Multilateral Comovement in a New Keynesian World: A Little Trade Goes a Long Way

Supplementary Material

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November 17, 2022

Contents

A A New Keynesian Global Economy

There are N countries indexed by $n = 1, ..., N$.

A.1 Households

The representative household in country n has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_{n,t}/A_{n,t})^{1-\sigma_n} - 1}{1-\sigma_n} - \omega_n \frac{L_{n,t}^{1+\eta_n}}{1+\eta_n} \right\}.$$

Households in country n face the budget constraint,

$$P_{n,t}C_{n,t} + \frac{1}{\mathcal{E}_{rn,t}} E_t \left[V_{t+1}S_{n,t+1}\right] + E_t \left[P_{n,t}A_{n,t}\Theta_n \left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{rn,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t} \right) \right] + \frac{1}{R_{n,t}} B_{n,t}$$
$$= W_{n,t}L_{n,t} + \Pi_{n,t} + \frac{1}{\mathcal{E}_{rn,t}} S_{n,t} + B_{n,t-1} - T_{n,t}, \quad (1)$$

where $1/\mathcal{E}_{rn,t}$ denotes the nominal exchange rate that converts the reference currency into local currency. Thus, if reference country r uses the Euro, say, and country n is the U.S., $\mathcal{E}_{rn,t}$ is expressed in €/\$.[1]

Household optimization implies the following equations:

– The optimal choice of $S_{n,t+1}$ dictates

$$\left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} V_{t+1} \left[1 + \Theta_n' \left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{n,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t} \right) \right] = \beta \frac{\mathcal{E}_{n,t}P_{n,t}A_{n,t}}{\mathcal{E}_{n,t+1}P_{n,t+1}A_{n,t+1}} \left(\frac{C_{n,t+1}}{A_{n,t+1}} \right)^{-\sigma_n}.$$

– The optimal choice of $B_{n,t}$ implies that

[1] Anticipating our forthcoming discussion of trade, consider 2 regions, Europe (country r) and the U.S. (country n). Europe produces cheese and prices it in € while the U.S. produces movies and prices it in \$. The regions trade in cheese and movies. For the purpose of this example, and given our definitions above, the nominal exchange rate, $1/\mathcal{E}_{rn}$, converts prices in € into prices in \$,

$$\text{prices in \$} = 1/\mathcal{E}_{rn} \times \text{ prices in €},$$

or

$$P_n = 1/\mathcal{E}_{rn} \times P_r,$$

where $\mathcal{E}_{rn}$ is expressed in $\frac{€}{\$}$. Therefore, an increase in $\mathcal{E}_{rn}$ represents an appreciation of the \$ or a depreciation of the reference currency, the €. It also follows that we can define a real exchange rate, e_{rn}, measured in units of cheese per units of movies, $\frac{\text{cheese}}{\text{movies}}$, such that

$$e_{rn} = \mathcal{E}_{rn} \times \frac{P_n}{P_r}.$$

Therefore, an increase in e_{rn} represents a real appreciation of country n's goods, movies, or a real depreciation of the reference region's goods, cheese, (i.e., the reference region has to give up more cheese to watch a given quantity of movies).

$$\left(\frac{C_{n,t}}{A_{n,t}}\right)^{-\sigma_n} = \beta R_{n,t} E_t \left(\frac{C_{n,t+1}}{A_{n,t+1}}\right)^{-\sigma_n} \frac{P_{n,t}A_{n,t}}{P_{n,t+1}A_{n,t+1}}.$$

– The optimal choice of $C_{n,t}$ implies that

$$(\frac{C_{n,t}}{A_{n,t}})^{-\sigma_n} = \Lambda_{n,t} P_{n,t} A_{n,t}.$$

– Finally, optimal labor supply implies that

$$\omega L_{n,t}^{\eta_n} \left(\frac{C_{n,t}}{A_{n,t}}\right)^{\sigma_n} = \frac{W_{n,t}}{A_{n,t}P_{n,t}}.$$

