing on the intensity of use.

[27]The amount borrowed is calculated as the aggregate amount of daily borrowings outstanding over a quarter, including weekends.

4.2.4 Collateral composition

To build on the results presented in Table 7, we next take a closer look at the relationship between borrowing and types of collateral pledged at the discount window. As mentioned above, because collateral is observed only when a bank borrows or tests, the collateral data are observed less frequently than the balance sheet data for most banks. In light of this issue, we focus here only on the cross-sectional variation of posted collateral. We redefine the dependent variable to equal one if the bank borrows at any time in our sample, where borrowing is defined at the $1 million threshold used in Table 6. Similarly, we redefine the independent variables as sample averages of nonzero values for each category of collateral expressed as a share of total collateral. We include all institution types (commercial banks, credit unions, and foreign branches) for one part of the analysis, and limit the sample to commercial banks for the other.

Table 8 presents the results. Column (1) examines the correlation between total collateral to assets and the decision to borrow. We confirm our result in the previous table that indicates that institutions with more collateral posted, relative to assets, tend to borrow more frequently. For a one standard deviation increase in the collateral-to-assets ratio (around 9 percentage points), there is a roughly 7.5 percentage point increase in the probability of borrowing at the discount window. This overall result masks some heterogeneity at the bank level, however. As shown in column (2), FBOs tend to borrow less frequently when posting a higher share of collateral to assets; credit unions, on the other hand, do not appear to exhibit statistically different behavior from commercial banks in this case. This divergent strategy for FBOs is not unique to collateral, as we learned in our robustness specifications above. As such, we will focus on commercial banks in the remainder of the table.

Column (3) explores whether discount window borrowing is correlated with broad categories of collateral. We group collateral into three types: liquid securities, illiquid securities, and loans. We then re-evaluate our probit model on the domestic commercial banks sample. The results are striking: banks that borrow post more illiquid collateral. Specifically, while the estimated marginal effect of the probability of borrowing is insignificant for banks posting the most liquid securities, the probability of borrowing moves up 3 percentage points for every standard deviation increase in illiquid securities posted, and 8 percentage points for loans. The marginal effects of different types of collateral reported in columns (4) through (6) largely agree with the

Table 8: Borrowing and collateral

	Dependent variable: Borrowed ($DW_i = 1$)					
	All institution types		Banks only			
	(1)	(2)	(3)	(4)	(5)	(6)
Total collateral to assets	0.841***	1.191***				
	(0.215)	(0.195)				
Credit unions		-0.362				
		(0.291)				
FBOs		-1.135***				
		(0.229)				
Share of collateral						
Liquid securities			0.0124			
			(0.0317)			
Treasury securities				-0.148***		
				(0.0267)		
Municipal bonds				-0.0717*		
				(0.0296)		
Agency MBS				-0.0861**		
				(0.0277)		
Illiquid securities			0.133*			
			(0.0535)			
Corporate securities					0.0434	
					(0.0647)	
ABS					0.257	
					(0.161)	
Private MBS					0.210	
					(0.166)	
International securities					-0.350	
					(0.212)	
Loans			0.154***			
			(0.0363)			
CRE						0.108***
						(0.0316)
C & I						0.167***
						(0.0298)
Consumer loans						0.179***
						(0.0459)
Residential mortgages						0.0107
						(0.0598)
Number of banks	3,165	3,165	2,034	2,034	2,034	2,034
Pseudo R^2	0.0156	0.0445	0.0203	0.0152	0.0022	0.0208

Note: This table provides estimates from a probit model of the marginal effects of selected collateral ratios and types on the probability of borrowing at the discount window. Sample is restricted to those institutions that executed at least a test loan. Observations are at the institution level. The dependent variable equals 1 if an institution borrowed at the discount window at any time during the sample. Collateral variables are institution-level averages of nonzero values. The specification in column (2) includes separate intercept terms for credit unions and FBOs. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model. Robust standard errors are shown in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

results by categories; banks that post Treasury securities are less likely to borrow, while banks that post illiquid commercial real estate, C&I, or consumer loans are more likely. Taken together, these results support the insights from our model, which indicate that banks with less ability to use their assets to secure liquidity from the market (by selling the securities or using them as collateral) are more likely to borrow at the discount window.

4.2.5 FHLB advances

Our results in Table 6 indicate that FHLB advances and discount window borrowing are positively correlated at the bank level. Table 7 highlights some heterogeneity behind this result, with smaller banks being the main drivers. Here, we take a closer look at this interaction.

Panel A of Table 9 provides a breakdown of the banks in our sample by FHLB and discount window borrowing activity. Overall, banks are more likely to borrow from FHLBs than from the discount window. Of the 7,285 banks in the sample, more than three quarters of them are "FHLB-active," defined as reporting nonzero FHLB advances at some point our the sample period (line 2). By contrast, a little less than 10 percent of the banks in our sample borrow a meaningful amount (i.e., not a test loan) from the discount window (line 3). Interestingly, and consistent with Table 6, nearly all banks that actively borrow at the discount window (710) also borrow from FHLBs (649).

Panel B of Table 9 takes a closer look at borrowing frequencies for the 649 banks that tap both the FHLBs and the discount window. For these 649 banks, we calculate the percent of bank-quarter observations in which these banks borrow only from the FHLBs, only from the discount window, or borrow from both. Most of the time, these banks borrow from FHLBs but not the discount window (line 5).

Generally speaking, as discussed in Section 2.3, interest rates for FHLB advances tend to be lower than the primary credit rate for a comparable term. In addition, banks can obtain FHLB advances of longer tenor than is feasible at the discount window. A common term for FHLB advances is one year, but advances may be available for terms of up to 30 years.[28] In contrast, during our sample period, discount window loans were generally overnight.[29] Consistent with these observations, as shown in

[28] For more information, see https://fhlbanks.com/advances/.

[29] The maximum term of discount window loans is often increased during financial crises. This was

line 6 of the table, these banks rarely borrow solely from the discount window. But sometimes, banks borrow from both (line 7).

Table 9: FHLB advances and DW borrowing

Panel A: Number of banks	
1. All banks in sample	7,285
2. FHLB active	5,578
3. DW active	710
4. FHLB and DW active	649
Panel B: Frequencies for FHLB and DW active banks (percent of bank-quarter observations)	
5. Only FHLB borrowing	72
6. Only DW borrowing	1
7. Both FHLB and DW borrowing	8

Why would banks borrow from both the FHLBs and the discount window? As highlighted in our model, banks have multiple sources of funding available. For example, they could issue short term commercial paper, or they could go to the interbank market. Borrowing from the FHLBs is a form of funding that is perhaps closest to the discount window. FHLB advances are collateralized by mortgage-related assets (loans) as well as Treasury and agency securities. In addition, both the discount window and FHLB advances are generally accessible to a wider range of institutions than money market instruments are. Even so, there could be limits on the availability of FHLB advances to a bank. For example, FHLBs accept a narrower range of collateral than the discount window does. In addition, primary credit loans may be available later in the day than FHLB advances are. As a result, banks may exhaust borrowing opportunities at an FHLB earlier in the pecking order before turning to the discount window.

