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On Regional Borrowing, Default, and Migration

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Abstract

How do local government borrowing, default, and migration interact? We find in-migration results in excessive debt accumulation due to a key externality: Immigrants help repay previously-issued debt. In addition to providing direct IV evidence on this mechanism, we show cities are heavily indebted, near state-imposed borrowing limits, vulnerable to interest rate increases, and default even after periods of robust population and productivity growth. Our quantitative model reproduces these features of the data and reveals a bifurcation: in-migration strongly affects borrowing, but borrowing only weakly affects migration. The model predicts large interest rate declines in the Great Recession prevented a wave of municipal defaults.

JEL Codes: E21, F22, F34, R23, R51
Keywords: migration, debt, default, bankruptcy, cities, regional

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1 Introduction

Municipal debt is a \$1.9 trillion source of financing for local governments (Mayo et al., 2020). Normally considered a safe investment, yields exploded both in the Great Recession and COVID-19.[1] Since municipal bond yields reflect actual default risk (Schwert, 2017), their sharp increase suggests local government finances are more vulnerable than they appear in noncrisis periods. Both crises precipitated large-scale interventions, which at least for the COVID-19 crisis, seem to have worked (Haughwout et al., 2021). In addition to these cyclical patterns, default rates and municipal bond spreads have been increasing secularly over the last 30 years, suggesting local government finances are only getting worse.[2]

Local government finances depend not only on national economic conditions, but also on migration decisions, with each entrant to the city reducing debt per person and increasing the tax base (and each departure doing the opposite). This link can be clearly seen in major urban centers where population growth or decline can be tremendous. The leading example of the latter, Detroit, had a 35% reduction in population from 1986 to 2013, which undoubtedly contributed to its 2013 bankruptcy. The remote-work era that COVID-19 may usher in has potential to induce massive population shifts with commensurate implications for city finances. Despite this, there is currently limited empirical evidence and no model linking city finances, migration, and default. This paper fills these gaps in the literature.

We first use a two-period Lucas (1972)-type islands model to highlight a key mechanism, specifically, an overborrowing externality. Each island represents a local economy and has households who make migration decisions, a per person endowment, and a planner who issues debt in the first period (transferring the proceeds to households) and repays it in the second (using lump-sum taxes). The key assumption is that the local planner maximizes the welfare of *current* residents.[3] The model reveals that, relative to an economy-wide planner, local planners have an incentive to overborrow. The reason is simple: New arrivals in the second period will help repay debt issued in the first period, and the planner does not directly value their utility.

With this externality in mind, we turn to the data where we expect to find, and do find, excessive debt. Using comprehensive datasets on city finances, population, migration, and labor productivity, we establish five stylized facts. (While technically incorrect, we will refer to cities and municipalities interchangeably.)[4] First, cities of all types are heavily indebted. Second, cities of all types are near state-imposed borrowing limits. Third, cities respond to arguably exogenous variation (constructed using a Bartik-style shift-share instrument) in in-migration rates by increasing debt. Fourth, cities default even after booms in population and, to a lesser extent, productivity—a phenomenon we name *boom defaults*. Last, default risk is highly sensitive to interest rate movements.

[1] For instance, the BBB-AAA spread increased from 0.6% in 2007Q1 to a peak of 4.3% in 2009Q1; while the AAA spread over 10-year Treasuries rose from 0.71% on February 21, 2020, to 2.74% by March 23, 2020. (Authors' calculations using data from Haver.)

[2] We establish this fact in Section A.5.3 in the appendix.

[3] We show the results also apply when the planner takes future resident utility into account but favors current residents at least marginally more than new entrants.

[4] A municipality is a city, town, or village that is incorporated into a local government.

We then extend the two-period model to a full-blown quantitative model with an infinite horizon, production, government services, housing, borrowing limits, and default. After showing the economy can be centralized (at a local level), we demonstrate the calibrated model reproduces the data's stylized facts. The model reveals an important dichotomy: In-migration strongly affects borrowing, but borrowing only weakly affects migration. This obtains in the calibration because the model needs a large amount of idiosyncratic noise to rationalize migration patterns in the data, in particular the level of out-migration rates and correlation between population and productivity. This makes migration decisions not respond much to changes in debt. In contrast, the overborrowing externality plays a huge role, implying an effective discount of 6.5 cents on the dollar on average.[5]

We then use the model to understand the observed path of the economy in the Great Recession. The model predicts that absent the data's large decline in real interest rates, default rates would have spiked to more than 2% because of the large and persistent decline in productivity. However, the persistent decline in real interest rates greatly improved the ability of local governments to service debt, preventing any substantial increase in municipal default.

Related literature

Our paper builds on the large sovereign default literature begun by Eaton and Gersovitz (1981), which has focused almost exclusively on nation states. Some of the key references are Arellano (2008), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), and Mendoza and Yue (2012). (The handbook chapter Aguiar et al. (2016) provides a thorough description of the literature.) Epple and Spatt (1986) is an exception that argues states should restrict local debt because default by one local government makes other local governments appear less creditworthy. Such a force is not at work in our model because we assume full information. We contribute to this literature by showing migration strongly influences debt accumulation and can result in boom defaults.

Our work also connects to a vast literature on intranational migration. The empirical work and to a lesser extent theoretical is surveyed in Greenwood (1997). Two seminal papers in this literature, Rosen (1979) and Roback (1982), employ a static model with perfectly mobile labor. This implies every region provides individuals with the same utility. While this indifference condition allows for elegant characterizations of equilibrium prices and rents, it also means government policies are completely indeterminate: every debt, service, or tax choice results in the same utility. Our model breaks this result by assuming labor is imperfectly mobile, which lets it match both the sluggish population adjustments and the small correlations between productivity and migration rates observed in the data.

More recently, Armenter and Ortega (2010), Coen-Pirani (2010), Van Nieuwerburgh and Weill (2010), Kennan and Walker (2011), Davis et al. (2013), and Caliendo et al. (2017) have analyzed determinants of migration and its consequences in the U.S. Kennan and Walker (2011) use a

[5]The overborrowing externality implies a discount of $1 - (1 - o)/(1 - o + i)$ where o and i are the out- and in-migration rate, respectively. Because both rates are approximately 6.5% at a local level, the implied discount is around 6.5 cents on average.

structurally estimated model of migration decisions and find expected income differences play a key role, providing external evidence of the model's productivity-driven migration decisions. Outside the U.S., recent research has been focused on migration in the EU (Farhi & Werning, 2014; Kennan, 2013, 2017). All these papers abstract from debt. To our knowledge, ours is the first quantitative model of regional borrowing and migration, let alone having default.

A few papers in this literature have discussed the potential for local governments to overborrow because of migration. Bruce (1995) and Schultz and Sjöström (2001) prove that overborrowing generally does occur. However, both of their models are two-period models with costless moving, and our theoretical results imply this is not an innocuous assumption. Additionally, we show empirically and quantitatively the role of overborrowing in reproducing many of the data's features. One beneficial reason for cities to issue debt is that debt can make those who benefit from capital investment in the future pay for it (Bassetto & Sargent, 2006). While we abstract from this mechanism, Schultz and Sjöström (2001) show that overborrowing occurs even with durable goods. In fact, given our result that in-migration effectively makes governments impatient, there is good reason to think extending our model to public investment would result not only in excessive debt issuance but also underinvestment in durable goods (which is borrowing in a different guise).

Finally, building on earlier versions of this paper, Alessandria et al. (2020) features a sovereign government (Spain in their calibration) facing in-migration and issuing debt. Similar to our finding that in-migration induces overborrowing, they find in-migration causes increased indebtedness relative to a no-in-migration economy. A key difference from our paper is that in-migration, in their model, is exogenous and out-migration is not allowed: In contrast, in-migration and out-migration in this paper are jointly determined given the entire distribution of available locations.

The rest of the paper is organized as follows. Section 2 presents a simple model that highlights the overborrowing mechanism inherent in models with in-migration and borrowing. Section 3 documents key stylized facts. Section 4 lays out the quantitative model, and Section 5 describes the calibration. Section 6 shows the quantitative model reproduces the stylized facts and analyzes the Great Recession shock. Section 7 concludes. The appendices report data details, the computational algorithms, proofs, and omitted results.

2 The overborrowing mechanism

First, we highlight how migration influences borrowing decisions and efficiency using a two-period model. To focus purely on the roles of borrowing and migration, we assume there is full commitment to repay debt and, hence, no default.

The economy is comprised of a unit measure of islands and a unit measure of households. Assume islands are homogeneous, and consider an arbitrary one. In the first (second) period, the island has a per person nonstochastic endowment of y_1 (y_2). The local government issues $-b_2$ debt per person ($b_2 > 0$ means assets) at price $\bar{q}$. Total debt issuance is $-b_2 n_1$, where n_1 is the initial measure of households on the island. At the beginning of the second period, households draw an idiosyncratic utility cost of moving $\phi \sim F(\phi)$ with a density f and then decide whether to migrate.

If they migrate, they pay ϕ and obtain expected utility J, which is an equilibrium object.

Households value consumption according to $u(c_1)+\beta u(c_2)$, where c_1 (c_2) is consumption in the first (second) period. Household utility in the second period is $u(c_2)$ if they stay and $J-\phi$ if they move, so migration decisions follow a cutoff rule in ϕ with indifference at $J-u(c_2)$. Consequently, the outflow rate is $o_2 = F(J-u(c_2))$. The inflow rate is given by $i_2 = \bar{i}\mathbf{I}(u(c_2))$, where $\mathbf{I}$ is a differentiable, increasing function, and $\bar{i}$ is an equilibrium object that ensures aggregate inflows equal aggregate outflows. (Consequently, inflows can depend on the distribution of utility across islands, but that information must be summarized in $\bar{i}$.) The population law of motion is $n_2 = (1+i_2-o_2)n_1$. We assume the migration decision is noisy in the sense that $F(0) > 0$, so that some people will move even if $u(c_2) = J$.

After all migration has taken place, the government pays back its total obligation, $-b_2n_1$, by taxing the n_2 households lump sum. Consequently, per person consumption in the second period is $c_2 = y_2 + b_2n_1/n_2$. The government's problem may be written

$$\begin{gathered}\max_{b_2} u(c_1) + \beta \int \max\{u(c_2), J-\phi\}\, dF(\phi) \\ \text{s.t. } c_1 + \bar{q}b_2 = y_1, \quad c_2 = y_2 + b_2\frac{n_1}{n_2}, \quad n_2 = n_1(1-o_2+i_2), \quad c_1, c_2 \geq 0, \\ i_2 = \bar{i}\mathbf{I}(u(c_2)), \quad o_2 = F(J-u(c_2)).\end{gathered} \tag{1}$$

An *equilibrium* is a pair $\{\bar{i}, J\}$ with optimal migration, consumption, and borrowing decisions such that

1. total inflows equal total outflows, $\bar{i}\int \mathbf{I}(u(c_{2,j}))n_{1,j}dj = \int F(J-u(c_{2,j}))n_{1,j}dj$, and
2. the expected utility of moving is consistent, $J = \int u(c_{2,j})\frac{\bar{i}\mathbf{I}(u(c_{2,j}))n_{1,j}}{\int \bar{i}\mathbf{I}(u(c_{2,i}))n_{1,i}di}dj$

(where j indexes islands).

Proposition 1 gives the Euler equation for government bonds (all proofs are in Appendix C).

Proposition 1. *The local government's Euler equation is*

$$u'(c_1)\bar{q} = \beta\frac{1-o_2}{1-o_2+i_2}u'(c_2)\left(1-\frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right). \tag{2}$$

The Euler equation reflects two competing forces. One is an externality seen in the term $\frac{1-o_2}{1-o_2+i_2} \leq 1$. Because the planner does not value the utility of new entrants and because new entrants bear $\frac{i_2}{1-o_2+i_2}$ of the debt burden (which is their share of the second period population), the marginal cost associated with an additional unit of debt—holding fixed migration rates—is $1-\frac{i_2}{1-o_2+i_2}$ or $\frac{1-o_2}{1-o_2+i_2}$. Else equal, higher in-migration lowers the effective discount factor ($\beta\frac{1-o_2}{1-o_2+i_2}$) and increases borrowing. Clearly, then, the assumption that the planner only values current residents plays a key role. But note that it is also the most natural assumption: If households could vote on the planner's policy in the first period, they would unanimously approve it because it maximizes their welfare.[6]

[6]The social welfare function (SWF) in (1) is the welfare of current residents. Suppose instead that the SWF is

$$n_1u(c_1) + \beta n_1^{1-\theta}n_2^{\theta}u(c_2).$$

Before discussing the second force, we emphasize that this externality is really about in-migration, not out-migration. Consider an extreme case where half the population leaves $o_2 = 1/2$ but no one arrives $i_2 = 0$. In that case, the overborrowing term $\frac{1-o_2}{1-o_2+i_2}$ equals 1, i.e., there is no extra discounting in the Euler equation. The reason is that while half of current residents leave and pay nothing, the half who remain must pay double, and this offsets in the Euler equation. On the other hand, if no one leaves $o_2 = 0$ but the population doubles through in-migration $i_2 = 1$, then the overborrowing term $\frac{1-o_2}{1-o_2+i_2}$ equals 1/2, implying an effective 50% discount on debt issuance. In intermediate cases where o_2 and i_2 are positive, in-migration has a first-order effect while out-migration has only a second-order effect. While the externality is primarily driven by in-migration, out-migration still has a first-order effect on debt per person and hence will play a key role in default decisions.

The Euler equation's other, potentially offsetting force, is seen in the term $1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}$, which is one minus the elasticity of the next period's population with respect to savings. It reflects that for each person attracted to the island through less borrowing, the overall debt burden per person falls. (Conversely, if $b_2 > 0$, each additional entrant reduces assets per person, which discourages savings.) Hence, a rational government, internalizing the effects of city finances on migration decisions, should exercise more financial discipline (else equal) to attract individuals to the islands to reduce debt per person. While this force is present in the quantitative model, the overborrowing term dominates it since migration decisions must be noisy to match the data's migration patterns.

To this point, we have claimed that cities overborrow, implicitly having in mind the solution to a social planner problem, which we now state. Let $\hat{c}_{1,i}, \hat{c}_{2,i}$ denote the optimal consumption (in periods 1 and 2, respectively) of household $i \in [0, 1]$, and let ϕ_i denote the moving cost shock realization the household receives. With homogeneous islands, the endowments are the same irrespective of moving decisions, with y_1 (y_2) the first (second) period endowment. Taking migration decisions m_i as given, the planner's objective function is

$$\max_{\hat{c}_{1,i}\geq 0, \hat{c}_{2,i}\geq 0} \int \alpha_i (u(\hat{c}_{1,i}) + \beta(u(\hat{c}_{2,i}) - m_i\phi_i))di, \tag{3}$$

where α_i is the Pareto weight on household i. The resource constraint is given by

$$\int \hat{c}_{1,i}di + \bar{q}\int \hat{c}_{2,i}di = y_1 + \bar{q}y_2. \tag{4}$$

Here, the parameter $\theta \in [0, 1]$ controls how much the planner cares about the welfare of future residents, and $\theta = 1$ results in the SWF of Hall and Jones (2007) and Alessandria et al. (2020). The Euler equation becomes

$$u'(c_1)\bar{q} = \beta\left(\frac{1}{1-o_2+i_2}\right)^{1-\theta} u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right) + \psi,$$

where ψ is a term capturing a "value of statistical life" effect, which equals zero if $u(c_2) = 0$. When $\theta = 1$, migration does not induce overborrowing. But when policy weights current residents even marginally more, $\theta < 1$, more in-migration gives extra incentive to borrow else equal. The externality is amplified as $\theta \to 0$.

Note this SWF fails to recognize increasing b_2 has no effect on outgoing-resident utility, $J - \phi$, with only newcomers and the $n_1(1 - o_2)$ residents who stay benefiting. Accounting for this with a SWF $n_1 u(c_1) + \beta[n_1(1-o_2)]^{1-\theta} n_2^{\theta} u(c_2)$ results in an overborrowing term of $(\frac{1-o_2}{1-o_2+i_2})^{1-\theta} \leq 1$.

We say an allocation is *constrained efficient* if it solves the planner problem with migration decisions given for some Pareto weights.

Optimality requires that marginal rates of substitution must be equated across individuals, i.e., $\beta u'(\hat{c}_{2,i})/u'(\hat{c}_{1,i}) = \beta u'(\hat{c}_{2,j})/u'(\hat{c}_{1,j})$ for almost all i, j. Using the resource constraint, it is easy to show these must also equal $\bar{q}$, i.e.,

$$u'(\hat{c}_{1,i})\bar{q} = \beta u'(\hat{c}_{2,i}). \tag{5}$$

In comparing (5) with the local government's Euler equation (2), it is clear that overborrowing will occur if the optimal bond choice b_2 is close to zero: in that case, the incentive to attract people—reflected in the term $1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}$—is close to zero, while the externality of new entrants shouldering the burden—reflected in $\frac{1-o_2}{1-o_2+i_2}$—is not. With $\bar{q} = \beta u'(y_2)/u'(y_1)$, implementing the constrained efficient allocation requires $b_2 = 0$, which results in overborrowing as formalized in Proposition 2:[7]

Proposition 2. *If $\bar{q} = \beta u'(y_2)/u'(y_1)$, equilibrium is not constrained efficient. Moreover, at the constrained efficient allocation, governments would strictly prefer to borrow.*

To summarize, we showed cities have an incentive to overborrow. The mechanism is an externality created through in-migration: new entrants will help repay debt issued today. We now turn to the data.

3 Empirical patterns of debt, migration, and default

In this section, we will establish five stylized facts using a variety of data sources described in Appendix A, where we also provide additional analysis of the data and facts. First, virtually all cities, large and small, productive and unproductive, are indebted. Second, almost all cities are close to state-imposed borrowing limits. Third, in response to in-migration, expenditures per person and debt per person increase. Fourth, cities default after both booms and busts in population and productivity. Last, municipal bond default risk is sensitive to interest rate changes.

3.1 Stylized fact #1: Cities of all types are indebted

Our first stylized fact is that all types of cities are indebted. This can be seen in Figure 1, which presents the empirical relationship of log debt per person (p.p.) and log revenues per person (p.p.) in California. One can see cities of all sizes *have* debt: there are not cities clustering at low debt levels. Additionally, the linear relationship is stark, with more expenditures strongly positively correlated with more debt. As we show in Figure 12 in the appendix, these patterns applies to all cities in the U.S., not just Californian ones. The quantitative model will generate both of these features of the data through (1) the overborrowing incentive already seen in the theoretical model paired with (2) borrowing constraints that relax as income and/or expenditures increase.

[7]This inefficiency result may be surprising in light of the seminal paper by Tiebout (1956). It arises because he assumes costless and fully-directed mobility, which we do not require. We prove in Proposition 4 in the appendix that efficiency can hold in this case, which requires an "infinite elasticity" of in-migration to debt.

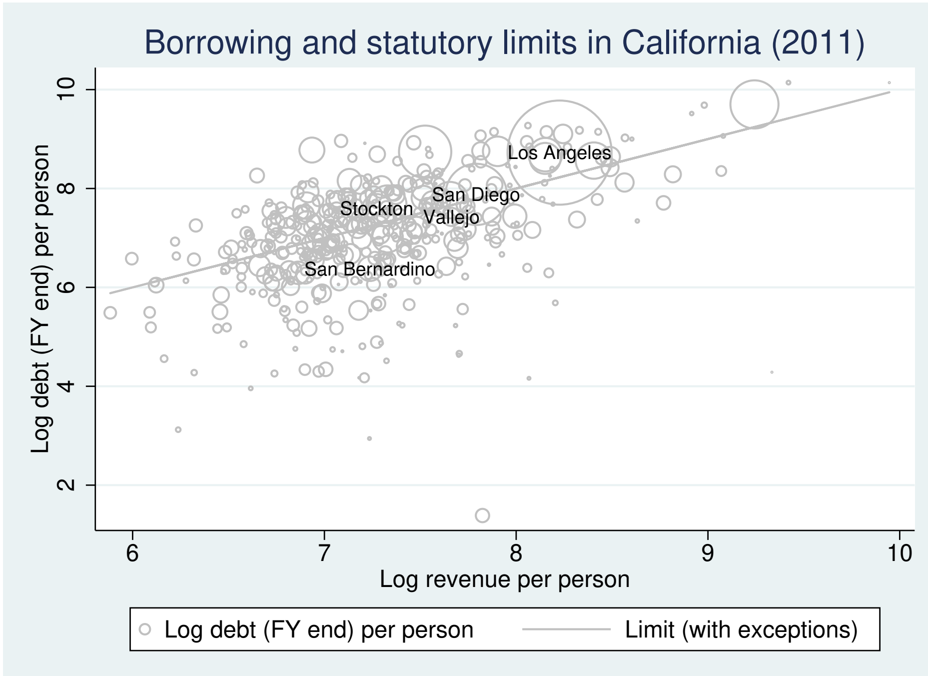


Figure 1: Statutory borrowing limits and closeness to the limits

3.2 Stylized fact #2: Cities of all types are close to borrowing limits

While we saw above that cities of all types are indebted, we have not established precisely how indebted they are relative to their state-imposed maximum borrowing capacity. Because most of the borrowing limits are expressed in terms of assessed property value (which is not in our dataset), in general it is hard for us to directly look at this question.