Domestic market clearing for bonds requires that $B_{n,t} = 0\ \forall n$ and t. The budget constraint equation then reduces to

$$P_{n,t}\left(C_{n,t} + E_t\left[A_{n,t}\Theta_n\left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{n,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right]\right) + \frac{1}{\mathcal{E}_{n,t}} E_t\left[V_{t+1}S_{n,t+1}\right]$$
$$= W_{n,t}L_{n,t} + \Pi_{n,t} + \frac{1}{\mathcal{E}_{n,t}} S_{n,t} - T_{n,t}.$$

A.2 Final Goods

Final goods, $\mathcal{C}_{n,t}$, are assembled by producers using differentiated varieties, $y_{n,t}(j)$, with the technology,

$$\mathcal{C}_{n,t} = \left[\int_0^1 y_{n,t}(j)^{\frac{\varepsilon_{n,t}-1}{\varepsilon_{n,t}}} dj\right]^{\frac{\varepsilon_{n,t}}{\varepsilon_{n,t}-1}},\ \varepsilon_{n,t} > 1.$$

Final goods firms are competitive and, given the constant-returns-to-scale (CRS) technology, make zero profits. Their (nominal) unit cost of production is equal to the price of final goods, $P_{n,t}$. Given a price $P_{n,t}(j)$ for variety j, cost-minimization implies the demand function,

$$y_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_n, t}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t},$$

where

$$P_{n,t} = \left[\int_0^1 P_{n,t}(j)^{1-\varepsilon_{n,t}} dj\right]^{\frac{1}{1-\varepsilon_{n,t}}},$$

such that $P_{n,t}\mathcal{C}_{n,t} = \int_0^1 P_{n,t}(j) y_{n,t}(j) dj$.

Final goods may be used by households for either consumption proper or to pay for portfolio re-balancing costs so that

$$\mathcal{C}_{n,t} = C_{n,t} + A_{n,t} E_t\left[\Theta_n\left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{rn,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right].$$

A.3 Intermediate Goods

Non-Traded Sticky-Price Intermediate Goods: Each variety j of sticky-price goods in country n is produced using the technology,

$$y_{n,t}(j) = A_{n,t} Q_{n,t}(j), \tag{2}$$

where $Q_{n,t}(j)$ denotes a composite input used by firms in domestic sector j. This input consists of a bundle of different varieties making up second type of intermediate goods, described below, potentially purchased from other countries. Let $P^Q_{n,t}$ denote the price of the input bundle, $Q_{n,t}(j)$. Producers of sticky-price goods minimize total costs, $P^Q_{n,t} Q_{n,t}(j)$, subject to its technology (2). It follows that

$$P^Q_{n,t} Q_{n,t}(j) = MC_{n,t} y_{n,t}(j),$$

where $MC_{n,t}$ denotes marginal cost, $\frac{P^Q_{n,t}}{A_{n,t}}$. Firms operate on the demand function,

$$y_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t},$$

which implies that their profits are given by

$$\frac{\Pi_{n,t}(j)}{P_{n,t}} = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t} \left((1+\tau_n)\frac{P_{n,t}(j)}{P_{n,t}} - mc_{n,t}\right),$$

where $mc_{n,t} = \frac{MC_{n,t}}{P_{n,t}}$ and τ_n is a subsidy given to sticky-price goods producers.[2] The marginal cost of production faced by sticky-price producers in country n, $MC_{n,t}$, depends on the price of an input bundle, $P^Q_{n,t}$, consisting of potentially imported varieties.

- Note: since $P^Q_{n,t} Q_{n,t}(j) = MC_{n,t} y_{n,t}(j)$, it follows that $P^Q_{n,t} Q_{nt} = MC_{n,t} \mathcal{C}_{n,t} \Delta_{n,t}$ where $\Delta_{n,t} = \int_0^1 \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} dj$ is a measure of price dispersion which, under flexible prices, is just equal to 1.