Against this backdrop, our next exercise empirically investigates the interaction of discount window borrowing and FHLB advances. We augment the specification used

the case during the 2008 financial crisis, although discount window terms had all but normalized by the beginning of our sample period. During the COVID-19 financial stress in March 2020, discount window loans were again made available at maturities of up to 90 days; see https://www.federalreserve.gov/regreform/discount-window.htm for more information.

in Table 6 with an indicator variable that equals one if the ratio of FHLB advances to liabilities is one of the "top 5" observations for this ratio for the bank over the sample. We take this measure to proxy for "intensity" of use for FHLB advances. Specifically, if a bank has a relatively high ratio of FHLB advances to liabilities, it may be more likely to be closer to exhausting its funding opportunity at the FHLB system. The dependent variable continues to be an indicator variable that equals 1 if a bank borrowed from the discount window that quarter.

Our results are reported in Table 10. Column (1) of the table repeats the estimation results from our baseline specification reported in column (5) of Table 6. Consistent with the summary statistics presented in Table 9, discount window borrowing is positively correlated with FHLB advances. The second column includes our new indicator variable. Here, we see that if FHLB advances are at the top of a particular bank's observed ranges of advances to liabilities, the probability of borrowing from the discount window increases by around 60 basis points, an economically meaningful although modest amount.

Table 10: Borrowing, conditional on access – The role of FHLB advances

Dependent variable	$DW_{it} = 1$					
	All		Large banks (gt \$10B)		Smaller banks (lt \$10B)	
	(1)	(2)	(3)	(4)	(5)	(6)
Reserves to assets	-0.176***	-0.174***	-0.180*	-0.176*	-0.186***	-0.184***
	(0.0328)	(0.0326)	(0.0799)	(0.0788)	(0.0343)	(0.0342)
FHLB advances to liabilities	0.0987***	0.0721***	0.0126	-0.104	0.0994***	0.0759***
	(0.0203)	(0.0214)	(0.108)	(0.116)	(0.0206)	(0.0217)
"Top 5" advances to liabilities		0.00589**		0.0254*		0.00527**
		(0.00188)		(0.0104)		(0.00191)
Number of observations	67,806	67,806	3,200	3,200	64,569	64,569
Number of banks	2,035	2,035	91	91	1,944	1,944
pseudo R-sq	0.021	0.021	0.036	0.044	0.025	0.025

Cluster-robust standard errors in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window. Sample is restricted to those banks that executed at least a test loan. The dependent variable is an indicator that equals one if a bank borrowed at the discount window in a quarter. An observation is a bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model.

Columns (3) through (6) repeat the analysis in columns (1) and (2), but break our results into subsamples for larger and smaller banks, using the same definitions as in Table 7. Looking at these columns, we see that the coefficients on reserves-to-assets are little changed across samples and specifications. However, there is some notable difference in the coefficients on FHLB advances to liabilities and our measure of FHLB advance intensity. Larger banks, shown in columns (3) and (4) appear not to shift borrowing behavior at the discount window until FHLB advances are towards the top of the range. If FHLB advances to liabilities are one of the top 5 observed over the sample for larger banks, the estimated implied probability of borrowing at the discount window leaps by 2 percentage points. By contrast, smaller banks, in columns (5) and (6), are more likely to borrow at the window even if FHLB advances to liabilities are in more normal ranges.

Taken together, the results point to a few possible explanations for the coincidence of borrowing at the discount window and at the FHLBs. Overall, banks that borrow from FHLBs are more likely to borrow from the discount window. But the motivations for borrowing may be different according to whether banks have access to other sources of funding. Larger banks, that presumably have better access to money markets and other funding sources, only borrow from the discount window once FHLB advances are high relative to liabilities. Smaller banks, on the other hand, tend to borrow from both programs simultaenously, and not significantly differently when advances are relatively high. Still, a key determinant of borrowing at the window–the level of reserve balances relative to assets–is little changed, suggesting that borrowing is usually prompted by the overall need for liquidity.

4.2.6 Alternative definitions of borrowing

So far, we distinguish decisions to borrow from the discount window from decisions to test access by using a rule-of-thumb based on loan size, with $1 million as the threshold. As explained above, the extra funding cost of an overnight loan of this size for the majority of our sample period was only about $15 (assuming a 50 basis point premium on the discount window interest rate). Because this funding cost is modest, the decision to borrow may not respond strongly to the financial motivations that we aim to identify. Furthermore, given the modest cost, it may be the case that our test threshold is sometimes too low —banks may execute a larger test loan or may test more than once per quarter. The risks of choosing a threshold that is too low are likely minimal; smaller loans, even if they are not tests, are not the ones that would best showcase the economic forces interesting for our study.

Table 11: Borrowing–conditional on access, higher threshold

	Dependent variable: Borrowed ($DW_{it} = 1$ and $loan_{it} >$ \$10 million)				
	(1)	(2)	(3)	(4)	(5)
Asset composition					
Reserves to assets	-0.298***	-0.293***	-0.259***	-0.278***	-0.254***
	(0.0425)	(0.0432)	(0.0406)	(0.0415)	(0.0409)
CRE to assets		0.0167			0.0157
		(0.0103)			(0.0112)
C & I to assets		0.0707***			0.0680***
		(0.0176)			(0.0191)
Treasury securities to assets		-0.0599			-0.0563
		(0.0627)			(0.0649)
Short loans to total loans		0.0187			0.0126
		(0.00969)			(0.0103)
Liability composition					
Transaction deposits to total liabilities			-0.0303**		-0.00605
			(0.0117)		(0.0122)
Federal funds borrowed to liabilities			0.172*		0.169
			(0.0873)		(0.0868)
Repos to liabilities			0.155***		0.159***
			(0.0463)		(0.0479)
FHLB advances to liabilities			0.113***		0.120***
			(0.0232)		(0.0230)
Balance sheet size and capital					
Log(assets)				0.00844***	0.00634***
				(0.00121)	(0.00133)
Tier 1 capital ratio				-0.0270	0.0134
				(0.0281)	(0.0286)
Unused commitments to total assets				-0.00666	-0.00489
				(0.00586)	(0.00592)
ROA				0.202	0.159
				(0.149)	(0.156)
Other controls					
District 2	0.0304**	0.0264**	0.0285**	0.0204*	0.0211*
	(0.0113)	(0.00939)	(0.00952)	(0.00948)	(0.00961)
District 6	0.0157*	0.0131	0.0239**	0.0179*	0.0206**
	(0.00735)	(0.00774)	(0.00795)	(0.00779)	(0.00795)
District 9	0.0226*	0.0181*	0.0302**	0.0284**	0.0296***
	(0.00957)	(0.00859)	(0.00921)	(0.00877)	(0.00883)
Real GDP growth	-0.000708	-0.000742	-0.000682	-0.000796	-0.000804
	(0.000436)	(0.000435)	(0.000435)	(0.000439)	(0.000438)
σ_{TGA}	-0.000211***	-0.000219***	-0.000191***	-0.000235***	-0.000223***
	(0.0000377)	(0.0000375)	(0.0000369)	(0.0000385)	(0.0000376)
VIX	0.000158	0.000193	0.0000300	0.000255	0.000184
	(0.000167)	(0.000166)	(0.000167)	(0.000166)	(0.000164)
Number of banks	2,035	2,035	2,035	2,035	2,035
Number of observations	67,806	67,806	67,806	67,806	67,806
Pseudo R^2	0.0130	0.0152	0.0177	0.0171	0.0222

Note: This table provides estimates from a random effects panel probit model of the marginal effects of selected bank characteristics on the probability of borrowing at the discount window. Sample is restricted to those banks that executed at least a test loan. The dependent variable is an indicator that equals one if a bank borrowed at least \$10 million on an overnight equivalent basis from the discount window in a quarter. An observation is a bank-quarter. The pseudo R^2 statistic is calculated as $1 - \frac{LL_m}{LL_0}$, where LL_m is the log-likelihood of the model and LL_0 is the log-likelihood of a constant-only model. Standard errors (shown in parentheses) are clustered at the bank level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

To support this logic and better understand the impact of the choice of threshold, we re-evaluate here the specifications in Table 6 with a higher borrowing threshold. A bank is defined as (meaningfully) borrowing from the discount window in an individual quarter if the total amount borrowed exceeds \$10 million. This threshold captures the possibility that banks execute more than one loan per quarter or that test loans are larger. For most of our sample period, the overnight funding costs of \$10 million are around \$300, still a moderate amount. The drawback is, of course, that we may be leaving out from the estimation loans that are not tests, and in that way compromising the informativeness of our data sample.