However, we can directly assess how close cities are to their borrowing limits for two states. California uses expenditures to constrain debt, and so we can tell how close cities are to their limit directly from Figure 1. The graph reveals that many cities—including very large ones—are borrowing beyond the revenue per person limit (the limit allows exceptions for spending on special projects and borrowing authorized by referendum). For Michigan, we have taxable-valuation data from Kleine and Schulz (2017), and in the appendix we show cities there are also close to their statutory borrowing limits. This finding suggests borrowing limits are binding for many cities in the data, consistent with the model prediction that cities have a strong incentive to borrow.

3.3 Stylized fact #3: In-migration increases expenditures and debt

So far we have shown cities are highly indebted, but we have not looked at the link between migration and debt accumulation. We now examine this mechanism directly using an instrumental variable approach. In particular, we construct an instrument for in-migration using the shift-share approach, a common technique in empirical studies of migration (see, e.g., Altonji and Card, 1991 and Card, 2001). For each county c, we construct the share of individuals at a reference time $\tau > 0$

periods ago who live in c but arrived from county o at some time in the past.[8] Then, with the share $\theta_{c,o,t-\tau}$, we compute the total outflows $O_{o,t}$ from each county o at time t. Given this aggregate shift from o, we then apply the share $\theta_{c,o,t-\tau}$ to get expected inflows $\theta_{c,o}O_{o,t}$ at time t from county o to county c. Summing across all other counties gives the total expected inflows and a total expected in-migration rate:

$$\hat{i}_{c,t} = \sum_{o \neq c} \theta_{c,o,t-\tau} O_{o,t}/N_{c,t}, \tag{6}$$

where $N_{c,t}$ is the population in county c at the start of time t. If shocks to county o are orthogonal to shocks in c (after controlling for time effects), the expected inflow rate $\hat{i}_{c,t}$ should only affect local variables through its effect on $i_{c,t}$, as required for a valid instrument.

Given this, we run the specification

$$y_{c,f+t} = \alpha + \beta i_{c,t} + \gamma y_{c,t-l} + \zeta \varpi_c + \sum_{\tilde{t}} \mu_{\tilde{t}} \mathbf{1}[t = \tilde{t}] + u_{c,t}, \tag{7}$$

where ϖ_c is a productivity fixed effect and μ_t are time effects. We instrument the actual in-migration rate $i_{c,t}$ with the predicted one $\hat{i}_{c,t}$. We use $\tau = 5$ in (6); in (7), we use $l = 5$ with f ranging from 0 to 20.[9]

Figure 2 plots the semi-elasticity coefficient β at different horizons (f in equation 7) for each of debt, expenditures, and revenue as the dependent variable (always in log per person). Full estimates are reported in tables in Appendix C. The top (bottom) panels give the IV (OLS) estimates, and the gray bands provide 95% confidence intervals from robust standard errors. Evidently, the IV estimates are far from the OLS and, for expenditures and revenue, differently signed (indicating the presence of endogeneity and the necessity of IV).

Since debt—whose maturity is around ten years—is mostly predetermined, on impact debt should change little with a valid instrument, and one can see that it does.[10] In contrast, ten years ahead there is a statistically significant effect, which is also economically significant: The standard deviation of $i_{c,t}$ is 0.022, so the point estimate implies a one standard deviation increase in $i_{c,t}$ increases debt per person by approximately 31% ($= e^{12.39 \times 0.022}$). There are also significant effects for revenue and expenditures per person, with a semi-elasticity of around 10 and 12.5, respectively. The larger increase in expenditures, paired with expenditures typically being larger than revenue, eventually results in debt growth, consistent with the theory.[11]

3.4 Stylized fact #4: Cities default after busts and booms

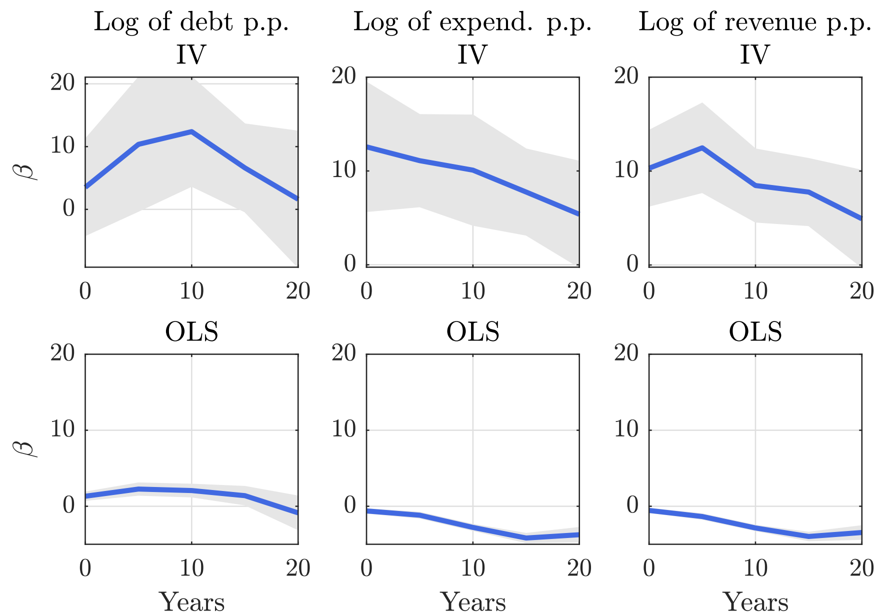
We now seek to establish a key, surprising, and novel finding, which is that cities default not only after busts but booms as well. To this end, our sample is Detroit (MI), Flint (MI), Harrisburg (PA),

[8] See the appendix (Section C.7) for details.

[9] We focus on 5-year intervals because the Census of Governments data is most comprehensive in years ending in 1 or 6.

[10] In the Census of Governments data, the median (mean) of the ratio of retired to total long-term debt is 0.09 (0.14), implying a maturity of 7 to 11 years.

[11] We do not report estimates for deficits because they are very noisy. The noise is due in large part to lagged deficits containing little information, unlike lagged debt stocks or expenditures.



Note: Gray bands denote 95% confidence intervals.

Figure 2: Estimated effects due to the overborrowing externality

San Bernardino (CA), Stockton (CA), Vallejo (CA), Chicago (IL), and Hartford (CT), cities that have defaulted or been reported as having financial difficulties in the last few years.[12]

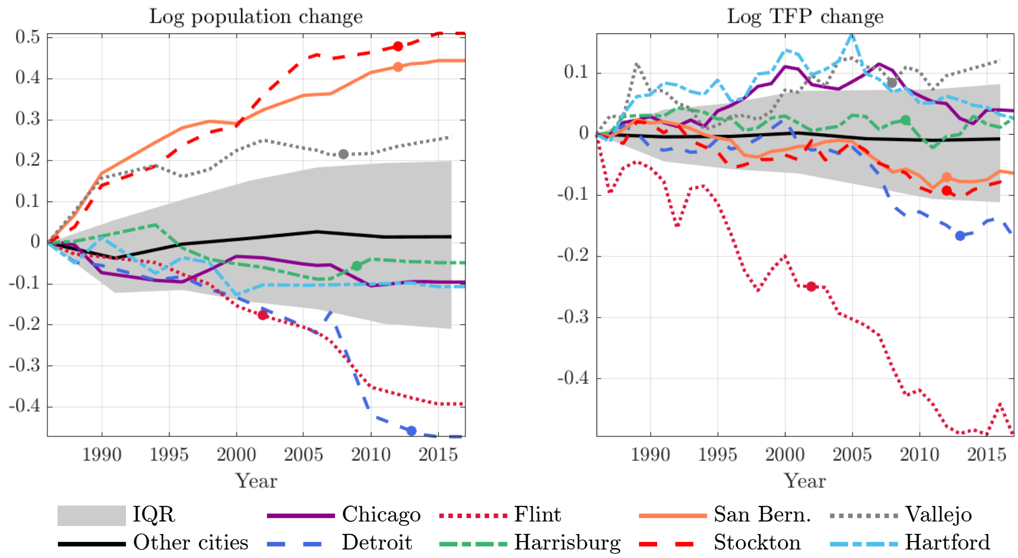
The data point to heterogeneous paths to default or, more generally, fiscal stress. Figure 3 reveals some cities experience unusually large population growth before default, while others experience large losses. Given that population growth directly reduces debt per person, it may be surprising that default could occur after such large booms. However, the model will generate this through the overborrowing externality that keeps debt per person high especially when cities are growing. Productivity growth has a similar though less drastic pattern. Unsurprisingly, some cities experience fiscal stress after adverse productivity shocks. However, other cities experience fiscal stress after either stable or positive productivity gains (Vallejo, Hartford, Chicago). From the lens of consumption-smoothing models, the latter observation is surprising: In response to positive productivity shocks, agents should deleverage. However, we will show the model can generate defaults after productivity booms (as well as busts) due to overborrowing.

3.5 Stylized fact #5: Default risk is elastic to interest rate changes

One of the quantitative model's predictions is that interest rates strongly influence default risk. So for our last stylized fact, we investigate whether this is the case in the data and find that it is. In the main text, we do this using a piece of narrative evidence. But the appendix uses a structural vector autoregression (SVAR) model to establish the same result.

The left panel of Figure 4 reports the spread of BBB, A, and AA municipal bond yield to maturity (YTM) over AAA municipal bonds. To the extent that default risk is tail risk, one should

[12]News coverage on these and other cities is listed in Appendix A.



Note: Changes are log differences relative to 1986 except for Chicago, which is relative to 1987; the TFP measure is net of time effects; circles denote periods of acute fiscal stress such as defaults, bankruptcies, or emergency manager takeovers (the last only for Flint); "other cities" is the universe of cities in the data, which covers 64% to 74% of the U.S. population over the time range; IQR stands for interquartile range.

Figure 3: Case Study – Cities under Financial Stress

expect increases in default risk to primarily show up in larger BBB spreads, and one can clearly see the spikes in 2009 and 2020. There is one other noticeable spike, which occurs in the latter part of 2013 into 2014. What was its cause?

The most likely culprit for this spike is the sharp increase in long-term interest rates known as the *taper tantrum*. This period, beginning in May 2013 (and seemingly coinciding with the May FOMC statement), saw the ten-year Treasury note rise from 1.6% to 2.9% in four months at a time when the fundamentals of the economy appeared relatively stable (and short-rates were unchanged). This rapid increase in interest rates, both in the ten-year Treasury and in AAA municipal bonds, is evident in the linearly-detrended series in the right panel of Figure 4. In our view, these nearly-doubled debt-servicing costs resulted in increased default risk that was concentrated specifically in the riskier municipal bond tranches.

In case this anecdotal evidence is unsatisfactory, we provide an analysis based on an SVAR in Section A.5.6 of the appendix. There we show that an exogenous increase in the AAA yield leads to a persistent increase in the spreads of the returns on AA, A, and BBB bonds, with the spread on the more risky BBB bonds increasing the most.

4 The quantitative model

We first provide an overview of the model and its timing. Then, we describe the household, firm, and government problems. Finally, we define equilibrium and show how the model can be centralized at a local level.

To simplify the exposition and avoid unnecessary notation, we describe the model in terms of

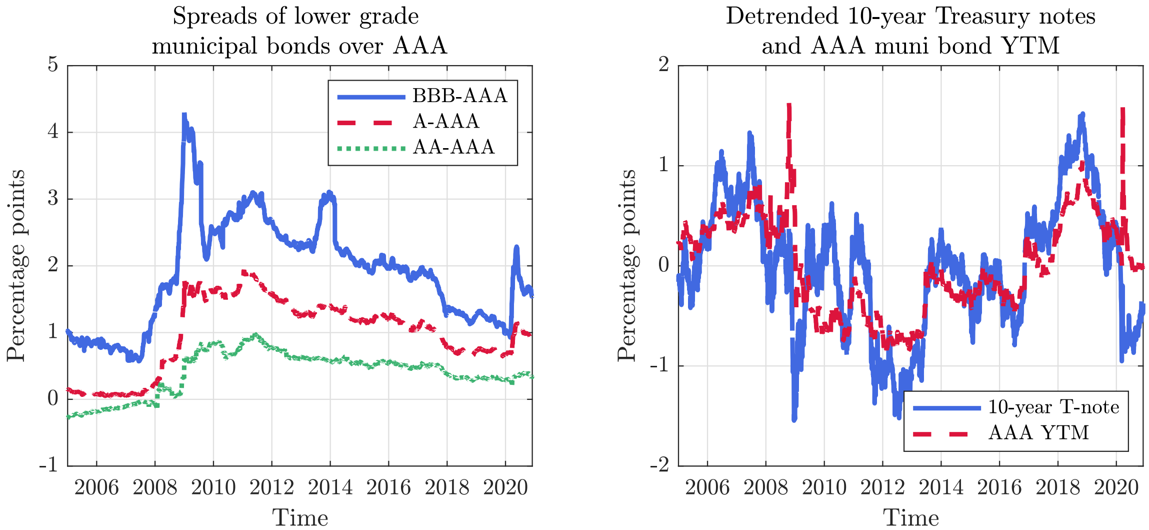


Figure 4: Municipal bond yields and spreads over time

household and government problems that must be consistent with each other and satisfy certain equilibrium properties. In Appendix C.2, we show how to cast the model in terms of an extensive form game, and show the model's equilibrium corresponds to the Markov perfect equilibrium of this extensive form game.

4.1 Overview and timing

We model municipalities as a unit measure of islands. Each island consists of a continuum of households (whose measure in the aggregate is one), a local government, and a neoclassical firm. Each local government is a sovereign entity that issues debt, taxes its residents, and provides government services. Households consume, work, and decide whether to stay on the island or migrate to another one.

The timing of the model is as follows. At the beginning of the period, all shocks are realized. Upon observing them, households make migration decisions. After migration occurs, the local government chooses its policies, including debt issuance. This timing means that unanticipated, one-period deviations in government policies do not alter the population.[13] Finally, households make consumption and labor decisions simultaneously with firms while taking prices and government policies as given.

4.2 Households

Define the state vector of a generic island as $\mathbf{x} := (b, n, z, \omega)$, where b is assets per person measured before migration, n is the population before migration, z is the island's productivity, and ω is a permanent island type that we will call "weather." The weather variable is a location-specific *fixed effect* that captures (in reduced form) climate/weather, location, history, or any other immutable, location-specific factor. (We call ω weather, rather than amenities, because local government services

[13] We view this assumption as reasonable in that migration is a time-consuming process that often involves searching for a new job, finishing a school year, selling an existing home, and finishing rental agreements.

are also an amenity.) Including it allows the model to match the variance of population across cities without producing a counterfactually large correlation between productivity and in-migration. We assume z follows a finite-state Markov chain.

Households, knowing $\mathbf{x}$, decide whether to stay $m = 0$ or move $m = 1$ after drawing an idiosyncratic utility cost of moving, $\phi \sim F(\phi)$. If they stay, they expect to receive lifetime utility $S(\mathbf{x})$ (specified below). If they move, they are assigned to another island, receive J in expected lifetime utility, and pay ϕ. Their problem is

$$V(\phi, \mathbf{x}) = \max_{m \in \{0,1\}} (1 - m)S(\mathbf{x}) + m(J - \phi). \tag{8}$$

The moving decision follows a reservation strategy $R(\mathbf{x})$ with $m = 1$ when $\phi < R(\mathbf{x})$.

The utility conditional on staying is

$$\begin{aligned} S(\mathbf{x}) = \max_{c \geq 0, h \geq 0} \ & u(c, g(\mathbf{x}), h, \omega) + \beta \mathbb{E}_{\phi', \mathbf{x}' | \mathbf{x}} V(\phi', \mathbf{x}') \\ \text{s.t. } & c + r(\mathbf{x})h = w(\mathbf{x}) + \pi(\mathbf{x}) - T(\mathbf{x}), \end{aligned} \tag{9}$$

where $w(\mathbf{x})$ is the island's wage; $\pi(\mathbf{x})$ is the per person profit from the island's firm; $g(\mathbf{x})$ is government services; $T(\mathbf{x})$ are lump-sum taxes (which we will show is virtually equivalent to using property taxes); and h is a housing good, owned by the firm and rented to households at price $r(\mathbf{x})$. The expectation term $\mathbb{E}_{\phi', \mathbf{x}' | \mathbf{x}}$ embeds household beliefs about the local government's policies, as well as others' migration decisions. In particular, since $\mathbf{x}' = (b', n', z', \omega)$, households must have beliefs on migration decisions of others (to pin down n') and the debt issuance policy (to pin down b') as functions of the shock z' (ω is time-invariant). We will assume default's effects on households operate only through government budgetary impacts, which is why they will not need to have beliefs regarding default, d.

If a household decides to move, they migrate to island $\mathbf{x}$ at rate $i(\mathbf{x})$ and must stay there for at least one period. The inflow rate at an island of type $\mathbf{x}$ is

$$i(\mathbf{x}) = \left(\int nF(R(\mathbf{x})) d\mu(\mathbf{x}) \right) \frac{\exp(\lambda S(\mathbf{x}))}{\int \exp(\lambda S(\mathbf{x})) d\mu(\mathbf{x})}, \tag{10}$$

where μ is the invariant distribution of islands.[14] This inflow rate is a continuous analogue of a logit-style, discrete choice framework.[15]

By construction, the measure of households leaving equals the measure entering in aggregate, $\int i(\mathbf{x}) d\mu(x) = \int nF(R(\mathbf{x})) d\mu(\mathbf{x})$. If $\lambda = 0$, households are uniformly assigned to each island ("random search"). As $\lambda \to \infty$, the city with the largest utility $S(\mathbf{x})$ receives all the inflows ("directed search"). Given these inflows, the expected value of moving in equilibrium is

$$J = \int S(\mathbf{x}) \frac{i(\mathbf{x})}{\int i(\mathbf{x}) d\mu(\mathbf{x})} d\mu(\mathbf{x}), \tag{11}$$

[14] This rule has the same form as in the two-period model. In particular, one can take $\mathbf{I}(S(\mathbf{x})) = \exp(\lambda S(\mathbf{x}))$.

[15] With a finite number of choices indexed by $\mathbf{x}$, the usual specification would be written $\max_{\mathbf{x}} S(\mathbf{x}) + \varepsilon_{\mathbf{x}}/\lambda$ where each $\varepsilon_{\mathbf{x}}$ is i.i.d. with a Type 1 extreme value distribution. Then the probability of choosing $\mathbf{x}$ is proportional to $\exp(\lambda S(\mathbf{x}))$. The problem that arises with a continuum of choices is that $\mathbb{E}[\max_{\mathbf{x}} S(\mathbf{x}) + \varepsilon_{\mathbf{x}}/\lambda]$ becomes infinite since $\varepsilon_{\mathbf{x}}$ has unbounded support. What we need in the continuous case is the notion of a *Gumbel process*. As the technical details for this are quite involved, we discuss how to micro-found (10) in Appendix C.

and the law of motion for population is

$$\dot{n}(\mathbf{x}) = n(1 - F(R(\mathbf{x}))) + i(\mathbf{x}), \tag{12}$$

where $\dot{n}$ denotes the population after migration has taken place.

4.3 Firms

Each island has a firm that operates a linear production technology zL and owns the island's housing stock $\bar{H}$. Alternatively, $\bar{H}$ may be thought of as the island's land. We assume $\bar{H}$ is in fixed supply and homogeneous across islands to prevent adding an extra state variable, but our inclusion of weather ω will capture some of this fixed heterogeneity across islands. Firms solve

$$\dot{n}(\mathbf{x})\pi(\mathbf{x}) = \max_{L,H \leq \bar{H}} zL - w(\mathbf{x})L + r(\mathbf{x})H, \tag{13}$$

taking w and r competitively, and the solution of this problem gives labor demand, L^d (and the housing supply, $H = \bar{H}$). Since $\dot{n}(\mathbf{x})$ denotes the number of households remaining after migration and each household inelastically supplies one unit of labor, labor market clearing requires

$$L^d(\mathbf{x}) = \dot{n}(\mathbf{x}). \tag{14}$$

It is worth making a few observations about the firm problem. First, in equilibrium, per person profits π equal $r\overline{H}/\dot{n}$. Consequently, by making local residents the firm shareholders, we are effectively assuming each gets the rent associated with owning an equal share of the housing/land stock. Second, if there were property taxes, say via $\tau r(\mathbf{x})\overline{H}$ for $\tau \in [0, 1]$, the taxes would reduce these rents by $\tau r\overline{H}/\dot{n}$ in the same way that the lump-sum tax T in (9) does. For this reason, we can interpret the lump-sum tax T as a property tax. Last, we have assumed there are no agglomeration or congestion effects in the production function (or that they are both present and cancel).[16] Their absence could result in the model under- or over-predicting the relationship between population and productivity. However, the model generates a signficant positive correlation between city density and productivity like that found in the data (Glaeser, 2010). Also, the model has congestion externalities in the form of reduced housing per person and agglomeration effects in that local governments provide a partly nonrival service, as will be discussed shortly.