In each period, a firm in country n is able to choose or reset its price optimally with probability $(1-\theta)$. Firms that adjust their prices all choose the same price, $P^*_{n,t}$, since all firms are identical within countries. With probability θ, a firm is unable to reset its price optimally. Its nominal price is then partially indexed to lagged inflation, denoted $1+\pi_{n,t-1} = P_{n,t-1}/P_{n,t-2}$, and long-run/steady state inflation, denoted $1+\pi_n$. In particular, for the latter firms,

$$P_{n,t}(j) = (1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n} P_{n,t-1}(j), \;\; \varrho_n \in [0,1],$$

where ϱ_n and $1-\varrho_n$ indicate the degree of indexation to lagged inflation and steady state inflation

[2] This subsidy can be used to eliminate any effects associated with monopolistic pricing distortions in steady state.

respectively in country n. The aggregate price index in country n, therefore, is given by

$$\begin{aligned} P_{n,t}^{1-\varepsilon_{n,t}} &= \int_0^1 P_{n,t}(j)^{1-\varepsilon_{n,t}} dj \\ &= (1-\theta)\left(P_{n,t}^*\right)^{1-\varepsilon_{n,t}} + \theta(1+\pi_{n,t-1})^{\varrho_n(1-\varepsilon_{n,t})}(1+\pi_n)^{(1-\varrho_n)(1-\varepsilon_{n,t})} P_{n,t-1}^{1-\varepsilon_{n,t}}, \end{aligned}$$

Alternatively,

$$1 = (1-\theta)\left(\frac{P_{n,t}^*}{P_{n,t}}\right)^{1-\varepsilon_{n,t}} + \theta\left((1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}(1+\pi_{n,t})^{-1}\right)^{1-\varepsilon_{n,t}}.$$

Firms that are able to reset their price optimally choose a price, $P_{n,t}^*$, that satisfies,

$$\begin{aligned} P_{n,t}^* = \arg\max E_t \sum_{k=0}^{\infty} \beta^{t+k} \frac{\Lambda_{n,t+k}}{\Lambda_{n,t}} \theta^k & \left[(1+\tau_n) \left(\frac{P_{n,t}^* \prod_{s=1}^{k} (1+\pi_{n,t+s-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}}{P_{n,t+k}} \right)^{1-\varepsilon_{n,t+k}} \right. \\ & \left. - mc_{n,t+k} \left(\frac{P_{n,t}^* \prod_{s=1}^{k} (1+\pi_{n,t+s-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}}{P_{n,t+k}} \right)^{-\varepsilon_{n,t+k}} \right] \mathcal{C}_{n,t+k}, \end{aligned}$$

where $\beta\frac{\Lambda_{n,t+k}}{\Lambda_{n,t}}$ is the discount factor from t to $t+k$. Alternatively,

$$P_{n,t}^* = \frac{1}{1+\tau_n} \frac{E_t \sum_{k=0}^{\infty} \beta^{t+k} \frac{\Lambda_{n,t+k}}{\Lambda_{n,t}} \theta^k \varepsilon_{n,t+k} \left(\frac{P_{n,t+k}}{\prod_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}} \right)^{\varepsilon_{n,t+k}} mc_{n,t+k}\mathcal{C}_{n,t+k}}{E_t \sum_{k=0}^{\infty} \beta^{t+k} \frac{\Lambda_{n,t+k}}{\Lambda_{n,t}} \theta^k (\varepsilon_{n,t+k}-1) \left(\frac{P_{n,t+k}}{\prod_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}} \right)^{\varepsilon_{n,t+k}-1} \mathcal{C}_{n,t+k}}.$$

Substitute for Λ_t and multiply the expression by $1/P_{n,t}$ to obtain,

$$p_t^* = \frac{1}{1+\tau_n} \frac{E_t \sum_{k=0}^{\infty} \beta^k \theta^k \varepsilon_{n,t+k} \left(\frac{P_{n,t+k}}{P_{n,t}\prod_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}} \right)^{\varepsilon_{n,+k}-1} mc_{n,t+k}\mathcal{C}_{n,t+k} C_{n,t+k}^{-\sigma} A_{n,t+k}^{\sigma-1}}{E_t \sum_{k=0}^{\infty} \beta^k \theta^k (\varepsilon_{n,t+k}-1) \left(\frac{P_{n,t+k}}{P_{n,t}\prod_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}} \right)^{\varepsilon_{n,t+k}-2} \mathcal{C}_{n,t+k} C_{n,t+k}^{-\sigma} A_{n,t+k}^{\sigma-1}},$$

where $p_{n,t}^* = \frac{P_{n,t}^*}{P_{n,t}}$ and $\mu_{n,t} = \frac{\varepsilon_{n,t}}{\varepsilon_{n,t}-1}$ is a time-varying markup.