Results are displayed in Table 11. Reassuring, the results are a sharper version of Table 6, with qualitatively similar but larger marginal effects. For example, with our baseline threshold, a one standard deviation increase (about 4.5 percentage points) in the level of reserves to assets leads to a 1 percentage point decline in the probability of borrowing. With the higher borrowing threshold, the effect is about a third larger, as a one standard deviation increase in the level of reserves to assets leads to a 1.4 percentage point fall in the probability of borrowing. The coefficients related to asset and liability composition are similar, both in statistical and economic significance, to those obtained with the lower threshold. Overall, even with the new definition of borrowing, the amount of variation in borrowing explained by the specification is close to that in Table 6, suggesting that our choice of threshold is not compromising the validity of our results.

4.3 Unobserved factors and endogeneity

Our previous empirical exercises put aside some of the most difficult issues that come from having unobserved factors affecting the choice to borrow from the discount window and from the joint endogeneity of borrowing and balance sheet choices. That is, so far we have proceeded under the presumption that, conditional on access, our coefficients suffer little bias. Some of the robustness results, however, suggest that discount window borrowing responds to unobserved factors, and our model in Section 3 explains how a bank's decision to borrow is jointly determined with its choice of portfolio composition.

In order to assess the robustness of our baseline results to the possibility of endogeneity effects, we take here an instrumental variables approach. The main focus is on reserves holdings because our model suggests reserves are first in the pecking order when a bank experiences a liquidity shock, and so might be the most likely to be endogenous.

We incorporate a range of variables and approaches that align with previous research addressing the endogeneity of reserve balances. We do not take a stand on the preeminence of any single approach, but rather, interpret them collectively in a way consistent with settings in which no one strategy obviously dominates (Keane, Krutikova, and Neal (2022)).

4.3.1 Description of the instruments

We use two groups of instruments. One group is comprised of bank-specific factors, and the other of aggregate Federal Reserve balance sheet items.[30]

We start with the bank-specific group. For the first instrument, we take advantage of pre-crisis variation in reserve balance holdings, similar to the approach used in Stevenson (2010), Bertrand et al. (2018) and Rodnyansky and Darmouni (2017). We set the pre-crisis comparison to June 2008, before the start of the QE programs. To construct the instrument, we interact a bank's 2008 reserves-to-assets ratio with indicator variables for each year in our sample. For the second instrument, we interact a bank's 2008 reserves-to-assets ratio with the aggregate four-quarter growth rate for reserves of large or small banks, depending on the size of the bank. For the third instrument, we interact the four-quarter lag of a bank's reserves-to-assets ratio with the aggregate four-quarter growth rate of reserves for large or small banks. This formulation is in the spirit of a Bartik instrument; Goldsmith-Pinkham, Sorkin, and Swift (2020). In all cases, we rely on predetermined reserves-to-assets ratios, as well as aggregate growth rates, to identify changes in individual bank reserves-to-asset ratios associated with broader shifts in reserve-to-assets, with an aim to eliminate bank-level unobserved factors that could bias our estimates.

Our second group of instruments contains aggregate measures of Federal Reserve balance sheet items. We use three: the aggregate reserves-to-assets ratio for all commercial banks, the FBO share of reserve balances, and the ratio of the level of the TGA to total Federal Reserve liabilities. These aggregate instruments reflect distinct, although related, phenomenon. The aggregate reserves-to-assets ratio reflects banking

[30] While the effect of reserve balances on foreign branch borrowing behavior is potentially interesting, given the divergence seen in predictors of borrowing illustrated in Table 7, we focus our analysis on domestic banks.

system liquidity relative to overall intermediation; it can also be interpreted as the common factor in all banks' individual reserves-to-assets ratios. The FBO share of reserve balances in part reflects less stringent leverage ratio requirements for FBOs relative to domestic banks; consequently, FBOs generally absorbed a disproportionate amount of reserve balances from quantitative easing.[31] And, as a dollar more in the TGA represents a dollar less in reserves, movements in the TGA also affect bank reserves.[32]

Importantly, no individual bank in our sample can meaningfully affect any of the aggregate ratios we consider, making them suitable for exploration as instruments. That said, there may be differences in how larger banks and smaller banks react to changes in items on the Federal Reserve's balance sheet. With that in mind, we allow for the coefficient on these instruments to differ according to whether a bank is large or small.

We use a specification analogous to that considered in Table 6, where the dependent variable is an indicator variable that equals 1 if the bank borrowed at the discount window and 0 if not. All regressions include the same controls as in Table 6. Maximum-likelihood is used to evaluate our instrumental-variable probit models, and we conduct a cluster-bootstrap procedure to calculate parameter estimates, marginal effects, and associated standard errors.[33] Other test statistics use standard cluster-robust techniques. First-stage estimates for each of these instruments are presented in tables A2 and A3 in the Appendix.

4.3.2 Results

Table 12 displays second stage parameter estimates. Columns (1) through (3) report estimates for our bank-specific instruments; columns (4) through (6) report estimates for aggregate instruments. Overall, the results are qualitatively similar to those that do not instrument reserve balances to assets. Specifically, discount window borrowing is associated with lower reserves holdings, particularly for larger banks. Considering the estimates in row (1), the baseline effect of a one standard deviation increase in reserves-to-assets ranges from a decrease in the probability of borrowing of 2.3

[31]Specifically, domestic banks were required to meet the leverage ratio on a daily basis, while FBOs faced a binding leverage ratio only on the quarter-end.

[32]Using unexpected flows into the TGA as an instrument dates to Hamilton (1997); Judson and Klee (2010) and Correa, Du, and Liao (2020) have also used versions of this instrument.

[33]We use 500 replications for our bootstrap procedure.