4.4 Local governments

Each local government decides the level of amenities/services $g \geq 0$ it wishes to provide. These services are potentially nonrival in that, to provide g services to each of the $\dot{n}$ households, the government must only invest $\dot{n}^{1-\eta}g$ units of the consumption good where $\eta \in [0, 1]$ is a parameter. The government pays for these services using tax revenue $T\dot{n}$ or, potentially, debt issuance. The government chooses a new level of debt per person $-b'$, implying a total obligation next period of $-b'\dot{n}$. (Since the current period's post-migration population $\dot{n}$ coincides with its pre-migration population next period n', next period debt is also $-b'n'$.) The discount price it receives on this pledge is $q(b', \dot{n}, z, \omega)$, which depends on the debt level, population after migration $\dot{n}$ (which equals

[16] A simple way to introduce agglomeration is with the modified production function $zL\dot{n}^{\varpi}$, where N is population and $\varpi > 1$. Duranton and Puga (2004) provide microfoundations for this type of agglomeration.

the next period's population before migration n'), productivity, and weather, as all of these potentially influence repayment rates.[17] We assume a portion γ is nondefaultable, and we will calibrate it to match the share of unfunded pension obligations.

In keeping with the statutory borrowing limits discussed in Section 3, we impose a borrowing limit

$$-b' \leq \delta g \dot{n}^{-\eta}, \tag{15}$$

where $\delta \in \mathbb{R}^+$ controls how tight the limit is. Hence, we require total debt issued in a period $-b'\dot{n}$ to be less than a fraction δ of total expenditures $g\dot{n}^{1-\eta}$. While this limit is qualitatively closer to the CA limit than most other states' limits, government expenditures are very positively correlated with housing rents in the model, so it also effectively captures a limit based on housing value. Additionally, exemptions in many states allow for spending on projects, which this form permits. Given the large variation in laws across states, we will choose δ to match observed debt levels rather than trying to choose it based on statutory law.

To define the government's problem, we need to specify how the economy will respond to deviations in government policies. To this end, we assume that wages and the rental rate adjust dynamically in response to the government policies (d, g, b', and T), clearing the labor and housing markets, and that households and firms optimize given those prices and implied profits. Formally, we assume that (c, h, r, w, π, L^d) always solve the following equations:

$$\underbrace{\begin{array}{c} u_c r = u_h \\ c + rh = w + \pi - T \end{array}}_{\text{Household optimization}}, \quad \underbrace{\begin{array}{c} w = z \\ \dot{n}\pi = zL^d - wL^d + r\overline{H} \end{array}}_{\text{Firm optimization}}, \text{ and } \underbrace{\begin{array}{c} \dot{n} = L^d, \\ \dot{n}h = \overline{H} \end{array}}_{\text{Market clearing}}. \tag{16}$$

Note that bankruptcy d and debt issuance b' only affects households indirectly through T and g. Letting U denote the indirect flow utility associated with g and T, it is easy to show

$$U(g, T, \dot{n}, z, \omega) = u(z - T, g, \overline{H}/\dot{n}, \omega). \tag{17}$$

To receive bankruptcy protection in the U.S., local governments must be insolvent and negotiate in good faith with creditors, as discussed in Section A.5.1 of the appendix. We interpret these statutory requirements as follows. First, we assume a municipality is *insolvent* if its debt service exceeds a fixed fraction κ of the expected lifetime income of residents. Specifically, the municipality is insolvent if debt service per person exceeds $\kappa\bar{z}(z)$ for $\bar{z}(\cdot)$ defined recursively by

$$\bar{z}(z) = z + \bar{q}\,\mathbb{E}_{z'|z}\bar{z}(z'). \tag{18}$$

In Section A.5.2, we report legal opinions that explicitly support this forward-looking nature of insolvency. When insolvent and filing for bankruptcy, the city pays the greater of $\kappa\bar{z}(z)\dot{n}$ and $-\gamma bn$, making γ fraction of its debt nondefaultable. This is meant to capture pension obligations, which as we discuss in Section A.5.1 are usually viewed as nondefaultable. Additionally, we assume

[17] Capeci (1994) and Schwert (2017) provide empirical evidence on the link between municipal default risk and interest rates. Our use of short-term debt significantly simplifies the computation as long-term debt models suffer from convergence problems (Chatterjee & Eyigungor, 2012).

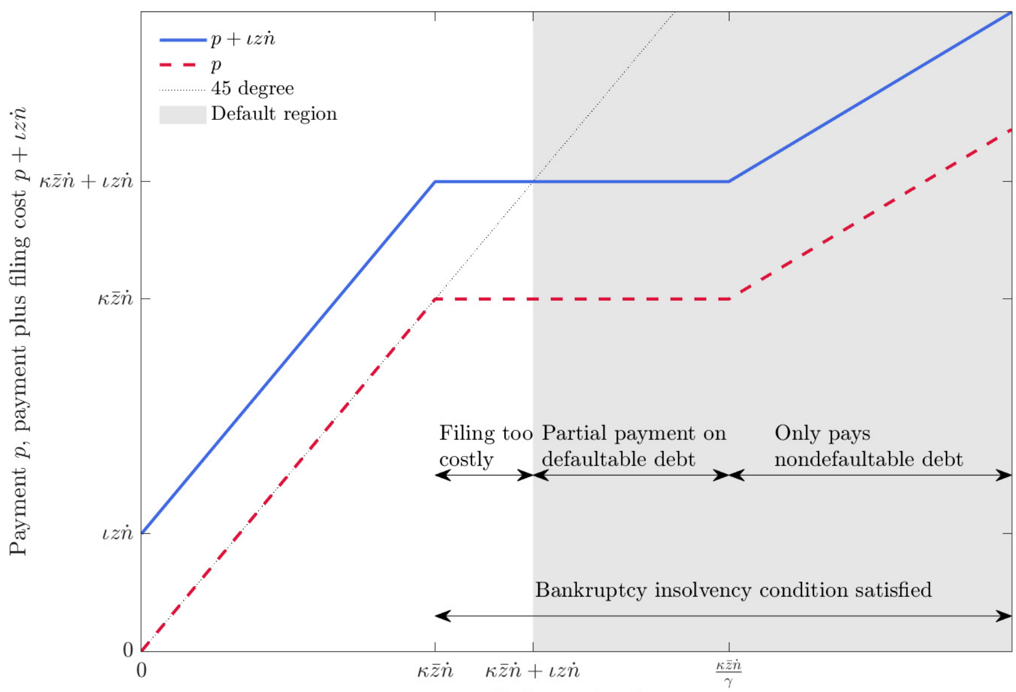
filing for bankruptcy entails a cost ι proportional to the municipality's total income. Consequently, the total payment required in bankruptcy is

$$p(b,n,z,\omega) := \max\{\underbrace{\gamma(-bn)}_{\text{Nondefaultable debt stock}}, \min\{\underbrace{-bn}_{\text{Debt stock}}, \underbrace{\kappa\bar{z}(z)\dot{n}(b,n,z,\omega)}_{\text{Insolvency threshold}}\}\}. \tag{19}$$

The total debt forgiveness from bankruptcy filing is thus $-bn-p(\mathbf{x})$. Since all the costs and benefits of default occur within a period, the default decision—conditional on endogenous variables—is static and given by

$$d(\mathbf{x}) = \mathbf{1}[\underbrace{p(\mathbf{x})}_{\text{Required payment in bankruptcy}} + \underbrace{\iota z\dot{n}(\mathbf{x})}_{\text{Bankruptcy filing fees}} < \underbrace{-bn}_{\text{Debt stock}}]. \tag{20}$$

Note that if the debt stock per person $-bn/\dot{n}$ is less than the insolvency threshold $\kappa\bar{z}$, then the whole debt stock must be paid $p = -bn$ and the city does not default $d = 0$. The bankruptcy process and default decision are illustrated in Figure 5.



Note: illustration holds $\dot{n}$ and n fixed while varying b

Figure 5: Illustration of the bankruptcy process

This modeling of bankruptcy has a number of important features. First, the financial gain possible in bankruptcy hinges on the relative bargaining power of sovereigns and creditors as captured by the parameter κ. Second, the use of "lifetime income" in (18) for determining solvency, rather than current income, means that temporarily low income does not make the city insolvent. This forward-looking component of insolvency, which captures provisions of bankruptcy law in the U.S.

(see Section A.5.2), is essential for preventing the model from predicting counterfactually-massive waves of default after large, but short-lived, shocks. Third, solvency hinges not just on the size of the debt stock, but also on its rollover cost. E.g., if real interest rates are zero, i.e., $\bar{q} = 1$, then the municipality is *never* insolvent, and it should not be because the municipality can roll over its debt at zero cost.[18]

Now we can state the government's problem as

$$\begin{aligned} \tilde{S}(b, n, z, \omega) = & \max_{d \in \{0,1\}, g \geq 0, T \leq z, b'} U(g, T, n', z, \omega) + \beta \mathbb{E}_{\phi', z'|z} V(\phi', b', n', z', \omega) \\ \text{s.t. } & g n'^{(1-\eta)} + q(b', n', z, \omega) b' n' = T n' + (1-d) b n + d(-p(b, n, z, \omega) - \iota z n') \\ & -b' \leq \delta g n'^{-\eta} \\ & n' = \dot{n}(b, n, z, \omega) \end{aligned} \tag{21}$$

We use the tilde on S to distinguish it from the household value function in (8), but equilibrium will require that the values coincide.[19]

4.5 Debt pricing

With risk-neutral debt pricing, bond prices must be given by

$$q(b', n', z, \omega) = \bar{q}\, \mathbb{E}_{z'|z}[1 - d(\mathbf{x}') + d(\mathbf{x}') p(\mathbf{x}')/(-b'n')]. \tag{22}$$

This links default rates and spreads very tightly, resulting in spreads being smaller than default rates, whereas in the data the reverse is true. Given this discrepancy, we will focus on matching small default rates rather than large spreads. To some extent, we could match both by using an extremely risk-averse pricing kernel. However, to properly match both would require incorporating aggregate risk and, perhaps especially, disaster risk. So we follow the bulk of the sovereign debt literature and use risk-neutrality.

4.6 Equilibrium

A steady-state recursive competitive equilibrium is value functions $S, V, \tilde{S}$; an expected value of moving J; household policies c, h, m; government policies g, T, b', d; prices and profit q, w, r, π; labor demand L^d; a law of motion for population $\dot{n}$; and a distribution of islands μ, such that (1) household policies c, h and migration decisions are optimal taking V, S, J, prices and government policies as given; (2) government policies g, T, b', d are optimal taking V, J, the population law of motion $\dot{n}(\mathbf{x})$, and prices q as given; (3) firms optimally choose $L^d(\mathbf{x})$ taking $w(\mathbf{x}), r(\mathbf{x})$ as given and optimal per person profits are $\pi(\mathbf{x})$; (4) bond prices are given by equation 22; (5) beliefs are consistent: $S = \tilde{S}$; (6) the distribution of islands μ is invariant; (7) and J and $\dot{n}$ are consistent with μ and household and government policies.

[18] That is, if they can roll over at the risk-free rate. A perhaps more theoretically appealing modification would be to use the discount rates implied by the sovereign's current and expected borrowing. However, such a modification also induces convergences problems as is typically encountered in long-term debt models (Chatterjee & Eyigungor, 2012).

[19] The appropriate continuation value is V because the government has no commitment, and so it takes future actions of itself and households as given.

4.7 Centralization

Proposition 3 shows the government, household, and firm problem may be centralized into a single problem, which we use as the basis for computation:

Proposition 3. *Suppose $\hat{S}$ satisfies*

$$\begin{aligned} \hat{S}(b,n,z,\omega) = \max_{c>0,g\geq 0,d,b'} & u(c,g,\overline{H}/n',\omega) + \beta \mathbb{E}_{\phi',z'|z} \max\{\hat{S}(b',n',z',\omega), J-\phi'\} \\ s.t.\ c + n'^{-\eta}g + q(b',n',z,\omega)b' &= z + (1-d)\frac{bn}{n'} + d(-\frac{p(b,n,z,w)}{n'} - \iota z) \\ n' &= \dot{n}(b,n,z,\omega) \\ -b' &\leq \delta g \dot{n}^{-\eta} \end{aligned} \tag{23}$$

with associated optimal policies $c(\mathbf{x}), g(\mathbf{x}), d(\mathbf{x}), b'(\mathbf{x})$. Then (1) $\hat{S}$ is a solution to the household problem, and c is its optimal consumption policy; (2) $\hat{S}$ is a solution to the government problem, and g, d, b' are its optimal policies; and (3) there exists prices r, w such that labor and housing markets clear and firms optimize.

In what follows, we will use S in place of $\hat{S}$.

We lack a proof of equilibrium uniqueness. However, we investigate uniqueness quantitatively by using 100 randomly drawn initial guesses for the equilibrium objects. Each guess converged to the same equilibrium values, and so at least computationally there is no evidence of indeterminacy at the calibrated values. See Appendix C.5 for more details.

5 Calibration

We now discuss the model's calibration and its fit of targeted and untargeted moments. A model period corresponds to a year in the data.

5.1 Productivity

As productivity (TFP) plays a vital role in the model, it is necessary to have a process that accurately captures location-specific productivity dynamics. For our TFP measure, we use real annual payrolls per employee. Let TFP for a county-year pair be denoted z_{it}. We specify

$$\log z_{it} = \varsigma_i + \varpi_t + \tilde{z}_{it} \tag{24}$$

and obtain the residual $\tilde{z}_{it}$ using a fixed-effects regression. To discretize the fixed effects ς_i, we nonparametrically break the estimates into bins corresponding to 0-10%, 10-50%, 50-90%, 90-99%, and 99-100%. The estimated fixed effects averaged within these bins are -0.34, -0.13, 0.09, 0.37, and 0.65, respectively. We discard the time effects ϖ_t as we will only consider steady states or specific paths for aggregate TFP.

For the residual TFP $\tilde{z}_{it}$, we use an AR(2) specification, which allows more persistent movements in TFP that better capture decade-long persistent movements in productivity such as what occurred in Detroit and Flint (see Figure 3). Restricting the sample to cities of at least one million residents, the estimated first and second AR coefficients are 0.73 (0.02) and 0.23 (0.03), respectively, with an innovation variance 0.001 ($= 2 \times 10^{-5}$).[20] We describe our discretization process in the appendix.

5.2 Preferences and moving costs

We set $\beta = 0.96$ and assume the flow utility exhibits constant relative risk aversion over a Cobb-Douglas aggregate of consumption, government services, and housing plus a taste shifter for weather:

$$u(c, g, h, \omega) = \frac{(c^{1-\zeta_g-\zeta_h} g^{\zeta_g} h^{\zeta_h})^{1-\sigma}}{1-\sigma} + \omega. \tag{25}$$

As ζ_g and ζ_h are relatively small, the constant relative risk aversion over consumption is approximately σ, which we take to be 2. The free parameters ζ_g and ζ_h are estimated jointly, strongly controlling the mean level of government expenditures and housing expenditures, respectively. We take the weather term ω—which is *fixed* over time for any given region but heterogeneous across regions—to be normally distributed with mean zero (a normalization) and standard deviation σ_ω. We discipline σ_ω by matching the standard deviation of log population across cities. We normalize the stock of houses $\overline{H}$ to one.

We assume the moving cost ϕ is distributed as $\underline{\phi}, -\underline{\phi}$, and $Logistic(\mu_\phi, \varsigma_\phi)$ with probability $p_{\underline{\phi}}/2, p_{\underline{\phi}}/2$, and $1 - p_{\underline{\phi}}$, respectively. Having the $\pm\underline{\phi}$ shock means that, for a sufficiently large $\underline{\phi}$, every island's departure rate is in $[p_{\underline{\phi}}/2, 1 - p_{\underline{\phi}}/2]$, which ensures some minimal stability in calibrating the model. We set $p_{\underline{\phi}} = 10^{-4}$ and take $\underline{\phi}$ arbitrarily large giving $\int V(\phi, \mathbf{x}) dF(\phi)$, the expected utility of being in an island with state $\mathbf{x}$, equal to

$$\frac{p_{\underline{\phi}}}{2}(J + S(\mathbf{x})) + (1 - p_{\underline{\phi}})(S(\mathbf{x}) + \varsigma_\phi \log(1 + e^{(S(\mathbf{x})-J-\mu_\phi)/\varsigma_\phi})) \tag{26}$$

plus a constant that we offset via a normalization.[21]

We jointly estimate the parameters controlling moving costs (μ_ϕ, ς_ϕ) and how directed moving is (λ). We identify them using three moments: the mean and standard deviation of out-migration rates, and the productivity-fixed-effect regression coefficient in a regression of log population on productivity residuals $\tilde{z}_{i,t}$, productivity fixed effects ς_i, and a constant.

We discipline how rival public goods are, as governed by η, using the coefficient of a regression of log population on log expenditures (1.118).

5.3 Borrowing and default

We set $\bar{q}$ to give a risk-free interest rate of 4%, the recent average for municipal bonds.[22] We choose the borrowing limit δ to match the data's total debt to GDP ratio of 0.125, equal to an explicit debt to GDP ratio of 0.089 plus unfunded pension debt to GDP of 0.036. The debt measure is gross in that we do not deduct the value of any assets because assets cannot be seized in a Chapter 9 bankruptcy (we discuss this in Section A.5.1 of the appendix). The default cost κ is chosen to match

[20] We use large cities to reduce the role of measurement error.

[21] The constant is $\underline{\phi} p_{\underline{\phi}}/2$. We subtract β times it from flow utility in (25) each period.

[22] In the Census of Governments data, current interest payments as a fraction of current total debt were 4.36% for cities and 3.81% for counties in 2016.

a 0.03% default rate.[23] To calibrate the cost of bankruptcy ι, we use information from Detroit's bankruptcy and arrive at $\iota = 0.125\%$.[24]

5.4 Fit of targeted and untargeted moments

Table 1 reports the targeted and untargeted statistics alongside the jointly calibrated parameter values. The model closely matches all of the targeted statistics. The estimated debt limit, δ, allows cities to borrow up to 153% of their expenditures, which is not very far from CA's statutory limit of 100% (plus exceptions).

Targeted Statistics	Data	Model	Parameter	Value
Default rate ($\times 100$) $\int d d\mu$	0.030	0.028	κ	0.006
Debt / GDP $\int -bn d\mu / \int z\dot{n} d\mu$	0.125	0.125	δ	1.528
Gov. expenditures / GDP $\int g\dot{n}^{1-\eta} d\mu / \int z\dot{n} d\mu$	0.084	0.082	ζ_g	0.069
Housing expenditures / GDP $\int r\bar{H} d\mu / \int z\dot{n} d\mu$	0.125	0.125	ζ_h	0.112
Std. deviation of $\log n$	1.843	1.848	σ_ω	0.331
*Out rate mean $\int F(R)/n d\mu$	0.065	0.064	μ_ϕ	22.445
*Out rate st. dev.	0.023	0.022	ς_ϕ	6.452
*Population reg. coef., $\log z$ FE	4.224	4.222	λ	0.603
Regression coef., log expenditures on $\log n$	1.118	1.034	η	0.316
Untargeted Statistics	Data	Model		
Autocorrelation of $\log n$	0.999	1.000		
Std. deviation of net migration rates	0.021	0.023		
Correlation of log expenditures and $\log n$	0.858	0.995		
Std. deviation of log expenditures	2.388	1.921		
*In rate mean	0.065	0.066		
*In rate st. dev.	0.022	0.030		
*In rate reg. coef., $\log z$	0.000	0.023		
*Out rate reg. coef., $\log z$	-0.006	-0.005		

Note: * means the data is county-level; the regressions are specified in Section C.6.

Table 1: Calibration targets and parameter values

Utility from weather plays a large role in location decisions, with a rough calculation giving the lifetime consumption equivalent variation of permanently moving from the median weather $\omega = 0$ to $2\sigma_\omega$ at 66%. The importance of weather for utility helps the model match the very low (in fact, negative) correlation between in-migration and productivity and, simultaneously, the large dispersion in population. The large value for μ_ϕ with a correspondingly large ς_ϕ makes out-migration largely dependent on moving cost shocks rather than fundamentals like productivity, letting the model match the weak relationships between productivity and migration rates.

The model gets most of the untargeted predictions qualitatively correct while missing on a few statistics. The model recreates the very slow population adjustments seen in the data with log population autocorrelation exceeding the data's 0.999. It also matches the small correlations between migration rates and productivity and migration rates and population. Last, it reproduces

[23] While high for Moody's rated bonds, it is not high for the Mergent dataset (see Figure 11 in the appendix).

[24] Detroit's cost the city \$178M on its \$18.5B bankruptcy (1% per unit of debt) (Reuters, 2014). Using Detroit's 1% legal cost per unit of debt and a 12.5% debt-gdp ratio, we set $\iota = 0.125\% = 1\% * 12.5\%$.

the dispersion in net migration rates and the large dispersion in government expenditures. Table 4 in the appendix provides additional untargeted moments.

6 Quantitative results

With the calibrated model in hand, we first show the model can replicate all the stylized facts we established in Section 3. Having established the empirical stylized facts hold in the model, we then turn to the model's predictions to determine why there were so few defaults in the Great Recession.