Traded Intermediate Goods: Inputs, $Q_{n,t}(j)$, in the production of sticky-price goods, $y_{n,t}(j)$, in country n consist of different varieties of traded goods that are potentially imported from different countries. In any sticky-price sector, j, $Q_{n,t}(j)$ is produced using the same technology. In particular, abstracting from the j index,

$$Q_{n,t} = \left[\int \left(\sum_{n'} Q_{nn',t}(\mathbf{z}) \right)^{\frac{\gamma-1}{\gamma}} d\Phi(\mathbf{z}) \right]^{\frac{\gamma}{\gamma-1}},$$

where $Q_{nn',t}(\mathbf{z})$ denotes a second layer of intermediate goods of different varieties, $\mathbf{z}$, produced in country n', and where the elements of $\mathbf{z}$ are jointly drawn from a Fréchet distribution, $\Phi(\mathbf{z})$, with shape parameter φ. Firms that produce inputs for the production of sticky-price goods solve,

$$\max P^Q_{n,t} Q_{n,t} - \sum_{n'} \int \kappa_{nn'} \mathcal{E}_{nn',t} p_{n',t}(\mathbf{z}) Q_{nn',t}(\mathbf{z}) d\Phi(\mathbf{z}).$$

where $p_{n',t}(\mathbf{z})$ is the price of traded variety $\mathbf{z}$ produced in country n', quoted in terms of the local currency, and $\mathcal{E}_{nn',t}$ is the nominal exchange rate that converts prices in n''s currency to n's currency.[3] Goods imported by country n from country n' are subject to iceberg shipping costs such that only a fraction $1/\kappa_{nn'} < 1$ of those goods make it to their destination.

Let $Q_{n,t}(\mathbf{z}) = \sum_{n'} Q_{nn',t}(\mathbf{z})$ be the total demand for traded variety $\mathbf{z}$ in country n. Then, we have that

$$Q_{nn',t}(\mathbf{z}) = \begin{cases} Q_{n,t}(\mathbf{z}) & \text{if } \kappa_{nn'} \mathcal{E}_{nn',t} p_{n',t}(\mathbf{z}) < \min_{n'', n'' \neq n'} \kappa_{nn''} \mathcal{E}_{nn'',t} p_{n'',t}(\mathbf{z}) \\ 0 & \text{otherwise} \end{cases}$$

Intermediate goods of a given traded variety $\mathbf{z}$ are produced in country n with the technology,

$$q_{n,t}(\mathbf{z}) = z_{n,t} \ell_{n,t}(\mathbf{z}),$$

where $\ell_{n,t}(\mathbf{z})$ denotes labor used in the production of variety $\mathbf{z}$ in country n. The lower-case $q_{n,t}(\mathbf{z})$ is used to denote the output of intermediate varieties of type $\mathbf{z}$, rather than its demand for it, the difference being what is lost in transit because of iceberg shipping costs. That is,

$$q_{n,t}(\mathbf{z}) = \sum_{n'} \kappa_{n'n} Q_{n'n,t}(\mathbf{z}).$$

Constant returns to scale and competitive pricing implies that the price of traded variety $\mathbf{z}$ in country n is equal to its marginal cost[4],

[3] Given our definition of nominal exchange rates, no-arbitrage in currency markets implies that $\mathcal{E}_{nn',t} = \mathcal{E}_{nn'',t} \mathcal{E}_{n''n',t}$.