Table 12: Borrowing – Instrumental variable approaches

	Dependent variable: Borrowed ($DW_{it} = 1$)					
	IV: Bank-specific instruments			IV: Aggregate instruments		
	2008—level	2008—growth	Lagged growth by size	Reserves growth & growth^2	TGA share	Fori share
	(1)	(2)	(3)	(4)	(5)	(6)
(1) Res.-to-assets (R/A)	-0.0442	-0.517*	-0.186	0.432	0.481*	0.990
	(0.100)	(0.201)	(0.148)	(0.294)	(0.228)	(0.799)
(2) Larger bank × R/A	-2.747**			-1.368**	-1.208**	-0.966**
	(0.921)			(0.474)	(0.168)	(0.132)
Balance sheet controls?	Y	Y	Y	Y	Y	Y
Other controls?	Y	Y	Y	Y	Y	Y
Number of banks	2,011	2,035	2,035	2,035	2,035	2,035
Number of observations	67,090	67,806	67,705	67,806	67,806	67,806
Weak instrument test						
$F-$stat	160.1	115.0	932.7	84.34	207.2	15.36
Hansen J test						
χ^2 stat	15.52	n.a.	n.a.	4.412	n.a.	
$p-$ value	0.626			0.110		
Anderson-Rubin F test						
$F-$stat	1.139	8.208	2.436	5.251	9.45	9.95
$p-$ value	0.301	0.00421	0.119	0	0	0

Notes: This table provides cluster-bootstrap estimates from an instrumental-variable probit model to control for the possible endogeneity of the reserves-to-assets ratio. Sample is all domestic banks that have borrowed at the discount window or have executed a test loan. Instruments for the first-stage regression include the ratio of the bank's reserves to that of its size group as of 2008Q2, and the four-quarter change in the share of reserve balances in total bank credit, allowing the coefficient to differ across size groups. Marginal effects presented in the second-stage results. The specifications include balance sheet and other controls shown in Table 6. Bootstrapped standard errors clustered by bank are in parentheses. Bootstrap-robust specification tests are shown in the lower rows of the table.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

percentage points to an increase of roughly 2 percentage points. Looking at row (2), the marginal effects are more pronounced for larger banks. A one standard deviation increase in the reserves-to-assets ratio for larger banks is associated with anywhere from a 16 percentage point decline in the predicted probability of borrowing at the discount window (column 1) to little effect (column 6).

Although the specifications reported in Table 12 reveal a wide range of elasticity estimates, the purpose of these exercises were not necessarily to provide a precise point estimate. Rather, our goal was to explore the robustness of the results in Table 6. The estimates in this section clearly encompass those, leading us to assess that our baseline estimates are not significantly affected by unobserved factors or simulteneity. And in general, the coefficients remain economically meaningful, pointing to a significant link between borrowing and reserves.

We report specification test statistics in the bottom rows of the table. Overall, the results indicate reasonable specifications with acceptable instruments. The first-stage F-statistics generally exceed typical thresholds, indicating that we do not use weak instruments, and our instruments typically satisfy overidentifying restrictions. More generally, although the IV results are not meaningfully different in economic terms from the reduced form results, most specifications suggest some endogeneity of reserves-to-assets.

Given the wide range of elasticity estimates reported in Table 12, we undertake additional exercises in Table 13 to further support robustness of our baseline result.

Table 13: Other approaches

	Dependent variable: Borrowed ($DW_{it} = 1$)				
	IV— all instruments	IV— Lasso	IV— Lasso	Fixed effects logit	Propensity score matching
	(1)	(2)	(3)	(4)	(5)
Reserves to assets	-0.00316	-0.138	0.0913	-1.484**	-0.145**
	(0.183)	(0.0742)	(0.081)	(0.553)	(0.050)
Large banks*reserves to assets	-0.973***		-1.098***		
	(0.234)		(0.117)		
Balance sheet controls?	Y	Y	Y	Y	Size and capital not incl.
Other controls?	Y	Y	Y	N	N
Number of banks	2,011	2,011	2,011	707	1,217
Number of observations	67,050	67,050	67,050	23,980	6,474
Pseudo R^2				0.044	0.023
χ^2 stat		3.48	92.98		
$p-$value		0.062	0.00		

Notes: This table provides estimates from a range of models to control for the possible endogeneity of the reserves-to-assets ratio. Sample is all domestic banks that have borrowed at the discount window or have executed a test loan. Instruments in column (1) include all those used in Table 12. Instruments in columns (2) and (3) are selected using a cross-fit partialing-out lasso algorithm for instrument and control variable selection. Propensity-score matching algorithm uses a nearest neighbor matching method with a logistic distance metric to match smaller banks to larger ones, based on asset and liability composition. Marginal effects presented in all specifications. The specifications include the balance sheet and other controls shown in Table 6 as indicated. Robust standard errors clustered by bank are in parentheses. Robust specification tests are shown in the lower rows of the table.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The first three specifications explore the instrumental variables jointly. Column (1) of Table 13 includes all instruments, while columns (2) and (3) use LASSO procedures to select instruments and exogenous controls.[34] The parameter estimates from using all instruments and from the LASSO results are economically and statistically close to each other, and the estimated magnitudes of the response of borrowing to changes in reserves-to-assets are similar to those seen in Table 12. They continue to suggest that larger banks discount window borrowing is sensitive to reserve balances, while the effect for smaller banks is not statistically different from zero.

[34]We use a cross-fit partialing-out LASSO procedure. Results with other procedures are similar.

Column (4) reports results from a fixed-effects panel logit model. Identification in this model derives from those banks that borrow from the discount window in some quarters, but not in others. The estimate of the sensitivity of borrowing to reserves-to-assets is larger than the one obtained on the full sample evaluated without fixed effects, and closer to that seen for the largest banks. We take this result to suggest that banks that actively manage their discount window borrowing decisions taking into account portfolio outcomes may be the most sensitive to changes in reserve balances.

Our final exercise builds on previous results and incorporates a propensity score matching estimator. Here, we construct a sample of banks in which large banks are matched to small banks with similar balance sheets using a nearest neighbor matching method with a logistic distance metric. The intuition behind this approach is that this should give us a clear picture of changes in borrowing that come from balance sheet items only, and eliminates unobserved differences from regulatory or other factors.

Our results are reported in column (5) of Table 13. Our marginal effect estimate of -0.145 is a little lower than that of our baseline in Table 6. That said, the standard deviation of reserves-to-assets in this subsample is a little higher than the overall standard deviation (6 percentage points versus 4.5 percentage points). With this difference, we find that the elasticity estimate of borrowing with respect to one standard deviation increase in reserves-to-assets is roughly 1 percentage point. This is nearly identical to our baseline results reported in Table 6, both in terms of economic and statistical significance.

5 Conclusions

This paper provides new evidence on the use of the Fed's discount window in normal times. Many banks in the U.S. tap the discount window even when financial markets are not in distress; i.e., in normal times. With data on discount window activity kept mostly confidential until recently, the conditions that move banks to borrow from the discount window in well-functioning financial markets are not yet well-understood.

In this paper, we show that a bank's discount window activity is tightly related to its holding of reserves balances at the Federal Reserve. Other components of banks' balance sheets, such as business lending and wholesale funding activity, are also important for understanding discount window borrowing. The amount and type of collateral pledged at the window is also a factor. We provide theoretical foundations for these links and investigate empirically their prevalence using transaction-level data that only became publicly available in the U.S. after the 2008 financial crisis.

There are multiple policy questions that hinge upon a better understanding of the mechanisms we study in this paper. For example, a long-standing question in central banking policy is whether the discount window should remain open at all times, not just during financial crises. Understanding how banks interact with the discount window outside of crises, as we do here, is a critical component in the search for a convincing answer to that question.

A second example involves monetary policy implementation and the costs and benefits of having a system with ample reserves. In this article, we show that banks' reserves holdings meaningfully affect the way, and the intensity, with which those banks interact with the central bank as provider of back-up liquidity in normal times. This is one aspect that, while not always duly emphasized, should be taken into consideration in any proper assessment of the ample-reserves system.