6.1 Stylized facts in the model

In establishing the stylized facts hold in the model, we begin with the key model mechanism, showing that in-migration induces more expenditures, revenue, and debt per person (stylized fact #3). We then show cities of all types are heavily indebted (fact #1); that they are close to their borrowing limits (fact #2); and that the model generates default after booms and busts (fact #4). Finally, we establish that default risk is sensitive to interest rate changes (fact #5).

6.1.1 The model mechanism in action

A direct way to inspect the model's overborrowing mechanism is examining how $b'(b, z, n, \omega)$ varies in (b, ω) for fixed (n, z): Weather is separable, so higher ω only affects borrowing decisions through its effect on migration. Figure 6 plots this policy.[25] Moving from $\omega = 0$ to $\omega = .66$ causes a dramatic increase in debt issuance. The top right panel shows this is driven by a drastic decrease in the overborrowing term, acting like a debt subsidy of 25 cents on the dollar. This change is attributable to in-migration rates moving from close to 0% to more than 30%. Moving from $\omega = -.66$ to $\omega = 0$ has some positive impact on borrowing as well, but to a much smaller extent, because the overborrowing term only declines marginally. The overborrowing term declines marginally despite a substantial change in out-migration rates because i has a first-order effect on $\frac{1-o}{1-o+i}$ while o only has a second-order effect. As discussed in Section 2, the overborrowing externality is potentially offset by an incentive to boost in-migration—thereby reducing debt per person—through improved fiscal discipline. But the figure shows the elasticity of in- and out-migration rates to debt, while qualitatively as expected, are very small. Migration rates respond much more to fundamentals like ω, z, and n, than debt b, resulting in an approximate dichotomy: In-migration strongly affects borrowing, but borrowing only weakly affects migration.

Another way to see the mechanism is by running an experiment that effectively reproduces the IV identification scheme of stylized fact #3. To this end, Figure 7 plots the response of a few key variables to two types of shocks: a shock to in-migration rates and a shock to interest rates (the latter will be considered later). To model an exogenous in-migration rate shock, we proportionately scale up or down the inflows $i(\mathbf{x})$ in (10) by $1+\bar{i}$, and we assume that $\bar{i}$ decays annually at rate 0.956, based on a five-year autocorrelation of in-migration rates equal to 0.8. The results for a positive (negative) shock are displayed in the blue solid (dashed) lines.

[25] We use the closest value to one on the n grid, the middle grid point for the TFP fixed effect, and a middling TFP shock.

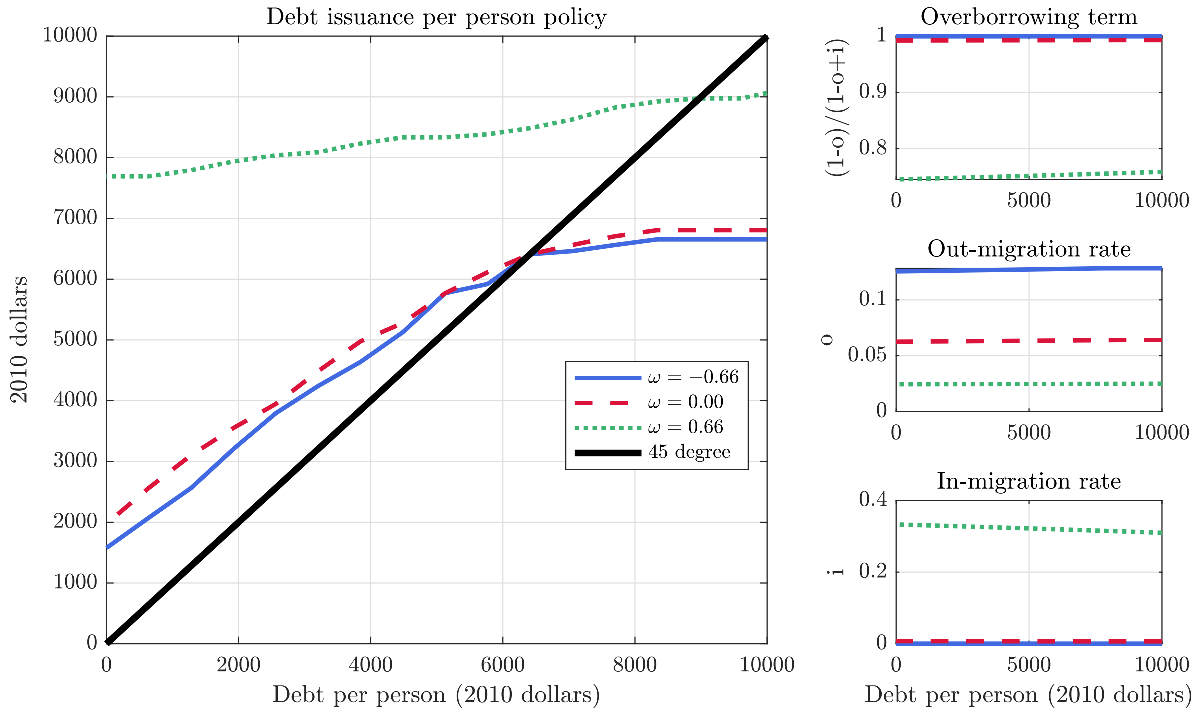


Figure 6: Debt policy function illustration

Consistent with the stylized fact #3, more in-migration leads to increases in debt, expenditures, and revenue per person. Additionally, the model predicts that higher in-migration rates reduce default rates on impact—reflecting the direct effect of less debt per person—but increase default rates and spreads in the future as the government's debt grows. This suggests the model will reproduce boom-defaults (stylized fact #4), which we will verify directly in Section 6.1.3.

6.1.2 All city types are heavily indebted and close to borrowing limits

The model reproduces the stark relationships among debt, expenditures, and borrowing constraints in stylized facts #1 and #2. This can be seen most clearly in Figure 8, which provides a scatter plot of cities debt and expenditures along with the borrowing constraint. It shows cities of all types—large and small, productive and unproductive, with great and poor weather—are indebted and close to their borrowing limits.

Why are all cities indebted and close to their borrowing limits? The answer lies in the high *level* of in-migration rather than in its dynamics. In the data and model, in-migration rates at a local level are around 6.5%. Loosely speaking, this means for every dollar of debt issued per person, 6.5 cents are paid for by new entrants. This is such a large "discount" on debt issuance that it dominates any consumption-smoothing motives, including a desire to save for a rainy day.

The model does have a restraining force in it, which is that debt accumulation deters entrants to the city (the term $\partial n_2/\partial b_2$ in equation 2). However, to match the data's large out-migration rates, most of the migration decision is attributed to idiosyncratic, person-specific factors—as seen in large μ_ϕ and ς_ϕ—rather than local fundamentals. Consequently, the elasticity of population to

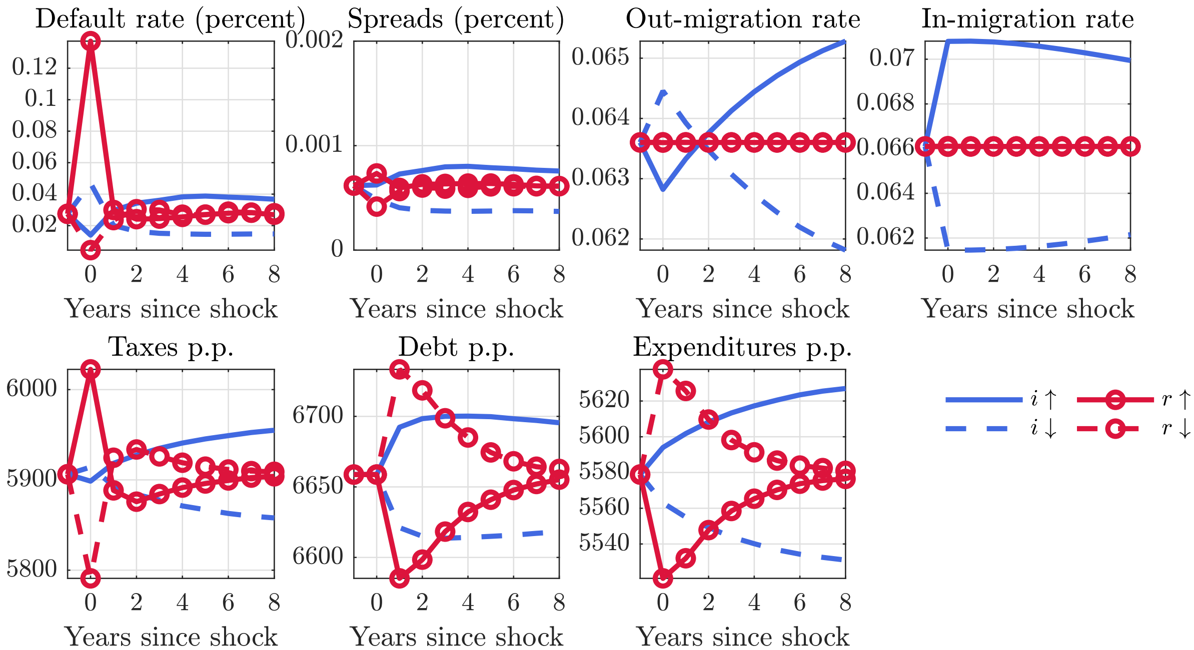
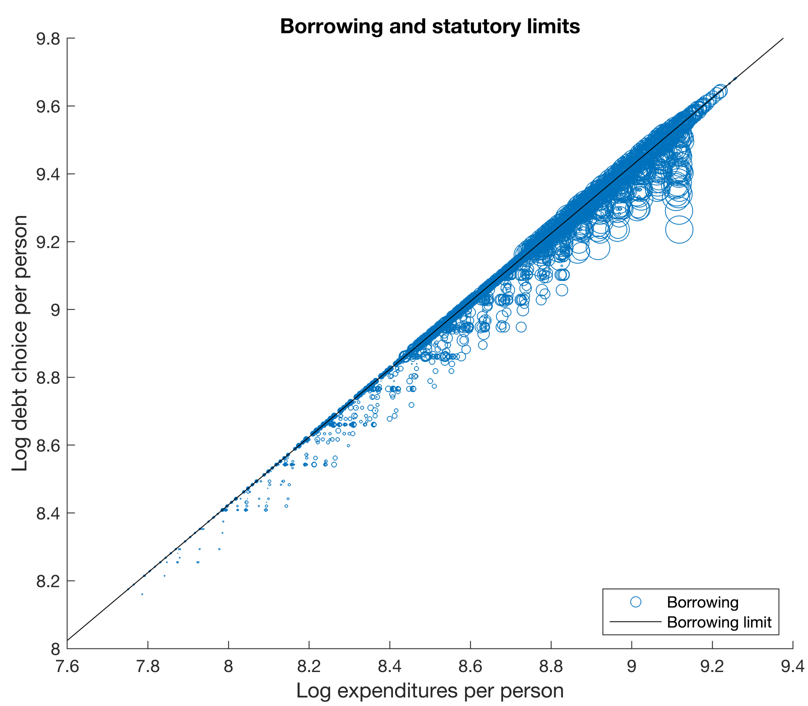


Figure 7: Model mechanisms



Note: circle areas are proportional to population.

Figure 8: Model distribution of cities relative to their borrowing limit

debt is quite small, and the model is almost bifurcated: migration strongly affects local government decisions, but local government decisions weakly affect migration. Moreover, this is reinforced in equilibrium because all governments over-accumulate debt, which depresses any incentive to move to a more fiscally responsible city—such cities simply do not exist in the model. Hence, the main restraining force in the model is essentially inoperable, resulting in cities of all types being nearly indebted as they can possibly be.

6.1.3 Cities default after busts *and* booms

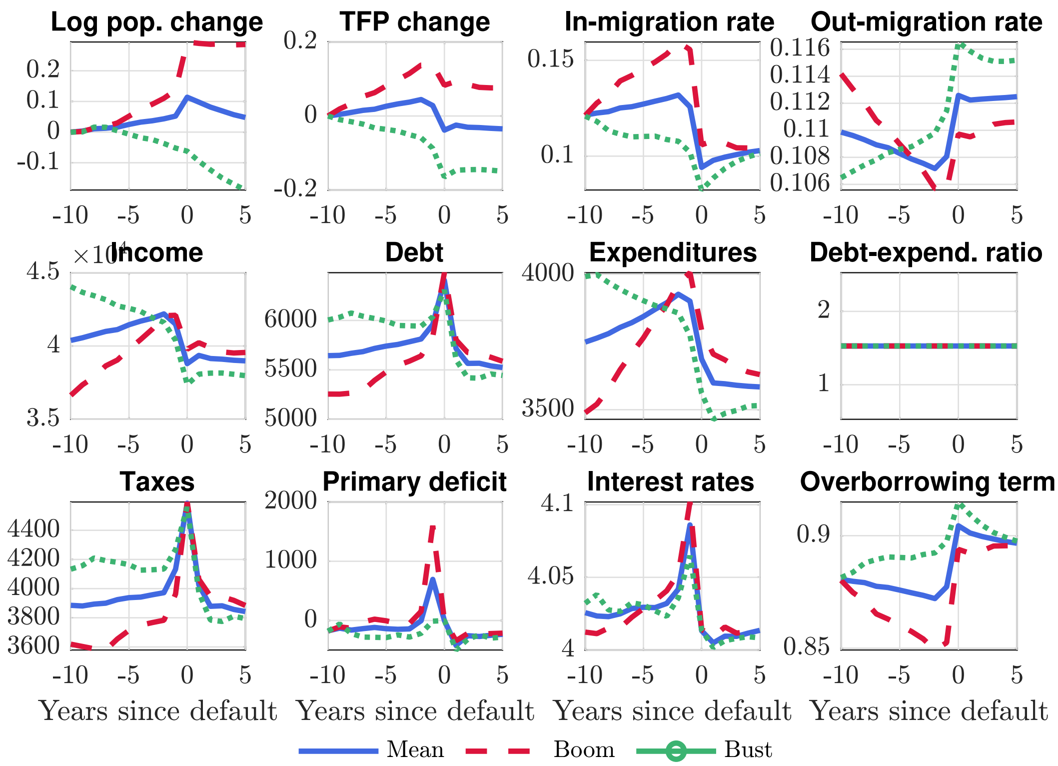
We now show the model generates boom and bust defaults, which was stylized fact #4. To do this, we use default episodes, looking at windows around the time of default. These are displayed in Figure 9, where we have broken default episodes into three cases: an average default event (blue line), a default during a technology boom (red dashed line), and a default during a technology bust (green dotted line). Formally, a boom (bust) is defined as having log productivity growth in the ten years preceding default above the 75th (below the 25th) percentile, and we compute the respective averages in the top and bottom quartile to construct the time series. Unsurprisingly, the model does generate default after long periods of productivity decline. But importantly, the "boom" defaults do in fact follow periods of substantial productivity growth—a feature that is *not* necessitated by our definition of booms.

Considering first average default episodes, one sees they coincide with slightly growing productivity followed by a sharp decline in productivity (a drop close to 10%) at the time of default. Additionally, the shock is such that the drop in TFP is expected to last a long time. Although on average population increases slightly predefault, cities see their population decline postdefault, losing about 5% of their inhabitants within five years.

Because the mean default episodes average over boom and bust defaults, they hide a large amount of heterogeneity. Looking at bust defaults, one finds a prolonged decline in population leading up to default, qualitatively similar to the experience of Flint and Detroit. Facing this shrinking population and persistently adverse productivity, the sovereign initially holds taxes and debt per person stable and cuts expenditures, resulting in a modest primary *surplus*. In the few years before default, interest rates increase, reflecting the increased default risk. Following default, the municipality deleverages by sharply reducing expenditures.

Looking at boom defaults, the population and productivity growth is strong until just a year before default, like in Vallejo. With the boom, the cities do not pay down debt, and even run a substantial deficit shortly before default. Consequently, debt per person grows as interest payments pile up, and this is despite substantial population growth that (else equal) reduces debt per person. The debt growth is paired with a noticeable increase in expenditures and taxes. Interest rates trend upward, showing that the city is taking on increasing (albeit small) amounts of risk. When a substantial negative productivity shock hits, in-migration plummets and debt per person increases, triggering default.

While boom defaults are triggered by a decline in productivity, a *necessary* ingredient is that the city must be leveraged enough to make default worthwhile, which is where overborrowing



Note: all financial variables are 2010 dollars per person.

Figure 9: Default episodes

plays a crucial role in generating boom defaults. To see this, consider the "overborrowing term" $(1-o)/(1-o+i)$ in the bottom right panel. For boom defaults, this falls to as low as 0.85, implying a massive 15% discount on debt issuance. This overborrowing incentive dwarfs the usual consumption smoothing motive, keeping cities in debt even after long periods of growing productivity and leaving them vulnerable to adverse shocks.

6.1.4 Default risk is sensitive to interest rate movements

We close this section by establishing the last stylized fact, which is that default risk is sensitive to interest rate. We turn again to Figure 7, which displays the response of a 1 percentage point risk-free rate increase (decrease) in the red circled (dashed) lines. The effect decays at rate 0.63 (which comes from a regression of detrended AAA yield to maturity on its twelve-month lag).

On impact, the increase in rates increases the default rate of bonds appreciably, as well as spreads. Default rates increase because larger interest rates increase debt service costs, which makes more cities insolvent and able to benefit from bankruptcy. Formally, a lower $\bar{q}$ lowers $\bar{z}(z)$, reducing—for insolvent or nearly insolvent cities—required debt payments and inducing bankruptcies. Spreads increase because of higher future default risk that sovereigns only partially undo through deleveraging. Of note, the plotted spreads series is ex-ante in that it is a function of $q(b', \mathbf{x})$. Consequently, the surprise losses incurred from the unanticipated shock are not displayed.

Cities rely on a combination of expenditure cuts and tax increases to deleverage. Using this combination is optimal from a utility perspective because c and g are complements, so reducing both c and g modestly is superior to letting one or the other fully absorb the impact. The impact on migration is not noticeable because (1) the interest rate only change taxes by a small magnitude, and (2) the interest rate change has no substantial redistributive effect since all cities are indebted.

6.2 The Great Recession

Having established the empirical stylized facts hold in the model, we now try to answer why there were so few municipal defaults in the Great Recession. We will model the Great Recession as a perfect foresight transition following a one-time, unanticipated shock.

The Great Recession resulted in a large and essentially permanent drop in real GDP per capita. Relative to the pre-2009 linear trend, the drop was 12% on impact and grew to almost 20% by 2020Q1, as seen in Figure 18 in Appendix D. The full impact of this decline, however, was not felt by local governments because of the American Reinvestment and Recovery Act of 2009 (ARRA). The ARRA provided large federal transfers that bolstered state and local government (SLG) revenue. In fact, as we document in Appendix D, transfers caused SLG total revenue to GDP to rise until 2011, despite large declines in tax revenue.

In light of this, we assume a pass-through from GDP declines to model TFP declines that is less than 100%. Specifically, since local government budgets are 8.2% of GDP and state to local transfers were 3.2% from 2006 to 2009, we assume a pass-through of $61\% = 1 - 3.2\%/8.2\%$. Combining the declines in real GDP deviations from trend with a 61% pass-through results in the aggregate TFP ϖ_t declines reported in Table 8 in the appendix.

Of course, the Great Recession also exhibited steep declines in risk-free real interest rates. These real rates, measured using five-year TIPS yields, fell by more than 3pp from 2006 to 2012 as can be seen in Appendix D. Given the empirical sensitivity of default to interest rates that we already established, we incorporate these declines as well. In particular, we compute two transitions, one incorporating these declines as reported in Table 8, and one holding the real rate constant.

Figure 10 plots the response of key variables over the transition path. The most glaring observation is that the default rate initially rises from a few basis points to 2 percentage points in the absence of real rate declines. Is this hypothetical prediction reasonable? As seen in Figure 11 in the appendix, in the early days of the 2007-2008 financial crisis, spreads of higher-yield municipal bonds relative to lower-yield were as little as 1%. As the spillover into unemployment and a housing crisis became clear, the spreads rose to 5% in anticipation of a wave of default. While default rates rose—from a low of 0.01% to as much as 0.16%—they never reached anything close to 2%. Viewed through the model lens, this is because the massive drop in real rates reduced default rates by a huge amount.

In contrast to default rates, spreads remain low and stable for many years. Part of this is a time aggregation issue: Spreads are purely forward-looking, so the high default rates on impact are decoupled from spreads on impact. More substantively, municipalities deleverage substantially, reducing debt by about 10% absent any interest rate decline. They do this by increasing taxes noticeably *when the shock hits*. This painful deleveraging runs counter to common policy advice that says governments should borrow more in downturns. And, indeed, governments have incentive to consumption smooth by borrowing more. Nevertheless, this effect is dominated by (1) spreads moving against them if they do not deleverage; (2) the exogenous borrowing limit, which effectively tightens in response to a decrease in optimal expenditures (see equation 15); and (3) effective impatience due to the overborrowing externality.