[4] This expression holds up to a scaling constant, $1/(\gamma^{\gamma}(1-\gamma)^{1-\gamma})$, that is unimportant for our purposes and that we leave out.

$$p_{n,t}(\mathbf{z}) = \frac{W_{n,t}}{z_{n,t}}.$$

In units of the local currency, the price paid for traded variety $\mathbf{z}$ by country n producers of inputs used in the production of sticky-price goods is given by,

$$P_{n,t}(\mathbf{z}) = \min_{n'} \left\{ \kappa_{nn'} \mathcal{E}_{nn',t} p_{n',t}(\mathbf{z}) \right\} = \min_{n'} \left\{ \frac{\kappa_{nn'} \mathcal{E}_{nn',t} W_{n',t}}{z_{n',t}} \right\}$$

Following the derivations described in Eaton and Kortum (2002), it follows that[5]

$$Q_{n,t}(\mathbf{z}) = \left(\frac{P_{n,t}(\mathbf{z})}{P^Q_{n,t}} \right)^{-\gamma} Q_{n,t},$$

where

$$\begin{aligned} P^Q_{n,t} &= \left[\int P_{n,t}(\mathbf{z})^{1-\gamma} d\Phi(\mathbf{z}) \right]^{\frac{1}{1-\gamma}} \\ &= \Gamma\left(\xi\right)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\kappa_{nn'} \mathcal{E}_{nn',t} W_{n',t} \right)^{-\varphi} \right)^{-\frac{1}{\varphi}}. \end{aligned}$$

Let $X_{nn',t}$ denote expenditures by country n on intermediate varieties from country n', $X_{nn',t} = \int \mathcal{E}_{nn',t} p_{n',t}(\mathbf{z}) \kappa_{nn'} Q_{nn',t}(\mathbf{z}) d\Phi(\mathbf{z})$, and $X_{n,t}$ country n's total expenditures on varieties. Then, letting $\varpi_{nn',t}$ denote the share of country n's expenditures on varieties from country n', we have that under the maintained assumptions,[6]

$$\varpi_{nn',t} = \frac{X_{nn',t}}{X_{n,t}} = \frac{\left(\kappa_{nn'} \mathcal{E}_{nn',t} W_{n',t} \right)^{-\varphi}}{\sum_{n''} \left(\kappa_{nn''} \mathcal{E}_{nn'',t} W_{n'',t} \right)^{-\varphi}}.$$

It also follows that the value of traded varieties produced by country n (in units of the local currency) is the sum of its exports to other countries (and to itself), $\sum_{n'} \mathcal{E}_{nn',t} X_{n'n,t}$. Moreover,

$$\begin{aligned} \sum_{n'} \mathcal{E}_{nn',t} X_{n'n,t} &= \sum_{n'} \int \mathcal{E}_{nn',t} \mathcal{E}_{n'n,t} p_{n,t}(\mathbf{z}) \kappa_{n'n} Q_{n'n,t}(\mathbf{z}) d\Phi(\mathbf{z}) \\ &= \int p_{n,t}(\mathbf{z}) q_{n,t}(\mathbf{z}) d\Phi(\mathbf{z}) \\ &= W_{n,t} \int \ell_{n,t}(\mathbf{z}) d\Phi(\mathbf{z}) = W_{n,t} L_{n,t}. \end{aligned}$$

[5] See Section I for explicit derivations.
[6] See Section I for explicit derivations.

Said differently, the sum of the demand from various countries for country n's varieties, in units of country n's currency, must equal the revenue of their producers, $\int p_{n,t}(\mathbf{z})q_{n,t}(\mathbf{z})d\Phi(\mathbf{z})$, which in turn must equal these producers' input costs (given constant returns and competitive markets), $W_{n,t}L_{n,t}$. Hence,

$$W_{n,t}L_{n,t} = \sum_{n'} \mathcal{E}_{nn',t}\varpi_{n'n,t}X_{n',t},$$

where $W_{n,t}L_{n,t}$ is then also the total value of intermediate varieties produced in and exported by country n.