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Appendix - Proofs

Consider the following constrained maximization problem, where $\lambda_i(\epsilon)$ for $i = 1, 2$ and $\beta_i(\epsilon)$ for $i = 1, 2, 3$ are Lagrange multipliers:

$$\begin{aligned} \max\ (1 + r_{IOR})f(\epsilon) - (1 + r_{FF})b^{FF}(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_o)o(\epsilon) \\ +\lambda_1(\epsilon)[s^L(\epsilon) + \theta s^I(\epsilon) - b^{DW}(\epsilon)] \\ +\lambda_2(\epsilon)[f - f(\epsilon) + b^{FF}(\epsilon) + b^{DW}(\epsilon) + o(\epsilon) - \Delta(\epsilon)] \\ +\beta_1(\epsilon)f(\epsilon) + \beta_2(\epsilon)b^{DW}(\epsilon) + \beta_3(\epsilon)o(\epsilon). \end{aligned}$$

The first-order conditions (FOC) are:

$$\begin{aligned} (1 + r_{IOR}) - \lambda_2(\epsilon) + \beta_1(\epsilon) = 0, \\ -(1 + r_{FF}) + \lambda_2(\epsilon) = 0, \\ -(1 + r_{DW}) - \lambda_1(\epsilon) + \lambda_2(\epsilon) + \beta_2(\epsilon) = 0, \\ -(1 + r_o) + \lambda_2(\epsilon) + \beta_3(\epsilon) = 0. \end{aligned}$$

Proof of Proposition 1. Using the second FOC, we have that $\lambda_2(\epsilon) = 1+r_{FF}$, and substituting in the fourth FOC and noting that $r_o > r_{FF}$, we have that $\beta_3(\epsilon) > 0$, and hence, by complementary slackness, $o(\epsilon) = 0$.

Using the third FOC and the fact that $\lambda_1(\epsilon) \geq 0$ and $r_{DW} > r_{FF}$, we have that $\beta_2(\epsilon) > 0$ and, by complementary slackness, $b^{DW}(\epsilon) = 0$.

Hence, we have that $f(\epsilon) = f + b^{FF}(\epsilon) - \Delta(\epsilon)$. Now, using the first FOC, when $r_{IOR} < r_{FF}$, we have that $\beta_1(\epsilon) > 0$ and $f(\epsilon) = 0$, which implies that $b^{FF}(\epsilon) = \Delta(\epsilon) - f$.

Proof of Proposition 2. To simplify notation, define $s \equiv s^L + \theta s^I$. Since the bank has no access to the interbank market, $b^{FF}(\epsilon) \equiv 0$ for all contingencies. As indicated in the statement of the proposition, we need to consider two cases, depending on whether the ex-ante amount of reserves held by the bank (f) is larger or smaller than the liquidity shock ($\Delta(\epsilon)$).

- We start with the case when $f \geq \Delta(\epsilon)$. Then, from the flow constraint, we have that:

$$f(\epsilon) = f + b^{DW}(\epsilon) + o(\epsilon) - \Delta(\epsilon),$$

and since $b^{DW}(\epsilon) \geq 0$ and $o(\epsilon) \geq 0$, we have that $f(\epsilon) \geq f - \Delta(\epsilon)$.
Claim 1. If $f = \Delta(\epsilon)$ then $b^{DW}(\epsilon) = 0$ and $o(\epsilon) = 0$. Proof: Suppose not. Since $f = \Delta(\epsilon)$, we have that $f(\epsilon) = b^{DW}(\epsilon) + o(\epsilon) > 0$. But then, $\beta_1(\epsilon) = 0$ and $\lambda_2(\epsilon) = 1 + r_{IOR}$, which implies that both $\beta_2(\epsilon)$ and $\beta_3(\epsilon)$ are positive and, hence, $b^{DW}(\epsilon)$ and $o(\epsilon)$ equal zero, which is a contradiction.■
Claim 2. If $f > \Delta(\epsilon)$ then $b^{DW}(\epsilon) = 0$ and $o(\epsilon) = 0$. Proof: If $f > \Delta(\epsilon)$ then $f(\epsilon) > 0$ and $\beta_1(\epsilon) = 0$ so that $\lambda_2(\epsilon) = 1 + r_{IOR} > 0$. Then, $\beta_3(\epsilon) = r_o - r_{IOR} > 0$ implies that $o(\epsilon) = 0$, and $\beta_2(\epsilon) = r_{DW} - r_{IOR} + \lambda_1(\epsilon) > 0$ implies $b^{DW}(\epsilon) = 0$.■

• When $f < \Delta(\epsilon)$ and $s > 0$, using the flow constraint, and since $f(\epsilon) \geq 0$, we have that:

$$b^{DW}(\epsilon) + o(\epsilon) = \Delta(\epsilon) - f + f(\epsilon) > 0,$$

which implies that either $b^{DW}(\epsilon) > 0$ or $o(\epsilon) > 0$ (or both).
Claim 3. If $o(\epsilon) > 0$ then $b^{DW}(\epsilon) > 0$. Proof: Suppose not. Suppose $o(\epsilon) > 0$ and $b^{DW}(\epsilon) = 0$. When $o(\epsilon) > 0$, we have that $\beta_3(\epsilon) = 0$ and $lambda_2(\epsilon) = 1 + r_o$. Also, when $b^{DW}(\epsilon) = 0$ and $s > 0$, we have that $\lambda_1(\epsilon) = 0$ and $\beta_2(\epsilon) = r_{DW} - r_o < 0$, which implies a contradiction (since β_2 is a Lagrange multiplier).■

Since either $o(\epsilon)$ or $b^{DW}(\epsilon)$ are greater than zero and when $o(\epsilon) > 0$, we also have that $b^{DW}(\epsilon) > 0$, then we conclude that $b^{DW}(\epsilon) > 0$.

Also, $b^{DW}(\epsilon) > 0$ implies $\beta_2(\epsilon) = 0$ and $\lambda_2(\epsilon) = 1 + r_{DW} + \lambda_1(\epsilon)$, which implies that:

$$\beta_1(\epsilon) = \lambda_2(\epsilon) - (1 + r_{IOR}) = r_{DW} - r_{IOR} + \lambda_1(\epsilon) > 0,$$

which implies that $f(\epsilon) = 0$.

We need to consider two cases now, depending on whether the collateral constraint is or is not binding.
Case 1. When $\Delta(\epsilon) - f \leq s$, given that $f(\epsilon) = 0$, from the flow constraint, we have that $b^{DW}(\epsilon) + o(\epsilon) = \Delta(\epsilon) - f \leq s$.
Claim 4. When $\Delta(\epsilon) - f = s$, we have that $o(\epsilon) = 0$. Proof: If $\Delta(\epsilon) - f = s$ then $b^{DW}(\epsilon) + o(\epsilon) = s$. In consequence, either $b^{DW}(\epsilon) = s$, which implies that $o(\epsilon) = 0$, or $b^{DW}(\epsilon) < s$, which implies $\lambda_1(\epsilon) = 0$. From the FOCs, then, we have $\lambda_2(\epsilon) = 1 + r_{DW}$ and $\beta_3(\epsilon) = r_o - r_{DW} > 0$, which by complementary slackness implies that $o(\epsilon) = 0$.■
Claim 5. When $\Delta(\epsilon) - f < s$, we have that $o(\epsilon) = 0$ and $b^{DW}(\epsilon) = \Delta(\epsilon) - f$. Proof: When $\Delta(\epsilon) - f < s$ then $b^{DW}(\epsilon) \leq b^{DW}(\epsilon) + o(\epsilon) = \Delta(\epsilon) - f < s$, so $b^{DW}(\epsilon) < s$ and $\lambda_1(\epsilon) = 0$, which implies that $\lambda_2(\epsilon) = 1 + r_{DW}$ and $\beta_3(\epsilon) = r_o - r_{DW} > 0$, which

by complementary slackness implies that $o(\epsilon) = 0$. Then, the flow constraint implies that $b^{DW}(\epsilon) = \Delta(\epsilon) - f$.■

Case 2. We consider now the case when $\Delta(\epsilon) - f > s$.