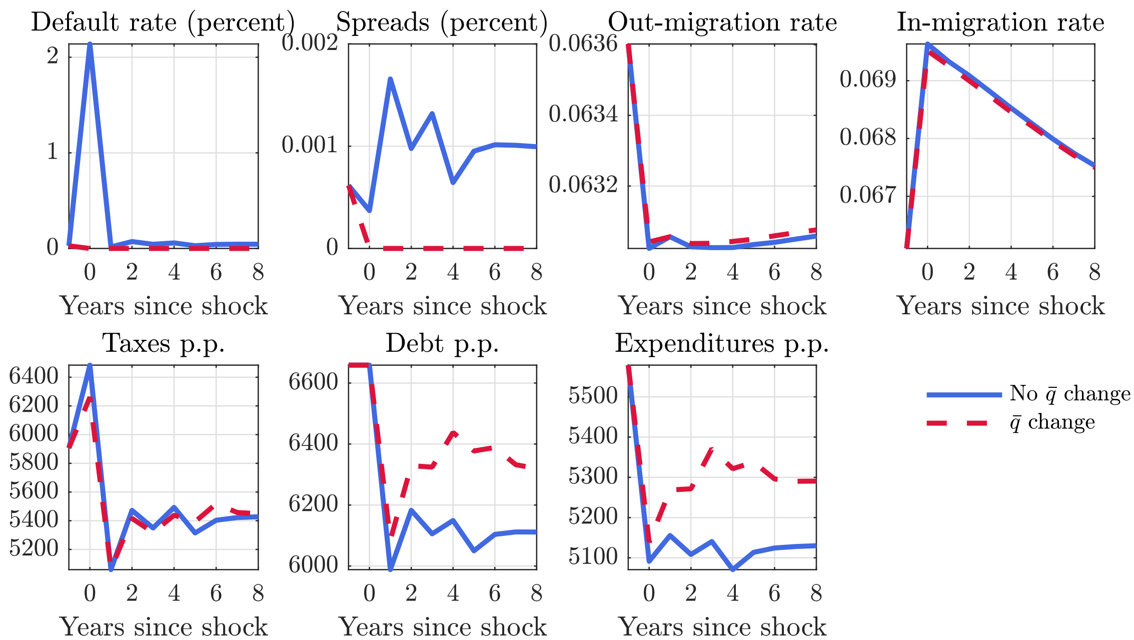


Figure 10: Great Recession transitions with and without interest rate reductions

A final aspect from the transition is that arrival rates (and departure rates, not pictured) are little changed from the shock. This is due to the nature of the aggregate shock, which effects all cities proportionally.

In Section C.8 of the appendix, we conduct a similar exercise of feeding in shocks to try to replicate COVID-19's effect on the economy's path from 2020 on. This entails one major modification relative to the Great Recession case: We modify the weather distribution to make the largest population centers (who have large ω values) less attractive, inducing outflows from them to smaller cities. We find this redistribute effect reduced overall default incidence while simultaneously shifting the population of defaulters towards larger cities. However, like it did in the Great Recession, the decline in real interest rates prevented any appreciable spike in default rates.

7 Conclusion

Borrowing, migration, and default are intimately connected. Theoretically, we demonstrated that migration tends to result in overborrowing. Empirically, we documented that defaults can occur after booms or busts in labor productivity and population, in-migration leads to indebtedness, that many cities appear to be at or near state-imposed borrowing limits, and that default risk is highly sensitive to interest rate movements. Our quantitative model was able to capture these stylized facts, in large part due to the overborrowing externality. Given the increase in debt and default over time, and the fiscal stress created by the Great Recession and continuing with COVID-19, understanding regional borrowing, default, and migration will remain a high priority.

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A Additional data details [Not for Publication]

This appendix describes our data sources, definitions of key variables, and cleaning procedures in Sections A.1, A.2, and A.3. Section A.4 gives a collection of statutory borrowing limits, and Section A.6 records newspaper headlines on local government finances.

A.1 Census County Business Patterns data

To construct TFP measures, we use data from the Census' County Business Patterns (CBP) database from 1986 to 2014. The main measures we use are the payroll variable `ap` (converted to constant dollars using the standard CPI series obtained from FRED) and the mid-March employment variable `emp`, along with the FIPS codes. In the CBP database, missing or bad values are assigned a value of zero, so we treat `ap` and `emp` as missing whenever they are 0. Our overall productivity measure z_{it} is `ap/emp`. The data includes disaggregated employment levels by sectors (NAICS and SIC), so we keep only the observations corresponding to aggregates. The panel includes 91,800 year-county nonmissing observations for z_{it}.

A.2 Annual Survey of State & Local Government Finances data

For our data on government finances and population, we use the Annual Survey of State & Local Government Finances (IndFin) compiled by the Census Bureau. Every five years (in years ending in two or seven), the aim is to construct a comprehensive record of state and local finances. (In practice, surveys are sent out for most cities and not all are returned, but the coverage is good enough to cover 64-74% of the U.S. population depending on the year.) In intervening years, a nonrepresentative sample is selected from the population. Some of the larger cities are "jacket units," and instead of surveys, the Census sends its own workers to record the data. The data are aggregated at different levels, with "cities"—i.e., municipalities and townships—counties, and states. We consider two samples: one corresponding to cities (`typecode` equal to 1 or 2) and the other to data aggregated at a county level (the aggregation of `typecode` values 1 through 5). Some of the data go back to 1967. However, the first population records begin in 1986 (survey year 1987), so we restrict ourselves to the 1987-2012 survey years.

The `population` is not recorded in each year (the data for it does not necessarily correspond to the survey year but are given by `yearpop`), and so we construct estimates. We restrict the sample so that each city/county has at least two population measures. We fill in missing observations using linear interpolation of the log population. We also allow for some extrapolation, but do not allow extrapolation beyond five years.

The raw sample consists of 390,557 year-county or year-city observations. We then use the sample restrictions as described in Table 2. We compute implied interest rates via the interest paid during the year over the total debt, short and long term: 100* `totalinterestondebt` / (`stdebtendofyear` + `totallongtermdebtout`). All financial variables are converted to real 2012 dollars using the CPI.

Sample selection condition	Obs.
Raw	390557
Require nonmissing `name`, require no id changes (`idchanged`=0), drop if "no data" (`zerodata>0`)	387963
Require `yearpop` not missing, drop if `datayearcode`="N", require at least two population observations	387219
Drop observations with nonmissing investment annual returns on trust funds exceeding 30%	386723
Dropping missing population estimates	386515
Dropping observations where population growth rates could not be estimated	386511
Dropping Louisville, KY observations before 2003	386494
Require annual population growth rates of less than 25%	386494
Require revenue per person of less than $25,000	386090
Require debt per person of less than $30,000	376778
Require accounting identity for the evolution of long-term debt to nearly hold	363365
Require estimated interest rates be less than 40% annually	362122

Table 2: Sample selection in IndFin

A.3 Migration data

Our migration data comes from the IRS. Up to 2010, we use the county-to-county migration flows as harmonized by Hauer and Byars (2019); from 2011 on, we construct our own.

A.4 State-imposed borrowing limits

Table 3 reports state-imposed borrowing limits for a collection of states.

A.5 More facts on municipalities

In this section we provide overview of municipal debt and default (Section A.5.1), highlight the forward-looking nature of the bankruptcy process (Section A.5.2), and establish debt and default have been increasing over time (Section A.5.3). We then provide additional evidence that for our stylized facts in Sections A.5.4, A.5.5, and A.5.6.

A.5.1 An overview of municipal debt and default

Two broad types of municipal debt exist: *general obligation* debt (GO) and *non-general obligation* debt (Non-GO). Non-GO debt is typically attached to a specific revenue stream, e.g., bonds for construction of a toll road that will be paid for using revenue from the toll road. In contrast, GO debt is fully backed by taxes. Additionally, Novy-Marx and Rauh (2011), Novy-Marx and Rauh (2009), and Rauh (2017) have shown state and local governments have a third large type of "debt," which is unfunded pension obligations.

How much debt local governments can accumulate is typically constrained by state-imposed borrowing limits that vary in type and degree. For example, California (CA) limits are tied to spending or revenue that year. In contrast, most of the states restrict debt based on a percentage of property valuations, but the percentages can differ substantially from as little as 0.5% (IL) to

State	Limit	Known exceptions
CA	Indebtedness less than revenue that year	Authorization by referendum, special projects, and public school spending.
IL	Limits range from 0.5%-3% of assessed value (1/3 of market value)	Schools have debt limits of 13.8% value of taxable property.
IN	2% of assessed value	Some revenue bonds (see note).
MA	5% of property valuation last year	
MI	10% (5% for townships and school districts) of assessed value	Approval of voters for more school district debt; most revenue bonds.
MN	"Net debt" less than 3% of market value of taxable property	Charter can increase to 3.67%, "first class" cities have a 2% limit.
NY	Roughly 10% of the property valuation over the previous five years	Debt related to water supplies and sewers.
OH	Net indebtedness less than 5.5% (or 10.5% with vote) of tax valuation	Self-supporting projects for water facilities, airports, etc.
WI	5% of taxable property value	Schools have a 10% debt limit, may issue $1 million without approval.

Sources are as follows: CA's is Harris (2002); MA's is MCTA (2009); MN's is Bubul (2017); IL, IN, MI, and WI's is Faulk and Killian (2017); NY's is ONYSC (2018); OH's is OMAC, 2013, p. 50. Revenue bonds are municipal bonds that are paid by revenue from a particular project (they are non-GO bonds).

Table 3: Sample of statutory borrowing limits by state

10% (NY). Table 3 in the appendix reports some of these, and reveals almost all the states have known exceptions, which usually include debt related to education, water supplies, and referendum-approved debt.

When GO and non-GO debt becomes excessive, local governments do have access, potentially, to Chapter 9 bankruptcy. This substantially differs from consumer and corporate bankruptcy, with one key distinction being that the municipality must be *insolvent*, either "unable to pay its debts" or "generally not paying its debts."[26] The latter statement, which in isolation would appear to be nonrestrictive, is strengthened by the additional requirement that filers must negotiate in good faith with creditors. A second key distinction is that local governments are allowed to keep essentially all their assets in default to prevent creditors from infringing on the local government's sovereignty.[27]

While bankruptcy allows discharge of GO and non-GO debt, pension obligations are commonly viewed as *nondefaultable*, either explicitly protected by state constitutions or otherwise protected by contract law (Brown & Wilcox, 2009). In practice, CA cities that went through bankruptcy did not have their pension obligations reduced (Myers, 2019). In Detroit, a worst-case example, there were in fact some pension cuts: 4.5% directly with cessation of Cost of Living Adjustments (COLAs) (Stempel, 2016). However, viewing pension obligations as nondefaultable seems to be a reasonable approximation.

[26] 11 U.S.C. §101(32)(C).

[27] Chapter 9 "is significantly different in that there is no provision in the law for liquidation of the assets of the municipality and distribution of the proceeds to creditors. Such a liquidation or dissolution would undoubtedly violate the Tenth Amendment to the Constitution and the reservation to the states of sovereignty over their internal affairs" (United States Courts, 2018).

A.5.2 Forward looking nature of insolvency

In the baseline model, we assume a city is insolvent if debt services per person is above $\kappa\bar{z}(z)$, where $\bar{z}(z) = z + \bar{q}\mathbb{E}_{z'|z}\bar{z}(z')$. This rule has a clear forward looking component. To justify it, note that according to Chapter 9, insolvency is defined as the "financial condition such that the municipality is i) generally not paying its debt as they become due unless such debts are the subject of a bona fide dispute; or ii) unable to pay its debts as they become due." We take this definition as a support of our modeling choice because it implies that future cash flows are assessed when determining whether a municipality is insolvent or not.

In fact, the following excerpt from Judge Christopher M. Klein's opinion on the Stockton's bankruptcy case (Case City of Stockton, 493 B.R. 772 (2013)) makes clear that future cash flows are part of the definition of insolvency:

> The language "unable to pay as they become due" in the municipal insolvency definition implicates the notions of time and projections about the future.

Additionally, two quotes from Gilson et al. (2022) highlight the forward looking nature of bankruptcy and how future cash flows enter the legal decision to approve a bankruptcy plan. These were made in the context of Detroit's bankruptcy.

The first quote (pages 14 & 15) establishes a concrete planning horizon of ten years. It emphasizes that the weak tax base resulted in small payments for unsecured creditors, while also showing some funds are reserved for local government spending:

> As part of the Plan of Reorganization, the city put forth a 10-year financial forecast .... Given Detroit's weak tax base and needed investments in public services, the plan showed minimal funds available from the General Fund to support unsecured creditors and retirees.

The second quote (page 15) shows the plan, in order to be approved, must be feasible and in creditors' best interest. Since creditors' best interest is getting paid back, this highlights the plan should be as much as the city can reasonably afford:

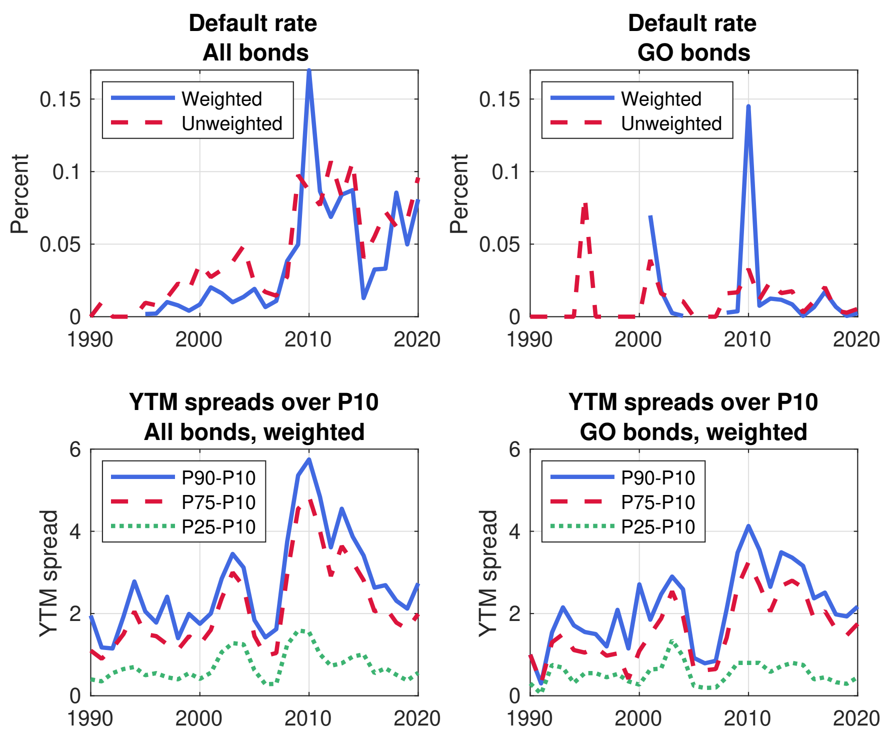
> One outstanding question was whether the latest plan proposed by the city was even feasible under the rules of federal bankruptcy law. For Judge Rhodes to confirm the plan, he needed to determine that the plan was in the "best interests of creditors" and that it was "feasible," or financially sound.

A.5.3 Debt and default are increasing over time

Historically, municipal bond default has been rare. For bonds rated by Moody's, Moody's (2013) report there have been 73 municipal bond defaults between 1970 and 2011, a default rate of 0.012%.

However, these low default rates belie the severity of the situation for several reasons. First, one reason is that default rates have been trending upward over time. Using a large municipal-bond dataset from Mergent, Figure 11 reports the time-series path of default rates, both as a frequency of outstanding bond counts (unweighted) and as a percent of outstanding debt (weighted). The top left panel uses all bonds, while the top right panel uses only GO bonds. Until the late 2000s, default rates for all bonds were generally below 0.05%. However, in the Great Recession, these doubled or even tripled and have remained elevated. For GO bonds, the upward trend is less obvious, but the period of relative stability from 2011 to 2019 saw default rates of around 0.01%, whereas during mid 2000s that rate was zero. It's also worth noting both series show substantial sensitivity of default rates to aggregate risk.

This increase in default risk is perhaps more clearly seen by looking at interest rate spreads over time. For instance, when we compare the weighted P90 yield to maturity (YTM) over a "riskfree" P10 YTM, the spread has risen from 2% during the Great Moderation to close to 4% in the 2010s.[28] A similar story appears looking at GO bonds, or the P75-P10 difference, or even to a lesser extent the P25-P10. Whether we look at default rates or interest rate spreads, the story is the same: Default risk is increasing over time.



Note: Default series excludes Puerto Rico.

Figure 11: YTM spreads and default rates over time

There are also other factors that have masked the amount of fiscal stress. As in Flint, cities can avoid default *but* come under state management and thereby lose fiscal autonomy. In fact, Kleine and Schulz (2017) report that Michigan (MI) had 11 cities (4%), one township, and one county

[28]We use the P10 YTM of municipal bonds as our measure of the riskfree rate because municipal bonds have a number of key differences from Treasuries that make them not very comparable. Chief among these differences are that municipal bonds tend to be callable (i.e., they can be refinanced at lower rates), and they tend to be tax-exempt. Of note, the increase in default risk post-2008 is also very evident for states (Arellano et al., 2015).

under state oversight due to a financial emergency (p. 9). Additionally, real interest rates—which we will show strongly effect municipal bond risk premia—have fallen secularly over time.

Lastly, debt has grown tremendously over the past few decades. For instance, the average debt per person across counties was \$3085 in 2016 but \$2107 in 1996 in constant 2012 dollars—nearly a 50% increase. Similarly, debt for cities grew by 51% from \$496 to \$750 per person. With the increased debt burden comes increased risk that local governments will not be able to repay.

A.5.4 Stylized fact #1 revisited: Cities of all types are indebted

Figure 12 presents the empirical relationship of log debt per person (p.p.) and log expenditures (p.p.) One can see cities of all sizes in the U.S. *have* debt: there are not cities clustering at low debt levels. Additionally, the linear relationship is stark, with more expenditures strongly positively correlated with more debt.

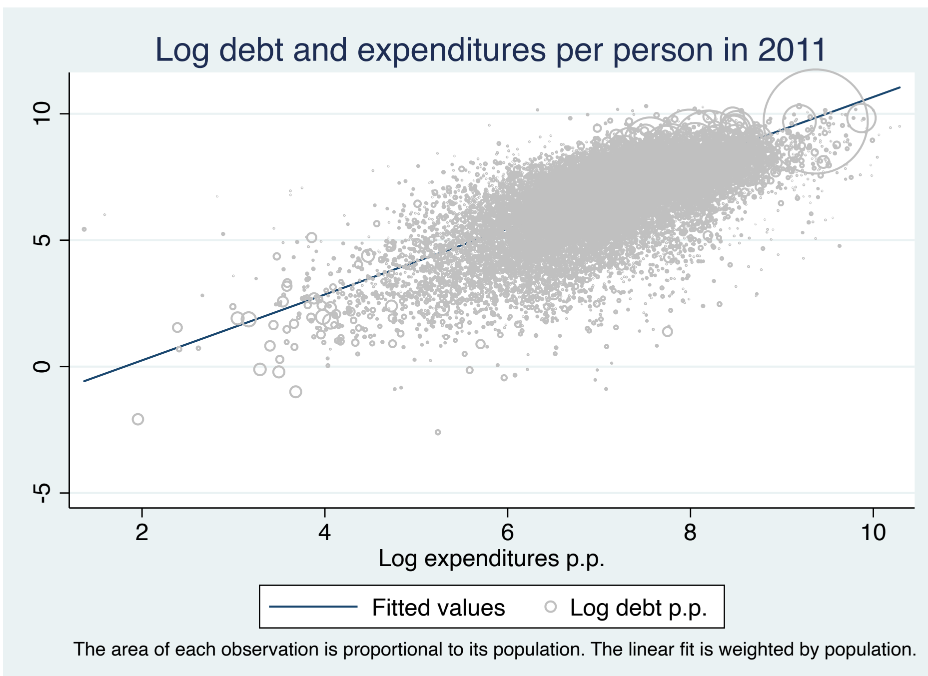


Figure 12: Debt and expenditures per person

A.5.5 Borrowing limits in Michigan

In the main text, we learned that most cities in California are close or at their borrowing limits. The other state we could check was MI using taxable-valuation data from Kleine and Schulz (2017), and Figure 13 displays how close MI cities are to their limits. Again, many cities are at, near, or above the limit including Detroit and Flint. Even the wealthiest cities (as measured by property values) are borrowing.

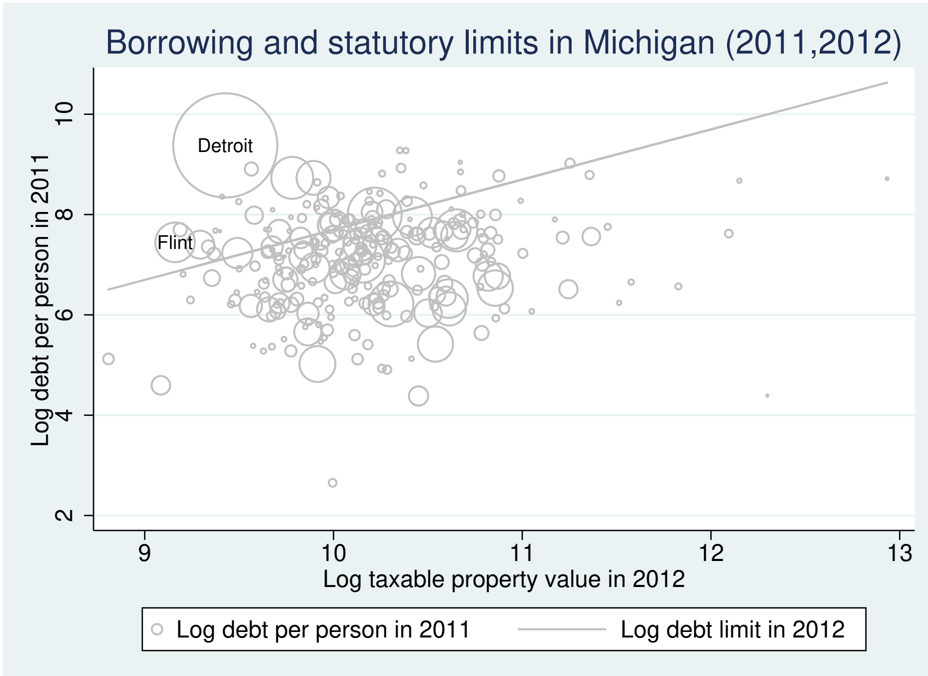


Figure 13: Statutory borrowing limits and closeness to the limits for Michigan

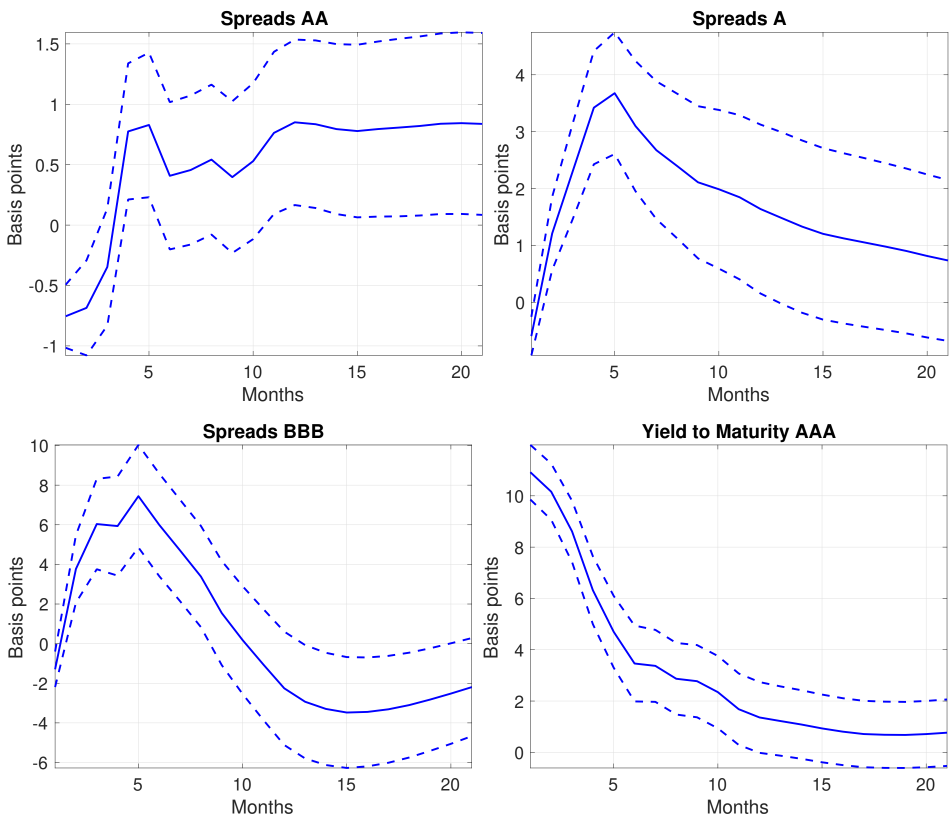
A.5.6 SVAR evidence on default risk and interest rate changes

While the anecdotal evidence about default risk and interest rates in the main text may be unsatisfactory, it is also born out in impulse responses from a structural vector autoregression (SVAR). To this end, we estimate a VAR using monthly data on industrial production, the yield-to-maturity for AAA (ytmAAA), the consumer price index, and the spread between the return on municipal bond with category A, AA, or BBB and the ytmAAA. We use the ytmAAA as our interest rate measure rather than a Treasury because municipal bonds are typically callable (i.e., they can be prematurely repayed) and are tax-exempt. The sample runs from January 2005 and October 2020, and the VAR uses 6 lags. Figure 14 shows the response to an orthogonalized shock to the ytmAAA.[29] It is not difficult to see that the exogenous increases in the AAA yield lead to a persistent response of the spreads. Consistent with interest rate increases inducing an increase in tail risk, BBB spreads are the most sensitive to the rise in interest rates, reaching a peak of 8 basis points around five months after the exogenous innovation. That is, the BBB rate almost doubles the initial jump in the risk-free rate (10 basis points). In contrast, the less risky spread AA barely increases, and its response is not statistically significant.

A.6 Cities making headlines

Here, we document some cities/municipalities experiencing financial difficulties as reported by different media outlets. In quotations, we include excerpts of these news. To retrieve the source, the

[29]More concretely, we use a Cholesky decomposition of the forecast errors from the VAR with the variables ordered as in the main text. Then we plot the IRFs to a shock to the second variable, i.e., the yield to maturity AAA.



Note: Dashed lines corresponds to bootstrapped 68% confidence intervals.

Figure 14: Impulse response functions to a risk-free rate (AAA) shock

interested reader should click on the city's name.

U.S. Virgin Islands: *"With just over 100,000 inhabitants, the protectorate now owes north of \$2 billion to bondholders and creditors. That is the biggest per capita debt load of any U.S. territory or state - more than \$19,000 for every man, woman and child scattered across the island chain of St. Croix, St. Thomas and St. John. The territory is on the hook for billions more in unfunded pension and healthcare obligations."*

Chicago: *"Chicago's finances are already sagging under an unfunded pension liability Moody's has pegged at \$32 billion and that is equal to eight times the city's operating revenue. The city has a \$300 million structural deficit in its \$3.53 billion operating budget and is required by an Illinois law to boost the 2016 contribution to its police and fire pension funds by \$550 million.*
Cost-saving reforms for the city's other two pension funds, which face insolvency in a matter of years, are being challenged in court by labor unions and retirees.
State funding due Chicago would drop by \$210 million between July 1 and the end of 2016 under a plan proposed by Illinois Governor Bruce Rauner."

Detroit: *"'It is indeed a momentous day,' U.S. Bankruptcy Judge Steven Rhodes said at the end of a 90-minute summary of his ruling. 'We have here a judicial finding that this once-proud city cannot pay its debts. At the same time, it has an opportunity for a fresh start. I hope that everybody associated with the city will recognize that opportunity.'*
In a surprise decision Tuesday morning, Rhodes also said he will allow pension cuts in Detroit's bankruptcy. He emphasized that he won't necessarily agree to pension cuts in the city's final reorganization plan unless the entire plan is fair and equitable. 'Resolving this issue now will likely expedite the resolution of this bankruptcy case,' he said."

Flint: *"Flint once thrived as the home of the nation's largest General Motors plant. The city's economic decline began during the 1980s, when GM downsized. In 2011, the state of Michigan took over Flint's finances after an audit projected a \$25 million deficit. In order to reduce the water fund shortfall, the city announced that a new pipeline would be built to deliver water from Lake Huron to Flint. In 2014, while it was under construction, the city turned to the Flint River as a water source. Soon after the switch, residents said the water started to look, smell and taste funny. Tests in 2015 by the Environmental Protection Agency (EPA) and Virginia Tech indicated dangerous levels of lead in the water at residents' homes."*

Hartford: *"Hartford's biggest bond insurer said it had offered to help the city postpone payments on as much as \$300 million in outstanding debt, in a move designed to help prevent a bankruptcy filing for Connecticut's capital. Under Assured Guaranty's proposal, debt payments due in the next 15 years would instead be spread out over the next 30 years without bankruptcy or default. The city* *would issue new longer-dated bonds and use the proceeds to make the near-term debt payments."*

Puerto Rico: *"The Puerto Rican government failed to pay almost half of $2 billion in bond payments due Friday, marking the commonwealth's first-ever default on its constitutionally guaranteed debt."*

New Jersey and other states: "*The particular factors are as diverse as the states. But one thing is clear: More states are facing financial trouble than at any time since the economy began to emerge from the Great Recession, according to experts who say the situation will grow more dire as the Trump administration and GOP leaders on Capitol Hill try to cut spending and rely on states to pick up a greater share of expensive services like education and health care.*"

On the State Crisis: "*States and cities around the country will soon book similar losses because of new, widely followed accounting guidelines that apply to most governments starting in fiscal 2018 – a shift that could potentially lead to cuts to retiree heath benefits."*

Illinois: *"After decades of historic mismanagement, Illinois is now grappling with $15 billion of unpaid bills and an unthinkable quarter-trillion dollars owed to public employees when they retire."*

B Computation [Not for Publication]

This appendix describes the computational algorithms used.

B.1 Discretization of the AR(2) process

To discretize the AR(2), we cast it in the form of a VAR(1) and then follow Gordon (2021). This method increases efficiency by dropping low-probability states and then suitably adjusting the transition matrix.[30] We use Tauchen (1986) as the underlying tensor-grid method with a "coverage" (i.e., a support) of two unconditional standard deviations. The algorithm delivers 58 discrete $(\tilde{z}_{it}, \tilde{z}_{it-1})$ states. These, combined with the five permanent productivity states and three weather states, make 870 exogenous states.

B.2 Equilibrium computation

To compute the equilibrium, we guess on two objects: the expected utility conditional on moving J and the average inflows over a "normalization" term for the logit probabilites,

$$\bar{i} := \frac{\int nF(R(\mathbf{x}))d\mu(\mathbf{x})}{\exp(\lambda(S(\mathbf{x}) - \max_{\mathbf{x}} S(\mathbf{x}))d\mu(\mathbf{x})}. \tag{27}$$

Subtracting off $\max_{\mathbf{x}} S(\mathbf{x})$ prevents overflows in the computation. Note that knowing $\bar{i}$, $i(\mathbf{x})$ can be obtained via

$$i(\mathbf{x}) = \bar{i} \exp(\lambda(S(\mathbf{x}) - \max_{\mathbf{x}} S(\mathbf{x}))). \tag{28}$$

B.2.1 Solving for the law of motion and value and price functions

With the $(J, \bar{i})$, we solve for the value function $S(\mathbf{x})$, the law of motion $\acute{n}(\mathbf{x})$, and the price schedule $q(b', \acute{n}, z)$ as follows:

1. Construct discrete grids of of debt per person $\mathcal{B}$, population $\mathcal{N}$, productivity $\mathcal{Z}$, and weather $\mathcal{W}$.

   For $\mathcal{B}$, we use 20 linearly spaced points from -0.2 to 0. Since average income across cities is normalized to 1 and the debt-output ratio is around .02, this allows for a given city to hold roughly four times as much debt as the average, and it is not binding in the benchmark. (This grid is coarse relative to those used in Bewley-Huggett-Aiyagari type models, but the dispersion in debt holdings is much more concentrated for cities.) For $\mathcal{N}$, we use 64 *log-linearly* spaced grid points over $\pm 5 * 1.8$ since the standard deviation of the log population is roughly 1.8. For $\mathcal{Z}$, we discretize the process as described in Section B.1 and tensor product it with the nonparametrically discretized permanent shocks. For $\mathcal{W}$, we use a three-point discretization $\{-2\sigma_\omega, 0, 2\sigma_\omega\}$ with Tauchen's method.

2. Fix tolerances (tol_q, tol_n, tol_S).

   We use $(tol_q, tol_n, tol_S) = (10^{-6}, 10^{-5}, 10^{-6})$.

[30] Specifically, we use the "TT0" refinement and the theshold value $\underline{\pi} = 10^{-6}$.

3. Guess on $S(\mathbf{x})$, $\dot{n}(\mathbf{x})$, $q(b', \dot{n}, z, \omega)$.

   The initial guess we use is $S(\mathbf{x}) = 0$, $\dot{n}(\mathbf{x}) = n$, and $q(b', \dot{n}, z, \omega) = \bar{q}$.

4. Solve for the implied $S^*(\mathbf{x})$ and associated policies.

   To determine the optimal value, we first do a grid search over the discrete bond states. When a discrete bond state, say $\mathcal{B}_i$, is less than 0.01% away from the maximum, we then search for a local maximum in $(\mathcal{B}_{i-1}, \mathcal{B}_i)$ and $(\mathcal{B}_i, \mathcal{B}_{i+1})$ (whenever applicable). In doing this search, we use Brent's method. Whenever we interpolate, we use linear interpolation.

   Conditional on debt and default outcomes, we solve for c, g, h using the analytic solution of the intratemporal problem.

5. Compute an update $q^*(b', \dot{n}, z, \omega)$.

6. Compute an update $\dot{n}^*(\mathbf{x})$ using $S^*(\mathbf{x})$ and J.

7. Determine whether the convergence criteria $||q^* - q||_\infty < tol_q$, $||\dot{n}^* - \dot{n}||_\infty < tol_n$, and $||S^* - S||_\infty < tol_S \cdot ||S||_\infty$ are satisfied. If so, stop. Otherwise, update the guesses and go to Step 4.

B.2.2 Solving for the invariant distribution and key equilibrium object updates

Given the converged values for $\dot{n}(\mathbf{x})$ and the bond policy $b'(\mathbf{x})$, we compute the invariant distribution $\mu(\mathbf{x})$ and updates $J^*, \bar{\imath}^*, \bar{q}^*$ as follows:

1. Fix a tolerance tol_μ.

   We use $tol_\mu = 10^{-10}$.

2. Guess on μ.

   Our initial guess is $\mu(0, 1, z, 1) = \mathbb{P}(z)$ with $\mu = 0$ elsewhere. (Consequently, the mass of households is 1 initially.) On subsequent invariant distribution computations, we use the previously computed μ.

3. Using $\dot{n}(\mathbf{x})$, $\mu(\mathbf{x})$, and the bond and default policies, compute an update on the invariant distribution $\mu^*(\mathbf{x})$.

   We use linear interpolation to distribute the mass from μ to μ^*. (An important advantage of linear interpolation is that it keeps the number of *households* the same on each iteration, i.e., $\int \mu(\mathbf{x}) n dx = \int \mu^*(\mathbf{x}) n dx$.)

4. Determine whether the convergence criteria $||\mu^* - \mu||_\infty < tol_\mu$ is satisfied. If so, continue to the next step. Otherwise, update the guess $\mu := \mu^*$ and go to Step 3.

5. For the updates J^* and $\bar{\imath}^*$, use the values associated with the computed invariant distribution μ.

B.2.3 Solving for the key equilibrium objects

With the initial guesses $J, \bar{i}$ and the updates $J^*, \bar{i}^*$, produce new initial guesses as follows:

1. Fix tolerances $(tol_J, tol_{\bar{i}})$.

   We use $(tol_J, tol_{\bar{i}}) = (10^{-6}, 10^{-6})$.

2. Check whether $|J^* - J| < tol_J \cdot |J|$ and $|\bar{i}^* - \bar{i}| < tol_{\bar{i}}$. If so, STOP: an equilibrium has been computed. Otherwise, go on.

3. Update the equilibrium values.

   Using the new guesses on $J, \bar{i}$, resolve for the value functions, price functions, law of motion, invariant distribution, and key equilibrium objects as described in Sections B.2.1 and B.2.2. Then go to step 2.

C Omitted proofs and results [Not for Publication]

This section contains additional theoretic, empirical, and quantitative results, as well as omitted proofs from the two-period model.

C.1 Microfounding inflow rates

To microfound inflow rates, we must use the notion of a Gumbel process. The beginning of the theory seems to be quite recent and due to Malmberg (2013). Here we follow the definition in Maddison et al. (2015):

Definition 1 (Maddison et al., 2015)**.** *Let $L(I)$ be a sigma-finite measure on sample space Ω, $I \subseteq \Omega$ measurable, and $G_L(I)$ a random variable. $\mathcal{G}_L = \{G_L(I) | I \subset \Omega\}$ is a* Gumbel process *if*

1. $G_L(I) \sim \textit{Gumbel}(\log L(I))$
2. $G_L(I) \perp G_L(I^c)$
3. *for measurable $A, B \in \Omega$, then $G_L(A \cup B) = \max\{G_L(A), G_L(B)\}$.*

Essentially, what the Gumbel process does is assign an infinitesimally small taste shock to any of the continuum of choices. The taste shock is small enough that the maximum over a continuum of choices is well-defined but large enough to influence the choices themselves.

For our purposes, $\mathbf{x} = (b, n, z) \in \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}^+ =: \mathbf{X}$. Let $\mathcal{X}$ denote the Borel σ-algebra of $\mathbf{X}$ with a Borel measure of islands μ. Then every $X \in \mathcal{X}$ is measurable with respect to μ. The sample space is $\mathbf{X}$.

To formalize the inflow rates, we do the following. Define $L : \mathcal{X} \to \mathbb{R}$ via $L(X) = \int_X \exp(\lambda S(\mathbf{x}) + c) d\mu(\mathbf{x})$ where c is a constant. (Then L is absolutely continuous with respect to μ.) In the Gumbel process, $L(X)$ will be the σ-finite measure on the sample space $\mathcal{X}$. Then a Gumbel process $\mathcal{G}_L$ with base measure L has random utility $G_L(X) \sim \textit{Gumbel}(\log L(X))$ for each $X \in \mathcal{X}$ with the additional restrictions that $G_L(X)$ is independent of $G_L(X^c)$ and $G_L(A \cup B) = \max\{G_L(A), G_L(B)\}$. The last restriction says, essentially, that if options in A and B are both available, whichever is best (taking into account random utility) will be chosen. In the finite case, this amounts to the optimal value being the maximum over the finite set. The probability that the optimum over $\mathbf{X}$ is contained in some set $X \in \mathcal{X}$ is equivalent to the event that $G_L(X) \geq G_L(X^c)$. Malmberg (2013) showed $\mathbb{P}(G_L(X) \geq G_L(X^c))$ is $L(X \cap \mathbf{X})/L(\mathbf{X})$ (and here $L(X \cap \mathbf{X})/L(\mathbf{X}) = L(X)/L(\mathbf{X})$). And $L(X)$ by definition is $\int_X \exp(\lambda S(\mathbf{x}) + c) d\mu(\mathbf{x})$. Therefore, the probability that the max is in X is $\int_X \exp(\lambda S(\mathbf{x})) d\mu(\mathbf{x}) / \int_{\mathbf{X}} \exp(\lambda S(\mathbf{x}) d\mu(\mathbf{x})$. Consequently, the argmax has a probability density (formally, a Radon-Nikodym derivative) of $\frac{\exp(\lambda S(\mathbf{x}))}{\int \exp(\lambda S(\mathbf{x})) d\mu(\mathbf{x})}$. Therefore, by a law of large numbers, the measure going to each island with type $\mathbf{x}$ is $\bar{\imath} \frac{\exp(\lambda S(\mathbf{x}))}{\int \exp(\lambda S(\mathbf{x})) d\mu(\mathbf{x})}$ where $\bar{\imath}$ is the total measure of in-migrants (equivalently, out-migrants).

C.2 The model in extensive form with Markov perfect equilibrium

In this section, we cast the model into an extensive form dynamic game and discuss how a Markov perfect equilibrium of that game maps directly into an equilibrium in the main text.

At the start of a period, nature moves, revealing z, as well as ϕ for each household. The payoff relevant state-space is $(\phi, \mathbf{x})$. Households simultaneously make migration decisions $m(\phi, \mathbf{x})$ to solve (8), knowing the decisions of the other households result in $\dot{n}(\mathbf{x})$ persons left after migration and anticipating a value $S(\mathbf{x})$ from staying, which will be the benevolent government's backward induction value when $\dot{n}(\mathbf{x})$ remain after migration.

After migration, there are n' households left (which will equal $\dot{n}(\mathbf{x})$ on the equilibrium path). The government then chooses its policies g, d, b', T with its relevant state-space being $\mathbf{x}_1 := (\mathbf{x}, n')$. This requires a modification of (21), making n' a state variable and not imposing $n' = \dot{n}(\mathbf{x})$:

$$
\begin{aligned}
\tilde{\tilde{S}}(\mathbf{x}_1) = & \max_{d \in \{0,1\}, g \geq 0, T \leq z, b'} U(\mathbf{x}_2) + \beta \mathbb{E}_{\phi', z' | z} V(\phi', \mathbf{x}') \\
& \text{s.t. } \mathbf{x}_2 = (\mathbf{x}_1, b', d, g, T), \ \mathbf{x}' = (b', n', z', \omega) \\
& g n'^{(1-\eta)} + q(\mathbf{x}_2) b' n' = T n' + (1-d) b n + d(-p(b, n, z, \omega) - \iota z n') \\
& -b' \leq \delta g n'^{-\eta}.
\end{aligned} \tag{29}
$$

So, let its optimal policies d, g, T, b' and value $\tilde{\tilde{S}}$ be functions of $\mathbf{x}_1$. In this problem, the flow utility $U(\mathbf{x}_2)$ is the backwards induction solution household flow utility, and likewise for $V(\phi', \mathbf{x}')$. The price $q(\mathbf{x}_2)$ will be set by the Walrasian auctioneer to make zero profits. In the main text, we took a shortcut, imposing the equilibrium path where $n' = \dot{n}(\mathbf{x})$, defined $\tilde{S}(\mathbf{x}) = \tilde{\tilde{S}}(\mathbf{x}, \dot{n}(\mathbf{x}))$, and then required the household expectation be consistent with that, $S(\mathbf{x}) = \tilde{S}(\mathbf{x})$. Likewise, we required q satisfy zero profits as an equilibrium condition.