Claim 6. When $\Delta(\epsilon) - f > s$, we have that $b^{DW}(\epsilon) = s > 0$. Proof: Suppose not. Suppose $b^{DW}(\epsilon) < s$. Then, $\lambda_1(\epsilon) = 0$ and, from the FOCs, we have that $\lambda_2(\epsilon) = 1 + r_{DW}$. This, in turn, also from the FOCs, implies that $\beta_3(\epsilon) = r_o - r_{DW} > 0$, which by complementary slackness implies that $o(\epsilon) = 0$. Now using the flow constraint, we have that $b^{DW}(\epsilon) = \Delta(\epsilon) - f > s$, which is a contradiction.■

To conclude, note that given that $b^{DW}(\epsilon) = s > 0$ implies that $\beta_2(\epsilon) = 0$ then, from the FOCs, we have that $\lambda_2(\epsilon) = 1 + r_{DW} + \lambda_1(\epsilon) > 0$, and the flow constraint holds with equality – with $f(\epsilon) = 0$, which we have shown before. This implies that $o(\epsilon) = \Delta(\epsilon) - f - s > 0$. This completes the proof.

Proof of Proposition 3. We present here only a sketch of the proof, since many of the details are straightforward.

We focus on how the bank would choose the level of securities and reserves in Problem 8. Given the linearity of the problem, we can evaluate those decisions independently from the lending decisions.

It is easy to see that whenever $r_{SI} = r_{SL} = r_D$, the bank will choose sufficient holdings of securities for the collateral constraint to not be binding (otherwise, the shadow return on holding securities would include the value of relaxing the collateral constraint and the return on holding an extra unit of securities would be higher than the deposit rate, which represents the cost).

The choice of the ex-ante level of reserves is more complex. We need to account for the changes in the ex-post decisions that are associated with different levels of reserves under different contingencies. These decisions were characterized in propositions 1 and 2.

When the bank has access to the interbank market (as in Proposition 1), we can reduce the ex-post payoff implied by Problem 8 to:

$$(1 + r_{IOR})f(\epsilon) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon) - (1 + r_{FF})b^{FF}(\epsilon).$$

Also from Proposition 1, we know that when $r_{IOR} = r_{FF}$, we have that $f(\epsilon) - b^{FF}(\epsilon) = f - \Delta(\epsilon)$, and when $r_{IOR} < r_{FF}$, we have that $f(\epsilon) = 0$ and $-b^{FF}(\epsilon) =$

$f - \Delta(\epsilon)$. Hence, in both cases, we can further reduce the ex-post payoff to:

$$(1 + r_{FF})(f - \Delta(\epsilon)) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon).$$

This will be the payoff when the bank has access to the interbank market for all possible values of the shock. This event happens with probability q.

With probability $1 - q$, the bank has no access to the interbank market. Ex-post decisions in this case are described in Proposition 2 and can be used to reduce the ex-post payoff implied by Problem 8 to:

$$(1 + r_{IOR})f(\epsilon) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon) - (1 + r_{DW})b^{DW}(\epsilon) - (1 + r_o)o(\epsilon).$$

If the bank were to choose $f \geq \Delta_2$ then $b^{DW} = 0$ and $o = 0$ for all values of the shock and the ex-post payoff would be:

$$(1 + r_{IOR})(f - \Delta(\epsilon)) - (1 + r_D)f + (1 + r_B)\Delta(\epsilon).$$

Combining these two payoffs, and given the probabilities associated with them, we can see that if $r_D < r^{T1} = qr_{FF} + (1 - q)r_{IOR}$, the bank would want to increase deposits and reserves indefinitely. Such a situation would not be consistent with equilibrium. When $r_D = r^{T1}$, the bank is indifferent between holding any quantity of reserves greater than or equal to Δ_2. This is a situation consistent with significant levels of excess reserves.

Now consider a situation where the bank chooses a level of reserves $f \in (\Delta_1, \Delta_2)$. In this case, when the bank has no access to the interbank market, if the shock is equal to Δ_2, then the bank will be short of reserves and will have to borrow from the discount window. If, instead, the shock is equal to Δ_1 or Δ_0, then the bank will have sufficient reserves to cover the shock and the leftover amount will earn the interest paid on reserves. As a consequence, the relevant portion of the expected payoff can be written as:

$$p_1(1 + r_{IOR})(f - \Delta_1) + p_0(1 + r_{IOR})f - (1 + r_D)f - p_2(1 + r_{DW})(\Delta_2 - f),$$

which, combined with the payoff when the bank has access to the interbank market (discussed above), results in a expected payoff equal to $(r^{T2} - r_D)f - X$, where X is a term independent of the choice of f.

Clearly, then, whenever $r_D \in (r^{T1}, r^{T2})$, the bank will choose to increase the level of reserves f until it reaches the value of the highest shock Δ_2 (at which point, the relevant payoff function changes to the one described in the case when $f \geq \Delta_2$).

Similarly, when the bank chooses a level of reserves lower than Δ_1, it will need to borrow from the discount window when $\Delta(\epsilon)$ equals either Δ_1 or Δ_2 and the relevant portion of the expected payoff function now equals $(r^{T1} - r_D)f - X'$. As a result, whenever $r_D \in (r^{T1}, r^{T3})$, the bank will choose $f = \Delta_1$.

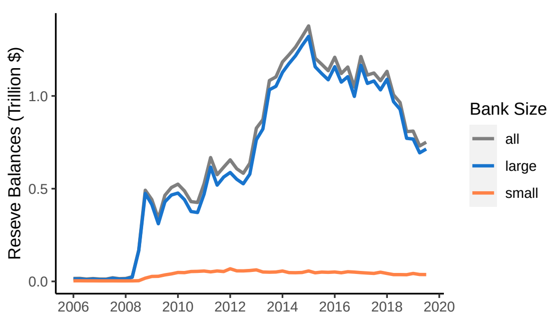
Finally, when $r_D > r_{T3}$, the cost for the bank of choosing an extra unit of reserves is higher than the return even when the bank needs to borrow from the discount window after any positive shock. Hence, in such case, the bank will choose to hold no reserve. The rest of the details of the proof are straightforward.

Proof of Proposition 4 The proof of Proposition 4 follows exactly the same logic as the proof of Proposition 3. The only difference is that when the bank has no access to the interbank market and the shock is larger than the level of reserves held by the bank, then the bank has to incur an overnight overdraft – which is more expensive than a discount window loan, since $r_o > r_{DW}$.