After the government makes its choices, a Walrasian auctioneer chooses w, r, q with the objective to clear markets and make zero profits on loans.[31] Taking its state space as $\mathbf{x}_2 := (\mathbf{x}_1, b', d, g, T)$, this results in policies $(w(\mathbf{x}_2), r(\mathbf{x}_2), q(\mathbf{x}_2))$. Then households and firms simultaneously work and produce, with state space $\mathbf{x}_3 := (w, r, \mathbf{x}_2)$, resulting in values c, h, L^d, π and a flow utility, say $\tilde{\tilde{U}}(w, r, \mathbf{x}_2)$. As $U(\mathbf{x}_2)$ in (29) must be given by backwards induction, $U(\mathbf{x}_2) = \tilde{\tilde{U}}(w(\mathbf{x}_2), r(\mathbf{x}_2), \mathbf{x}_2)$. In the main text, we took another shortcut: Rather than explicitly specifying the subgames for the auctioneer, households and firms, we simply assumed the backwards induction solution, supposing that c, h, r, w, π, L^d satisfy (16) and that q satisfies (22). This ends a period.

To close the model, we need to specify J, $i(\mathbf{x})$, and $\mu(\mathbf{x})$. Since households and governments are individually measure zero, these are independent of policies at any given island. So each island can take J, $i(\mathbf{x})$, and $\mu(\mathbf{x})$ as the result of other islands' policies, and then we can require consistency after the fact.

A symmetric (across islands) Markov perfect equilibrium are household migration decisions and values $m(\phi, \mathbf{x})$, $V(\phi, \mathbf{x})$, government decisions and values $g(\mathbf{x}_1)$, $b'(\mathbf{x}_1)$, $d(\mathbf{x}_1)$, $T(\mathbf{x}_1)$, $\tilde{\tilde{S}}(\mathbf{x}_1)$, auctioneer prices $w(\mathbf{x}_2)$, $r(\mathbf{x}_2)$, $q(\mathbf{x}_2)$, and consumer and firm policies and values $c(\mathbf{x}_3)$, $h(\mathbf{x}_3)$, $\tilde{\tilde{U}}(\mathbf{x}_3)$, $\pi(\mathbf{x}_3)$, $L^d(\mathbf{x}_3)$, $H(\mathbf{x}_3)$, and values $\dot{n}(\mathbf{x})$, J, $i(\mathbf{x})$, and $\mu(\mathbf{x})$, such that all the following hold:

1. Given $\mathbf{x}_3$, the choices $c(\mathbf{x}_3), h(\mathbf{x}_3)$ are optimal for each household and flow utility is $\tilde{\tilde{U}}(\mathbf{x}_3)$.

[31] One could of course add financial intermediaries as players and let them determine the price, but the outcome is the same.

2. Given $\mathbf{x}_3$, the choices $L^d(\mathbf{x}_3), H(\mathbf{x}_3)$ are optimal for the firm and result in per person profit $\pi(\mathbf{x}_3)$.

3. Given $\mathbf{x}_2$, the prices $w(\mathbf{x}_2)$ and $r(\mathbf{x}_2)$ are optimal for the auctioneer, which is guaranteed if $c(\mathbf{x}_3), h(\mathbf{x}_3), r(\mathbf{x}_2), w(\mathbf{x}_2), \pi(\mathbf{x}_3), L^d(\mathbf{x}_3)$ satisfy (16).

4. Given $\mathbf{x}_1$, the policies $d(\mathbf{x}_1), g(\mathbf{x}_1), T(\mathbf{x}_1), b'(\mathbf{x}_1)$ are optimal for the government given the strategies of the auctioneer, firms, and households, and the government's future self as summarized in $U(\mathbf{x}_2) = \tilde{\tilde{U}}(w(\mathbf{x}_2), r(\mathbf{x}_2), \mathbf{x}_2)$ and $V(\phi', \mathbf{x}')$.

5. Given $\phi, \mathbf{x}$, the migration decisions $m(\phi, \mathbf{x})$ and $V(\phi, \mathbf{x})$ are optimal for households taking as given the migration decisions of other households as summarized in $\dot{n}(\mathbf{x})$ and the backwards induction solution of the government as summarized in $S(\mathbf{x}) = \tilde{\tilde{S}}(\mathbf{x}, \dot{n}(\mathbf{x}))$.

6. The law of motion for migration is consistent with $i(\mathbf{x})$ and migration decisions $m(\phi, \mathbf{x})$, which is guaranteed if $\dot{n}(\mathbf{x})$ is given by (12).

7. J and $i(\mathbf{x})$ are consistent, which is guaranteed if (10) and (11) hold.

8. $\mu(\mathbf{x})$ is consistent with the policies and stochastic transitions of all the islands.

In such an equilibrium, the set of "*" value, prices, and policies defined recursively—such as $\tilde{S}^*(\mathbf{x}) = \tilde{\tilde{S}}(\mathbf{x}, \dot{n}(\mathbf{x}))$ and $w^*(\mathbf{x}) = w(\mathbf{x}_2) = w(\mathbf{x}_1, b'(\mathbf{x}_1), d(\mathbf{x}_1), g(\mathbf{x}_1), T(\mathbf{x}_1))$—will constitute an equilibrium of the model in the main text.

C.3 The centralized problem

We now give the proof that the model can be centralized at a local level:

Proof of Proposition 3. Consider an arbitrary choice (c, g, d, b') in the centralized problem. At this choice, household and firm optimization and market clearing will be satisfied if we take $r = u_h/u_c$, $w = z$, $L^d = \dot{n}$, $h = \bar{H}/\dot{n}$, π solving $\dot{n}\pi = z\dot{n} - w\dot{n} + r\overline{H}$, and T solving $\dot{n}c + rh\dot{n} = w\dot{n} + \pi\dot{n} - T\dot{n}$ (see equation 16). Eliminating profit from the consumption equation, one has $c = z - T$ (and clearly $h = \bar{H}/\dot{n}$). Hence, the flow utility associated with this allocation is $u(z - T, g, \bar{H}/\dot{n}, \omega)$, which according to (17) is the same as $U(g, T, \dot{n}, z, \omega)$. Hence, an arbitrary choice delivers $u(c, g, \bar{H}/\dot{n}, \omega)$ flow utility in the centralized problem, which—when supported using the above prices and allocations—is the same as $U(g, T, \dot{n}, z, \omega)$. Moreover, at these prices and allocations, b', d, T is feasible for the government as guaranteed by Walras's law.[32] Then, since the centralized planner is maximizing the same flow utility, discounting, and expectations as the government, optimal choices for the centralized planner must simultaneously solve the government's problem. Hence, the optimal choices from the centralized problem can be supported as a decentralized equilibrium using the prices r, w, firm allocation L^d, household housing consumption allocation h, firm profits π, and taxes T.

$\square$

[32] One can verify this easily. For instance, if $d = 0$, then the centralized budget constraint reads $\dot{n}c + \dot{n}^{1-\eta}g + qb'\dot{n} = z\dot{n} + b\dot{n}$. Using $c = z - T$ to eliminate c, one finds $\dot{n}^{1-\eta}g + qb'\dot{n} = T\dot{n} + b\dot{n}$, which is the government's budget constraint.

C.4 Two-period model proofs and omitted results

This subsection provides omitted proofs establishing the Euler equation and constrained inefficiency. It also includes a constrained efficiency result in the case of costless and fully-directed migration.

C.4.1 The Euler equation

Proof of Proposition 1. The objective function may be written

$$u(c_1) + \beta \left((1 - o_2)u(c_2) + \int_{-\infty}^{J-u(c_2)} (J - \phi) f(\phi) d\phi \right) \tag{30}$$

Using Leibniz's rule,

$$\begin{aligned}
0 &= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} + u(c_2)\frac{-\partial o_2}{\partial b_2} + \frac{\partial (J - u(c_2))}{\partial b_2}(J - \phi)f(\phi)\Big|_{\phi = J - u(c_2)} \right) \\
&= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} + u(c_2)\frac{-\partial o_2}{\partial b_2} + \frac{\partial (J - u(c_2))}{\partial b_2}u(c_2)f(J - u(c_2)) \right) \\
&= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} - u(c_2)\frac{\partial F(J - u(c_2))}{\partial b_2} + \frac{\partial (J - u(c_2))}{\partial b_2}u(c_2)f(J - u(c_2)) \right) \\
&= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} - u(c_2)f(J - u(c_2))\frac{\partial (J - u(c_2))}{\partial b_2} + \frac{\partial (J - u(c_2))}{\partial b_2}u(c_2)f(J - u(c_2)) \right) \\
&= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta(1 - o_2)u'(c_2)\frac{\partial c_2}{\partial b_2} \\
&= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\frac{\partial b_2 \frac{n_1}{n_2}}{\partial b_2} \\
&= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\left(\frac{n_1}{n_2} + b_2 n_1 \frac{\partial n_2^{-1}}{\partial b_2}\right) \\
&= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\left(\frac{n_1}{n_2} + b_2 n_1 (-1) n_2^{-2}\frac{\partial n_2}{\partial b_2}\right) \\
&= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\left(\frac{n_1}{n_2} - b_2 \frac{n_1}{n_2}\frac{1}{n_2}\frac{\partial n_2}{\partial b_2}\right) \\
&= -\bar{q}u'(c_1) + \beta\frac{n_1}{n_2}(1 - o_2)u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right) \\
&= -\bar{q}u'(c_1) + \beta\frac{n_1}{n_1(1 - o_2 + i_2)}(1 - o_2)u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right) \\
&= -\bar{q}u'(c_1) + \beta\frac{1 - o_2}{1 - o_2 + i_2}u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right)
\end{aligned}$$

Consequently, the Euler equation reads

$$\bar{q}u'(c_1) = \beta\frac{1 - o_2}{1 - o_2 + i_2}u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right). \tag{31}$$

□

C.4.2 Constrained inefficiency

Proof of Proposition 2. With no cross-sectional heterogeneity, constrained efficiency requires (5). If $\bar{q} = \beta u'(y_2)/u'(y_1)$, then this requires

$$\frac{u'(y_2)}{u'(y_1)} = \frac{u'(c_2)}{u'(c_1)} = \frac{u'(y_2 + b_2 \frac{n_1}{n_2})}{u'(y_1 - \bar{q} b_2)}. \tag{*}$$

Evidently, this requires $b_2 = 0$. However, $b_2 = 0$ is not compatible with the government's Euler equation. In particular, at $b_2 = 0$ and at $\bar{q}$, the government Euler equation can be written

$$\frac{u'(y_2)}{u'(y_1)} = \frac{1 - o_2}{1 - o_2 + i_2} \frac{u'(c_2)}{u'(c_1)}. \tag{**}$$

Hence, if $i_2 > 0$, then (*) and (**) cannot simultaneously hold. And in fact, some people will enter (i.e., i_2 is greater than 0) because in the constrained efficient allocation $c_2 = y_2$ for every island and so $J = u(c_2)$ and—given this—some people will move since $F(0) > 0$ (i.e., migration is noisy). Hence, the constrained efficient allocation cannot be supported as an equilibrium.

For the claim that at the constrained efficient allocation governments would strictly prefer to borrow, note the Euler equation at the constrained efficient allocation is not satisfied with

$$u'(y_1)\bar{q} > \beta u'(y_2) \frac{1 - o_2}{1 - o_2 + i_2} \Leftrightarrow 1 > \frac{1 - o_2}{1 - o_2 + i_2}.$$

The way to equate marginal utilities would then be to increase c_1 by borrowing. □

C.4.3 Constrained efficiency under costless and fully-directed migration

Our constrained inefficiency result established in Proposition 2 may be surprising in light of the seminal paper by Tiebout (1956). Tiebout showed that, under certain assumptions, equilibria are efficient when local governments compete for workers. One of his key assumptions, which is not met here, is that of costless and fully-directed mobility. In fact, the equilibrium can be efficient if migration is *perfectly* directed. To see why, consider trying to implement an efficient allocation that implies $b_2 = 0$. For the reasons described above, the Euler equation (2) would typically imply this is impossible. However, if inflow rates "punish" any debt accumulation by falling to zero in a nondifferentiable way, the Euler equation no longer characterizes the optimal choice, and the equilibrium can be efficient. We prove this in Proposition 4.

Proposition 4. *If migration is completely directed with (1) $\mathbf{I}(u(c_2)) = 0$ for $c_2 < y_2$, (2) the right-hand derivative of $\mathbf{I}(\cdot)$ at $u(y_2)$ infinite, and (3) $\mathbf{I}(\cdot)$ differentiable elsewhere, then an equilibrium with $\bar{q} = \beta u'(y_2)/u'(y_1)$ exists and it is constrained efficient.*

Proof of Proposition 4. Under these assumptions, the Pareto optimal allocation is $c_1 = y_1$, $c_2 = y_2$, with households moving whenever $\phi < 0$ and staying whenever $\phi > 0$ (with indifference elsewhere). Note that in contrast to the hypothesis of Proposition 2, inflow rates are assumed to be not differentiable at $b_2 = 0$, which means the Euler equation is not valid at that point.

We will prove the existence of a symmetric equilibrium with $\bar{q} = \beta u'(y_2)/u'(y_1)$ both of which have $b_2 = 0$ as optimal. We will do so by establishing that at this price $b_2 < 0$ is not optimal, that

$b_2 > 0$ is not optimal, and that an optimal choice exists (in which case it must be $b_2 = 0$). This will then support the allocation $(c_1, c_2) = (y_1, y_2)$ (and the migration decisions).

For use below, we note that whenever the derivative $\partial n_2/\partial b_2$ exists, one has

$$\begin{aligned} \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2} &= \frac{b_2}{n_2} n_1 \left(\bar{\imath}\mathbf{I}'(u(c_2)) + f(J - u(c_2))\right) u'(c_2)\frac{\partial c_2}{\partial b_2} \\ &= b_2 u'(c_2)\left(\frac{n_1}{n_2}\right)^2 \left(\bar{\imath}\mathbf{I}'(u(c_2)) + f(J - u(c_2))\right) \end{aligned} \tag{32}$$

Because $\mathbf{I}$ is increasing and f is positive, this has the same sign as b_2.

First we will show that $b_2 < 0$ is not optimal by showing the Euler equation does not hold there. Given no inflows for $b_2 < 0$, borrowing is not optimal because the Euler equation (which is valid everywhere except at $b_2 = 0$) requires

$$\beta\frac{u'(y_2)}{u'(y_1)} = \bar{q} = \beta\frac{u'(c_2)}{u'(c_1)}\underbrace{\frac{1-o_2}{1-o_2+i_2}}_{=1}\underbrace{\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right)}_{\geq 1 \text{ since } b_2<0} \geq \beta\frac{u'(c_2)}{u'(c_1)}. \tag{33}$$

However, with $b_2 < 0$, $c_1 > y_1$ and $c_2 < y_2$, which gives a contradiction.

Now we will show that $b_2 > 0$ is not optimal. The Euler equation in this case reads

$$\beta\frac{u'(y_2)}{u'(y_1)} = \bar{q} = \beta\frac{u'(c_2)}{u'(c_1)}\underbrace{\frac{1-o_2}{1-o_2+i_2}}_{\leq 1}\underbrace{\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right)}_{\leq 1 \text{ since } b_2>0} \leq \beta\frac{u'(c_2)}{u'(c_1)}. \tag{34}$$

However, with $b_2 > 0$, $c_1 < y_1$ and $c_2 > y_2$, which gives a contradiction.

Since $b_2 < 0$ and $b_2 > 0$ are not optimal, all that remains to show is that an optimal choice exists. Without loss of generality, we can restrict the choice set to $b_2 \in [-\delta, \delta]$ for δ arbitrarily small such that every choice is feasible. Then, with a continuous objective function being maximized over a compact set, a maximum exists, which must be $b_2 = 0$. □

C.5 Quantitative testing of indeterminacy

To test for indeterminacy, we proceed by drawing 100 random starting guesses for J and $\bar{\imath}$ uniformly distributed about ±50% of the benchmark's computed equilibrium values. (For the definition of $\bar{\imath}$, see Appendix B.2.) We then compute the implied equilibrium solution. Figure 15 shows a scatter plot of the guesses, and also reveals that they all converge to the same solution (up to small numerical differences). This suggests that the equilibrium is unique in a wide range about the computed benchmark equilibrium.

C.6 Additional calibration results

The cross-sectional regression specifications in Table 1 are as follows. For the row "Regression coef. log expenditures on $\log n$", the specification is

$$\log x_{i,t} = \alpha + \beta \log n_{i,t} + \epsilon_{i,t},$$

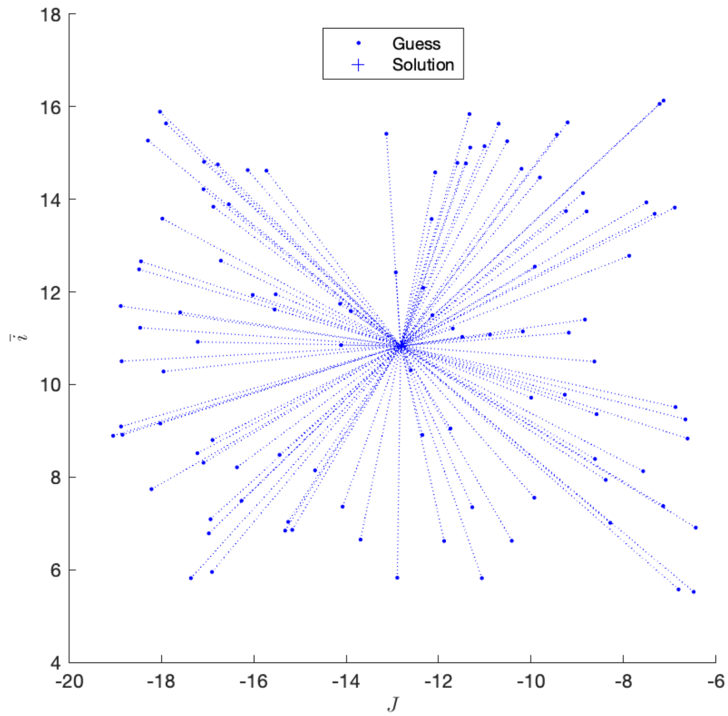


Figure 15: Quantitative testing of indeterminacy

where $x_{i,t}$ is total expenditures, and $n_{i,t}$ is population. For the rows with "rate reg. coef., log z," the specification is

$$y_{i,t} = \alpha + \beta \log z_{i,t} + \gamma \log n_{i,t} + \epsilon_{i,t}.$$

The dependent variable is either in-migration rates or out-migration rates, as specified in the row. In all regressions, the sample is restricted to $t = 2011$.

Table 4 provides additional untargeted statistics coming from richer regressions. The regression coefficients correspond to a regression of the form

$$y_{i,t} = \alpha + \beta\varsigma_i + \gamma\tilde{z}_{i,t} + \delta \log n_{i,t} + \epsilon_{i,t}$$

with ς_i and $\tilde{z}_{i,t}$ being the fixed effect and residual productivity from (24) (and $n_{i,t}$ population). Again, the sample is restricted to 2011. Overall, the underlying elasticities are not very different from those in the data.

C.7 Additional estimation results

We construct the share of individuals at a reference time $\tau > 0$ periods ago who live in c but arrived from county o at some time in the past as follows. Given inflows $I_{c,o,t}$, a population measured at the start of the period $N_{c,t}$, and an out-migration rate $\delta_{c,t}$, we construct $\theta_{c,o,t}$ under the assumption that every individual in c has the same probability δ_t of leaving. Let $N_{c,o,t}$ denote the stock of individuals in c from o and time t. First, we obtain $N_{c,o,1}$ by assuming the county's population

Untargeted Statistics	Data	Model
*In rate reg. coef., $\log n$	-0.000	-0.013
*In rate reg. coef., $\log z$ FE	-0.011	0.006
*In rate reg. coef., $\log z$ res	0.034	0.072
*Out rate reg. coef., $\log n$	-0.000	-0.011
*Out rate reg. coef., $\log z$ FE	-0.013	0.000
*Out rate reg. coef., $\log z$ res	0.015	-0.019
Note: * means the underlying data is county-level.		

Table 4: Additional untargeted moments

is in proportion to its inflows in $t = 1$, $N_{c,o,1} = \frac{I_{c,o,1}}{\sum_o I_{c,o,1}} N_{c,1}$. Second, we obtain $N_{c,o,t}$ for $t \geq 2$ recursively using $N_{c,o,t} = N_{c,o,t-1}(1 - \delta_{c,t-1}) + I_{c,o,t-1}$. The share is then $\theta_{c,o,t} = N_{c,o,t} / \sum_o N_{c,o,t}$.