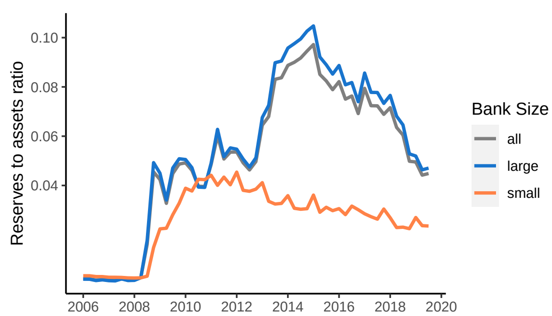
Appendix - Empirical results on endogeneity

To motivate our analysis, Figure A1 displays reserve balance holdings from 2006 to 2019 for our sample of domestic banks with discount window access, both in aggregate and as a ratio to assets. Before the 2007-2009 financial crisis, total reserve balances were low and a tiny fraction of bank balance sheets, both in aggregate and for large and small banks. During the financial crisis and subsequent quantitative easing programs, there was a substantial increase in reserve balances, reflecting the expansion in the Federal Reserve's balance sheet. While large domestic banks gained a considerable amount of reserve balances in absolute terms, smaller banks' holdings climbed less. However, in terms of the ratio of reserve balances to total assets, both large and small banks experience a rise in the share of reserves-to-assets until roughly 2012 or 2013. Thereafter, reserves-to-assets climbs considerably for the large banks, but declines modestly for small banks. Starting in 2015 or so, reserve balances decline for both bank sizes, but more rapidly for large ones.

Figure A1: Reserve balances over time



(a) Reserve balances



(b) Reserves to assets

These observations are substantiated by simple reduced-form regressions of reserves-to-assets holdings on a range of indicator variables for bank size and years. Column (1) of Table A1 displays estimated coefficients for all banks in the sample, and column (2) presents the interaction terms for the largest banks. Both groups of banks experienced increases in the reserve-to-assets ratio through 2016, but large banks saw even greater increases. This observation holds for changes in the reserves-to-assets ratio, reported in columns (3) and (4). There are statistically significant increases in this ratio for larger banks, but not for smaller ones.

Table A1: Pre-trends

Dependent variable:	Reserves to assets—level		Reserves to assets—change	
	Baseline	Large banks	Baseline	Large banks
	(1)	(2)	(3)	(4)
2011	0.00612***	-0.00237	0.00147*	0.00630*
	(0.000543)	(0.00258)	(0.000585)	(0.00280)
2012	0.0134***	-0.00445	0.00142*	0.00685*
	(0.000544)	(0.00258)	(0.000586)	(0.00279)
2013	0.00961***	0.00485	-0.00236***	0.0103***
	(0.000547)	(0.00258)	(0.000587)	(0.00279)
2014	0.00360***	0.0110***	-0.000657	0.00748**
	(0.000551)	(0.00258)	(0.000589)	(0.00279)
2015	0.00307***	0.0111***	-0.000320	0.00586*
	(0.000555)	(0.00259)	(0.000592)	(0.00280)
2016	0.00189***	0.00873***	-0.000560	0.00529
	(0.000559)	(0.00260)	(0.000594)	(0.00280)
2017	-0.000850	0.00761**	-0.000688	0.00638*
	(0.000564)	(0.00261)	(0.000596)	(0.00281)
2018	-0.00271***	0.00256	-0.000636	0.00556*
	(0.000569)	(0.00262)	(0.000599)	(0.00282)
2019	0.000856	-0.00872**	0.00146*	0.00325
	(0.000609)	(0.00278)	(0.000626)	(0.00292)
Number of banks	2,035		2,033	
Number of observations	67,812		65,771	
R^2	0.0178		0.0030	

Note: This table provides estimates from a panel regression of bank-level reserves-to assets levels and changes on yearly indicator variables. Sample is restricted to those banks that executed at least a test loan. The dependent variable is the reserves to asset ratio for a bank in a quarter, or the four-quarter change in the reserves-to-assets ratio. An observation is a bank quarter. Coefficients on the yearly indicator variables are allowed to vary for large banks relative to the baseline. Standard errors (shown in parentheses) are clustered at the bank level. $*p < 0.05$, $**p < 0.01$, $***p < 0.001$.

What drove this divergence? One potential reason is bank preference for liquidity. Pre-crisis, larger banks' preferred holdings of reserve balances and liquid assets tended to be higher than that for smaller banks, presumably based on expected payment flows, or other factors related to their individual portfolios and business models. In addition, the largest banks usually carried substantial daylight overdrafts, representing liquidity demand derived from payments activity. As the overall level of reserve balances climbed, daylight overdrafts were unnecessary and dropped considerably. But those banks that had higher demand for liquidity pre-crisis may have increased their reserve balances relatively more than other banks.

Another reason for the divergence could be regulatory. In early January 2013, the Basel Committee introduced the Liquidity Coverage Ratio (LCR), a framework under which banks are required to hold sufficient quantities of high-quality liquid assets (HQLA) to provide a buffer against the loss of less stable funding. Domestically, the final rule for the LCR was published in late 2014, with implementation on January 1, 2015. Under the LCR, reserve balances are considered HQLA, as are Treasury securities. Requirements are tailored to bank size, with the largest institutions subject to the strictest requirements and smaller banks somewhat less stringent ones. For example, bank holding companies with more than $250 billion in assets have the most stringent LCR requirements, while banks with less than $10 billion have no LCR requirement.[35]

In each column of tables A2 and A3, the dependent variable is a bank's reserve balances as a share of the bank's total assets in a given quarter, or that ratio interacted with a dummy variable that equals one if the bank is a larger bank. Starting with Table A2, the first four columns display the results using banks' 2008 level of reserve balances interacted with year and size indicator variables. Overall, the results are similar to those reported in Table A1: Large banks' reserve-to-asset ratios increase over time and to a greater degree than the ratios for smaller banks. In addition, current reserves-to-asset ratios appear to be positively correlated with reserves-to-assets observed in 2008. For example, summing the coefficients across columns (1) through (4), in 2014, the correlation of current reserves-to-assets ratios with the 2008 level was 1.12 for smaller banks and 2.31 for large ones.

Other instruments also demonstrate expected economic and statistical significance. As shown in columns (5) and (6), size group growth rates applied to this 2008 ratio are positively correlated with reserves-to-assets ratios, as are lagged relative group growth rates. The coefficients on these instruments are between zero and one, indicating significant, but not complete, correlation.

Turning now to the first-stage results in Table A3, we see similar patterns for the aggregate instruments as we did for the bank-level instruments. Looking at columns (1) and (2), we see some nonlinearity in banks' responses to reserves growth, particularly for larger banks. In particular, large banks increase their reserve balances relatively faster than smaller ones, and at an increasing rate as the Fed's balance sheet expanded. This pattern is also evident for increases in the ratio of the TGA to reserve balances. Large banks experienced a contemporaneous climb in reserves along with the TGA, and of notable magnitude, while smaller banks' reserves expanded less. The pattern for the FBO share is even more pronounced. As shown in columns (5) and

[35] Our estimation sample are commercial banks, not bank holding companies. In some cases, the bank falls below the $ 250 billion threshold while the holding company would be above. Inclusion or exclusion of these banks from the estimation sample does not meaningfully alter the results.

(6), while overall, smaller banks saw their reserves decline with aggregate increases in the FBO share, larger banks tended to experience contemporaneous climbs in their reserves-to-assets ratios. These differential patterns point to the need to allow for separate coefficients for each size group in the second-stage analysis.

Table A2: First-stage instrumental variable results—Bank level instruments

Instrument	2008 reserves-to-assets level interacted with year dummies				2008–growth	Lagged growth by size
Dependent variable:	Reserves to assets		Large banks*reserves to assets		Reserves to assets	
	Baseline	Large banks	Baseline	Large banks		
	(1)	(2)	(3)	(4)	(5)	(6)
					0.500***	0.843***
					(0.0914)	(0.0498)
2010	1.116***	-0.971**	-0.0500	0.503		
	(0.195)	(0.327)	(0.0298)	(0.281)		
2011	1.750***	-1.179***	-0.0628*	0.955***		
	(0.208)	(0.315)	(0.0278)	(0.254)		
2012	2.114***	-1.960***	-0.0752**	0.553***		
	(0.207)	(0.257)	(0.0253)	(0.162)		
2013	1.757***	-1.237***	-0.0796**	0.914***		
	(0.214)	(0.237)	(0.0253)	(0.103)		
2014	1.207***	-0.425	-0.0899**	1.190***		
	(0.207)	(0.262)	(0.0279)	(0.120)		
2015	1.253***	-0.670**	-0.107***	0.968***		
	(0.217)	(0.239)	(0.0300)	(0.0800)		
2016	1.050***	-0.865***	-0.138***	0.598***		
	(0.206)	(0.215)	(0.0291)	(0.0708)		
2017	0.633***	-0.508**	-0.128***	0.519***		
	(0.181)	(0.189)	(0.0280)	(0.0665)		
2018	0.264	-0.185	-0.132***	0.529***		
	(0.158)	(0.176)	(0.0290)	(0.0626)		
2019	0.588***	-0.684***	-0.148***	0.350***		
	(0.142)	(0.181)	(0.0287)	(0.0850)		
Number of banks	2,011				2,035	2,035
Number of observations	67,090				67,806	67,705
R^2	0.1971		0.3586		0.0475	0.0614

Table A3: First-stage instrumental variable results—Aggregate instruments

	Reserves to assets	Large banks* Reserves to assets	Reserves to assets	Large banks* Reserves to assets	Reserves to assets	Large banks* Reserves to assets
	(1)	(2)	(3)	(4)	(5)	(6)
Reserves growth	0.0615***	0.0150***				
	(0.00479)	(0.00163)				
Large banks	0.0141	-0.122***				
	(0.0274)	(0.0224)				
Reserves growth^2	-0.276***	-0.105***				
	(0.0237)	(0.0111)				
Large banks	-0.0216	1.506***				
	(0.217)	(0.193)				
TGA to reserves			-0.0655***	-0.0215***		
			(0.00543)	(0.00259)		
Large banks			0.0134	0.264***		
			(0.0363)	(0.0292)		
FBO share					-1.802***	-0.182
					(0.496)	(0.182)
Large banks					1.761	12.80***
					(1.579)	(1.340)
Number of banks	2,035	2,035	2,035	2,035	2,035	2,035
Number of observations	67,806	67,806	67,806	67,806	67,806	67,806
R-sq	0.0506	0.0232	0.0521	0.0111	0.0486	0.2057

Appendix - Discount Window Access

In general, the decision to gain access to the discount window also responds to basic cost-benefit evaluations. If the costs of gaining access to the discount window were zero, banks exposed to liquidity shocks would choose to have access (or be indifferent about it). However, there are some costs that a bank incurs when gaining access to the discount window, stemming from, for example, setting up the systems and collateral-pledging processes. In this appendix, we investigate further how access interacts with other decisions of the bank and in particular with the decision to hold bank reserves.

Suppose that bank i has a cost c_i^{DW} from gaining access to the discount window. Going back to expression (8), we have that bank i will choose to gain access to the discount window whenever

$$\widehat{V}_{iA} - c_i^{DW} \geq \widehat{V}_{iN},$$

where $\widehat{V}_{iA}$ is the optimized value of $\widehat{V}_i$ in expression (8) (when the bank has access to the discount window) and $\widehat{V}_{iN}$ is the optimized value of $\widehat{V}_i$ when the bank has no access to the discount window, as in Section 3.3.1. It is important here to realize that the choice of reserves (and securities) depends on whether the bank obtains access to the discount window. Corollary 4.1 illustrates this point.

Continuing with our leading example, we know from propositions 3 and 4 that the choice of reserves depends on the level of the interest rate on deposits relative to other relevant interest rates. For example, when $r_D \in (r^{T1}, r^{T2})$ we have that the bank would set the level of reserves to equal Δ_2 regardless of whether it has access to the discount window. In fact, the bank will not need to use the discount window or overnight overdraft in such case. When the cost $c_i^{DW} > 0$, the bank will choose not to pay it and hence will not have ready access to the discount window.

A more interesting case ensues when $r_D \in (r^{T2}, r^{T2'})$. In this case, a bank with access to the discount window would set its level of reserves equal to Δ_1, and a bank without access to the discount window would instead choose reserves equal to Δ_2 (see Figure 4). This, in turn, implies that the bank without access will not need central bank funding, while the bank with access to the discount window will borrow from the central bank when the shock is large (equal to Δ_2). Based on these patterns, after some algebra, it can be shown that the bank will choose to have access to the discount window if

$$(r_D - r^{T2})(\Delta_2 - \Delta_1) \geq c_i^{DW}. \tag{12}$$

The interpretation of this condition is simple. When a bank chooses to not have access to the discount window, it chooses to hold $\Delta_2 - \Delta_1$ extra reserves at a cost of r_D. A bank that chooses to have access, instead, would decide to hold lower reserves but would have to borrow from the interbank market or the discount window (by and amount $\Delta_2 - \Delta_1$) according to the probability that the interbank market is open (q), or not ($1 - q$), at the time when the bank needs the funds (this average funding cost is exactly reflected in the formula for r^{T2}). When the differential funding cost associated with the two alternatives is greater than the cost of obtaining access, the bank will choose to gain access.

A second interesting case is when $r_D \in (r^{T2'}, r^{T3})$. In this case, both the bank with access to the discount window and the one without it would choose the same level of reserves Δ_1. In that way, both banks would experience the same liquidity needs with the same probabilities. In particular, when the shock equals Δ_2 and the bank does not have access to interbank markets, it will need to seek funding from the central bank. Having access to the discount window lowers the cost of that funding (by avoiding a more expensive overdraft). Hence, whenever

$$(1 - q)p_2(r_o - r_{DW})(\Delta_2 - \Delta_1) \geq c_i^{DW}, \tag{13}$$

bank i will choose to obtain access to the discount window by paying the cost c_i^{DW}.

The rest of the cases (for higher values of r_D) are similar, with the bank saving in funding costs by lowering the cost of central bank liquidity in some contingencies (when the shock is large) even if in some cases the choice of lower reserve levels increases exposure to that liquidity risk.

With the set of interest rates taken as given by the bank, equations (12) and (13) (and their counterparts for the other cases) capture the factors that determine whether a bank will choose to gain access to the discount window. For example, banks with lower c^{DW} are more likely to choose to gain access to the discount window. These differences in cost may originate, for example, on a differential treatment of discount window requirements across Federal Reserve districts in the U.S.

Furthermore, banks facing different shock processes will, in principle, make different access decisions. The difference in shock process is reflected not only on the support of possible values of Δ, but also on the probabilities over those values and the probability that the bank could face the shock when the interbank market is closed (these probabilities determine r^{T2} in equation (12)).

For a given structure of the shock, the model suggests that a bank choosing to obtain access to the discount window would choose to hold no more reserves than a bank not choosing to have such access. However, note that the decision to gain access is driven (among other things) by the differential between values of Δ (the variability on the size of the shock) while the level of reserves chosen by the bank depends on the *level* of the different values of Δ (the size of the shocks). For this reason, when banks are heterogeneous over the shock process they face, some of those banks can be choosing to gain access to the discount window and also hold relatively high levels of reserves compared with other banks that face smaller and less variable shocks and choose to not have access to the discount window. With this in mind, we restrict the sample in some cases to banks with access to the discount window, to partially control for patterns of unobserved heterogeneity.