Tables 5, 6, and 7 report the estimates underlying Figure 2.

	Debt	F5.Debt	F10.Debt	F15.Debt	F20.Debt
In-migration rate	3.505	10.36	12.39	6.613	1.615
	(0.9)	(1.9)	(2.8)	(1.9)	(0.3)
Lagged debt	0.676	0.496	0.342	0.271	0.239
	(77.8)	(51.4)	(30.2)	(20.4)	(10.0)
Lagged out-migration rate	-2.619	-8.023	-8.720	-3.286	0.468
	(-0.8)	(-1.7)	(-2.3)	(-1.1)	(0.1)
Prod. FE	0.484	0.761	0.939	0.969	0.846
	(18.9)	(18.1)	(20.9)	(16.3)	(5.7)
Constant	2.398	3.628	4.887	5.460	5.786
	(41.6)	(49.8)	(63.3)	(57.1)	(33.0)
Observations	31830	20993	14303	8142	2196
R^2	0.609	0.414	0.269	0.222	0.161
First-stage F	14.78	32.32	30.47	36.30	13.75
Note: robust standard errors are used; year effects are included; "debt" is log of per person, real debt measured at the end of the fiscal year.					

Table 5: IV regressions capturing the externality effects on debt

C.8 COVID-19

We now turn to address an important contemporaneous question: what are the mid-term and long-term effects on municipalities from COVID-19 and the unprecedented, coincident policy interventions? Unfortunately, the ongoing uncertainty and lack of multiyear data on outcomes means we will not be able to provide a precise answer. Our approach is rather to feed in a few shocks that capture the defining features of 2020-2021: a drop in GDP, a decline in real interest rates, and a motive to relocate out of large cities. Given the rapid onset of COVID-19, we will also consider the

	Exp.	F5.Exp.	F10.Exp.	F15.Exp.	F20.Exp.
In-migration rate	12.57	11.10	10.09	7.754	5.383
	(3.6)	(4.4)	(3.4)	(3.3)	(1.9)
Lagged exp.	0.844	0.814	0.768	0.656	0.656
	(42.6)	(54.0)	(40.5)	(43.0)	(24.8)
Lagged out-migration rate	-10.88	-9.509	-8.415	-6.529	-4.350
	(-3.6)	(-4.4)	(-3.3)	(-3.4)	(-1.8)
Prod. FE	0.0215	0.0353	0.00232	-0.00792	0.0101
	(1.4)	(2.7)	(0.1)	(-0.3)	(0.2)
Constant	1.245	1.368	1.936	2.910	3.092
	(6.9)	(9.1)	(10.5)	(21.0)	(13.6)
Observations	32145	21172	14421	8221	2209
R^2	0.423	0.371	0.268	0.253	0.336
First-stage F	21.42	41.88	39.45	39.98	14.58

Note: robust standard errors are used; year effects are included; “expenditures” is log of per person, real expenditures.

Table 6: IV regressions capturing the externality effects on expenditures

	Rev.	F5.Rev.	F10.Rev.	F15.Rev.	F20.Rev.
In-migration rate	10.30	12.48	8.460	7.766	4.917
	(5.0)	(5.2)	(4.3)	(4.3)	(1.9)
Lagged rev.	0.850	0.838	0.779	0.667	0.679
	(62.3)	(47.6)	(52.1)	(44.2)	(25.1)
Lagged out-migration rate	-8.966	-10.75	-7.169	-6.510	-3.746
	(-5.0)	(-5.1)	(-4.2)	(-4.2)	(-1.7)
Prod. FE	0.0192	0.0312	0.00170	-0.0142	0.00546
	(1.7)	(2.2)	(0.1)	(-0.6)	(0.1)
Constant	1.220	1.181	1.917	2.840	2.941
	(10.1)	(7.1)	(13.8)	(21.6)	(12.8)
Observations	32145	21172	14421	8221	2209
R^2	0.531	0.334	0.345	0.260	0.359
First-stage F	21.69	43.88	41.08	41.01	14.48

Note: robust standard errors are used; year effects are included; “revenue” is log of per person, real revenue.

Table 7: IV regressions capturing the externality effects on revenue

consequences of cities being able to only sluggishly adjust their tax policies. We'll consider each of these in turn.

C.8.1 Aggregate productivity decline

We begin by assuming that aggregate productivity $\exp(\varpi)$ falls by 10%. Subsequently, we assume productivity reverts to its steady state at a rate of 0.95, that is, $\varpi_t = 0.95\varpi_{t-1}$. Given the uncertainty behind COVID-19 and its impact on the economy, we view this persistence as a reasonable starting point.

Following the shock, default rates rise by an order of magnitude, to nearly 0.4% (blue line in Figure 16). Like in the Great Recession, despite the consumption smoothing motive, taxes increase on impact—we will consider an alternative case where they cannot—to bring debt to a more sustainable level. Two years after the shock, debt has shrunk by 7 percent and remains protracted for many years following the drop in productivity. The reasons are as before: an effectively tightened borrowing limit from the optimal g decrease and spreads moving against the sovereign if they do not deleverage. Also as before, spreads remain low, mainly due to their forward-looking nature. (When we look at exogenous taxes, however, substantial recovery rates will play a key role in preventing large spread increases.)

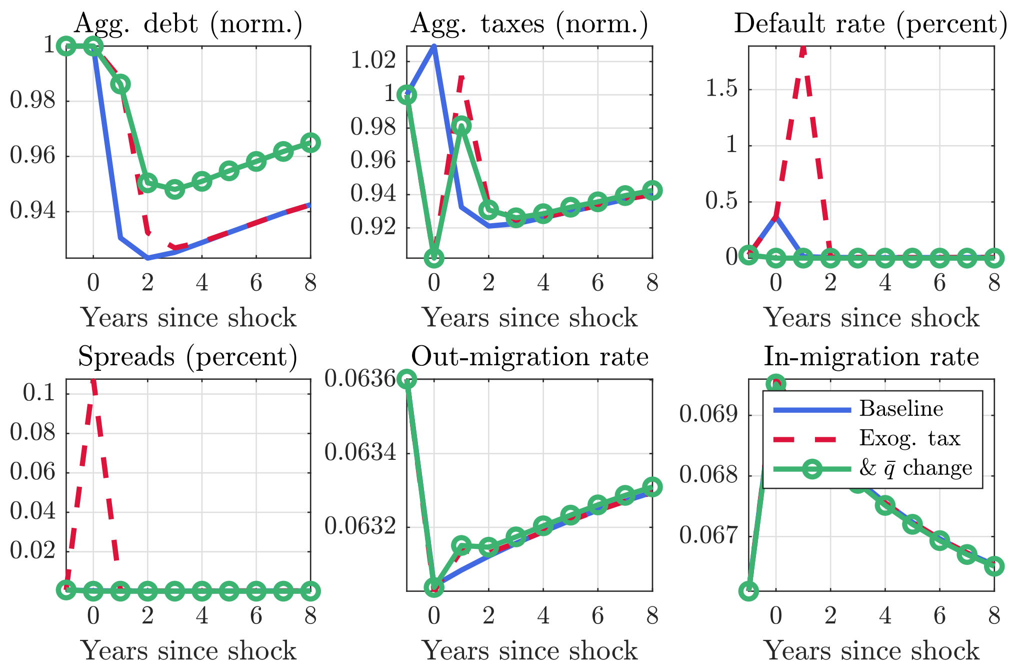


Figure 16: Transitions with and without exogenous taxes and interest rate declines

C.8.2 Exacerbating factor: Tax inflexibility

Given the rapid onset and unanticipated nature of the COVID-19 crisis, local government budgets could not adjust immediately to the shock. We now consider the possibility that taxes cannot adjust on impact. Specifically, we will think about the case where the first period tax rates must be proportional to housing expenditures, i.e., $T = \tau r h$. (We will continue to assume that residents take T—rather than τ—as given.) In this case, it is not hard to show that $c = z - T$ and $u_c/u_h = 1/r$ must still hold. Combining these equations, one has c as a function of τ, z, and parameters, with g given as a residual (conditional on a choice of b' and d) from the local government's budget constraint. We choose $\tau = 0.696$ to deliver similar allocations to the benchmark.[33]

As the optimal response to the shock is to raise taxes, the inability to do so leads to worse outcomes (red dashed lines in Figure 16). Here, default rates increase on impact but are much higher the next year as cities did not raise taxes and deleverage as much as they should have. Reflecting this, spreads also increase noticeably. The default rate increase of 180 bp is far larger than the 10 bp increase in spreads due to the large recovery rates, which are roughly 94% ($= 1 - 10/180$).

C.8.3 Mitigating factor #1: Real interest rate declines

As in the Great Recession, real interest rates declined substantially in 2020 and 2021. To capture this, we assume that the real interest rate falls by 1.5 pp, one-time and permanently. (While the permanent aspect is an exaggeration, given the massive decline in thirty-year rates, this may well approximate expectations.)

The decline in real rates drastically reduces default even when taxes cannot increase after the shock (green circled lines in Figure 16). The reasons are the same as before and are discussed in Section 6.1.4. Consequently, despite the very large and sudden shock, it is likely default rates will remain low due to the accommodation of real interest rates.

C.8.4 Mitigating factor #2: Redistributive effects

In addition to these aggregate shocks, COVID-19 has also reduced the attractiveness of living in densely populated regions. Unlike the other shocks, this has first-order redistributive effects. In the model, we can capture this by reducing the value of being in "high-weather" states—i.e., states with ω large. Because we discretized our ω state into three point $\{\underline{\omega}, 0, \overline{\omega}\}$, we implement this idea by reducing $\overline{\omega}$ by a given percentage. This decline lasts for a year and then recovers at the rate 0.95.

Figure 17 reports the transitions. A key takeaway is that larger declines in $\overline{\omega}$ lead to lower default rates. As people move from higher population to lower population cities, debt per person at the lower population cities declines. Because, as we already established, smaller to medium-sized cities are more likely to default than the largest cities, this shift from high population to lower population cities induces a composition effect that lowers default rates.

While default rates, measured as the rate at which cities default, decline, the migration shift from larger to smaller cities increases the typical size of bankrupt cities and the size of filings. The first claim follows immediately from the log population conditional on default in the bottom right

[33] Note $T\dot{n}$ is used to finance government spending $g\dot{n}^{1-\eta}$ and debt service, which on average is $(1/\bar{q} - 1)(-\dot{b}\dot{n})$. To not be too distortionary, we want $T\dot{n} = \tau r h \dot{n} = \tau r$ roughly equal to $g\dot{n}^{1-\eta} + (1/\bar{q} - 1)(-\dot{b}\dot{n})$. Expressed relative to GDP Y, we need $\tau \frac{rh\dot{n}}{Y} = \frac{g\dot{n}^{1-\eta}}{Y} + (1/\bar{q} - 1)\frac{-\dot{b}\dot{n}}{Y}$. These relative-to-GDP quantities are targeted, so we want $\tau \times 0.125 = 0.082 + 0.04 \times 0.125$, which implies $\tau = 0.696$. Note that this is 70% of housing *expenditures*, not property values.

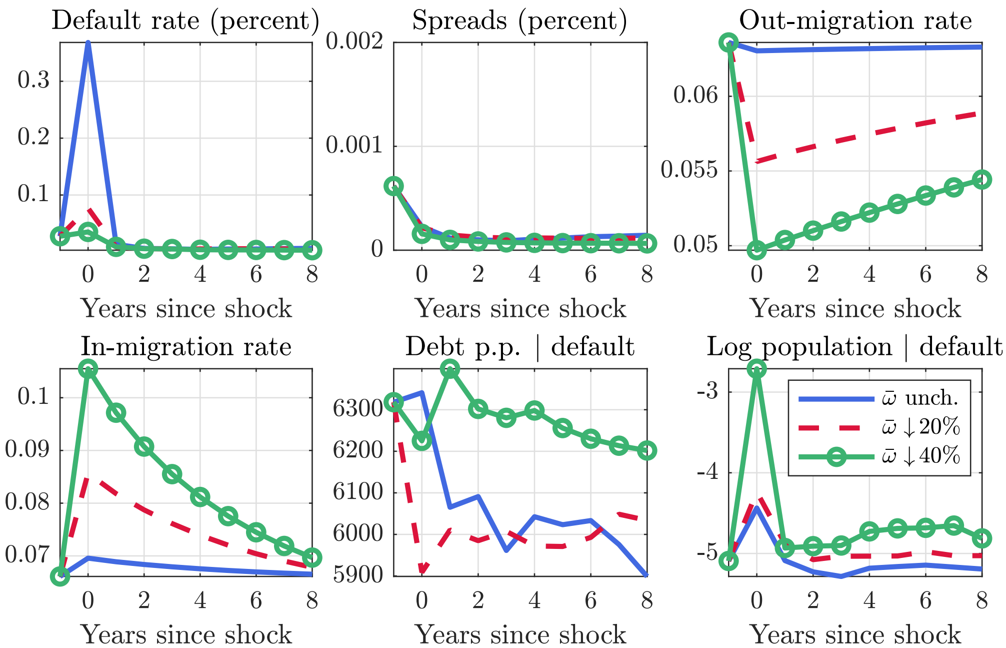


Figure 17: Negative TFP transitions with and without redistributive weather shock

panel. The second follows from the first in conjunction with an almost constant debt per person conditional on default (as seen in the bottom, middle panel). Hence, while default rates of cities in general are smaller, we may see defaults by larger cities involving correspondingly more debt.

D The Great Recession [Not for Publication]

In this appendix, we provide more background and rationale for the shocks we feed into the model for the Great Recession period.

To begin with, Figure 18 shows log real GDP per capita in relation to a pre-2009 linear trend. The drop in GDP p.c. was on the order of 12% on impact, and—relative to trend—continued through 2020Q1 reaching almost 20%. Against this headwind, the American Reinvestment and Recovery Act of 2009 (ARRA) was a sweeping legislation that included provisions to bolster state and local finances. This funding flowed directly to states and more indirectly to local governments. To see this, consider the time series of a few key state and local government (SLG) variables provided in NIPA in Figure 19. Expressed relative to GDP and in differences from 2006, federal transfers to state and local governments (SLG) rose from zero (i.e., they were at their 2006 levels) to almost 1% in late 2009 into 2011. This more than offset the decline in SLG tax revenue, which fell -0.5% relative to GDP, resulting in an overall boost of SLG revenue amounting to 0.25% that lasted into 2011. Unfortunately, this data does not separate the revenues of state and local governments with one exception, in that it reports the transfers from state to local governments. These rose slightly in 2009 and 2010 before falling substantially starting in 2011.

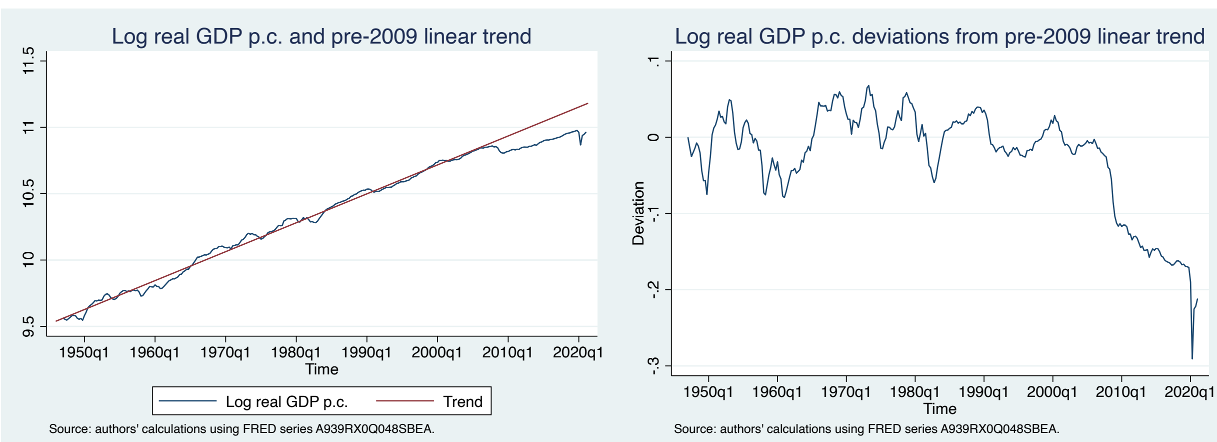


Figure 18: Log real GDP relative to trend

From a local government perspective, state to local transfers, while exhibiting some dip despite the ARRA, have been a steady percentage of GDP. Hence, in mapping this to the model, we assumed a pass-through from GDP declines to model TFP declines that is less than 100%. Specifically, we used that local government budgets are 8.2% of GDP, and state to local transfers were 3.2% from 2006 to 2009, suggesting a pass-through of $61\% = 1 - 3.2\%/8.2\%$.

Of course, the Great Recession also exhibits steep declines in risk-free real interest rates. These real rates, measured using five-year TIPS yields, are plotted in differences from 2006 in Figure 20. It reveals real rates declined by more than 3pp from 2006 to 2012. Table 8 summarizes the declines in productivity and interest rates that we feed into the model.

Table 8 reports the transition variables we use in the experiments.

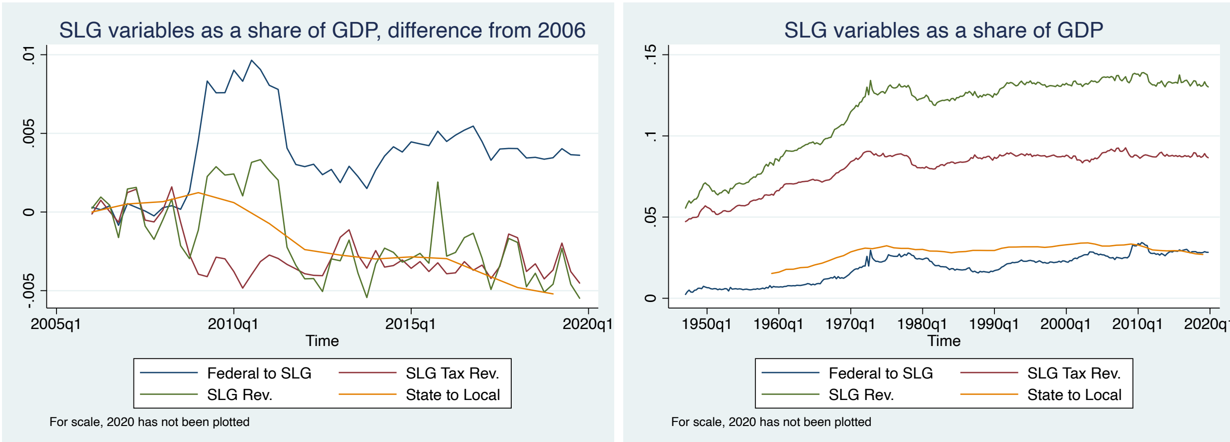


Figure 19: Federal, state, and local government transfers and revenue

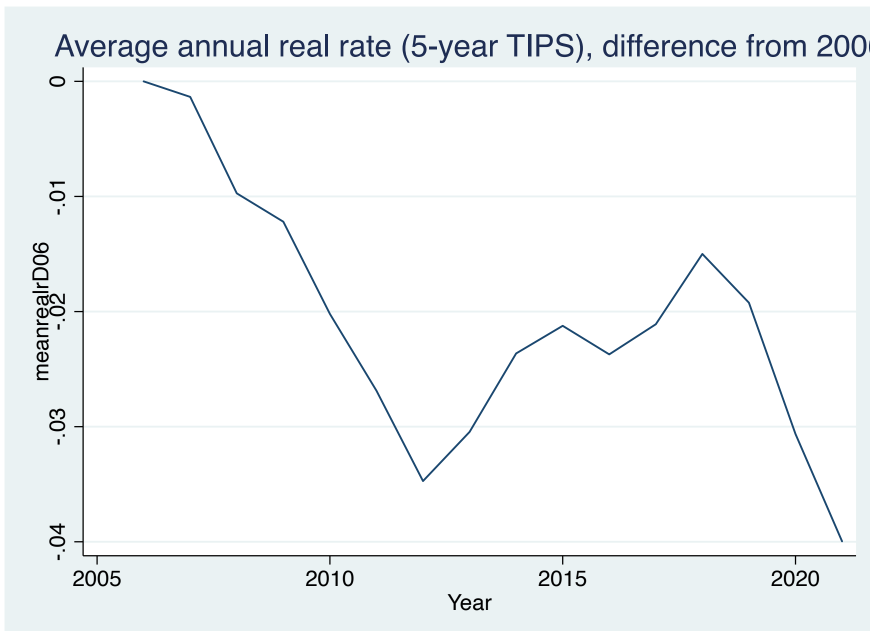


Figure 20: Real interest rates, difference from 2006

	Parameter	
Year	ϖ	$\bar{q}^{-1}-1$
2007 (steady state)	0	0.040
2008 (shock, 1st period)	-0.03	0.030
2009	-0.07	0.028
2010	-0.07	0.020
2011	-0.08	0.013
2012	-0.08	0.005
2013	-0.09	0.010
2014+ (new steady state)	-0.09	0.020

Table 8: Transition variables for the Great Recession

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