Finally, sticky-price producers sell all of their output as final goods for local use. Therefore,

$$\Pi_{n,t} = P_{n,t}\mathcal{C}_{n,t} + \tau_n P_{n,t}\mathcal{C}_{n,t} - P^{Q}_{n,t}Q_{n,t}.$$

Here, $P^{Q}_{n,t}Q_{n,t}$ represents total expenditures by sticky-price producers on inputs, $Q_{n,t}(j) = Q_{n,t}\ \forall j$, and thus also the revenue of firms producing those inputs. Under competitive markets, this revenue is exhausted by expenditures on varieties imported from abroad to produce $Q_{n,t}$, $P^{Q}_{n,t}Q_{n,t} = X_{n,t}$. Then, it follows that,

$$X_{n,t} = P_{n,t}\mathcal{C}_{n,t} - (\Pi_{n,t} - \tau_n P_{n,t}\mathcal{C}_{n,t}) = MC_{n,t}\Delta_{n,t}\mathcal{C}_{n,t}$$

where $\Delta_{n,t} = \int_0^1 \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}}$ is a measure of relative price distortions in country n. Market clearing for country n goods may then be expressed as:[7]

$$\underbrace{W_{n,t}L_{n,t}}_{\text{Total Value of Intermediate Varieties Produced in } n} = \underbrace{\sum_{n'} \mathcal{E}_{nn',t}\varpi_{n'n,t}MC_{n',t}\Delta_{n',t}\mathcal{C}_{n',t}}_{\text{Total Foreign Demand for Intermediate Varieties from } n}.$$

Under a balanced government budget, lump-sum taxes exactly finance production subsidies to firms producing sticky-price goods, $T_{n,t} = \tau_n P_{n,t}\mathcal{C}_{n,t}$. The household budget constraint, therefore, reduces to

$$MC_{n,t}\Delta_{n,t}\mathcal{C}_{n,t} + \frac{1}{\mathcal{E}_{rn,t}}E_t\left[V_{t+1}S_{n,t+1}\right] = W_{n,t}L_{n,t} + \frac{1}{\mathcal{E}_{rn,t}}S_{n,t}.$$

In the remainder of this appendix, we let $\mathcal{E}_{n,t} \equiv \mathcal{E}_{rn,t}$ for notational convenience and, without loss of generality, take country 1 to be the reference country, $(r = 1)$. Thus, under no arbitrage in currency markets, $\mathcal{E}_{1,t} = \mathcal{E}_{r,t} = \mathcal{E}_{rr,t} = 1$.

[7] Multiplying both sides of the market clearing equation by $\mathcal{E}_{rn,t}$ and summing over n, we recover the world resource constraint,

$$\sum_n \mathcal{E}_{rn,t}W_{n,t}L_{n,t} = \sum_{n'} \mathcal{E}_{rn',t}MC_{n',t}\Delta_{n',t}\mathcal{C}_{n',t},$$

using the fact that $\sum_n \varpi_{n'n,t} = 1$.

B Model Summary

The model may be summarized by the following system of equations in $\mathcal{C}_{n,t}, C_{n,t}, S_{n,t}, W_{n,t}, L_{n,t}, P_{n,t}, P^*_{n,t}, \mathcal{E}_{n,t}, R_{n,t}, MC_{n,t}, \varpi_{nn',t}, \Delta_{n,t}$, and $\Lambda_{n,t}$.

Household Budget Constraint:

$$MC_{n,t}\Delta_{n,t}\mathcal{C}_{n,t} + \frac{1}{\mathcal{E}_{n,t}} E_t\left[V_{t+1}S_{n,t+1}\right] = W_{n,t}L_{n,t} + \frac{1}{\mathcal{E}_{n,t}} S_{n,t}.$$

Household Optimality Conditions:

– International Financial Securities, $S_{n,t+1}$:

$$\left(\frac{C_{n,t}}{A_{n,t}}\right)^{-\sigma_n} V_{t+1}\left[1+\Theta_n'\left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{n,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right] = \beta\frac{\mathcal{E}_{n,t}P_{n,t}A_{n,t}}{\mathcal{E}_{n,t+1}P_{n,t+1}A_{n,t+1}}\left(\frac{C_{n,t+1}}{A_{n,t+1}}\right)^{-\sigma_n}.$$

– Domestic Bonds, $B_{n,t}$:

$$\left(\frac{C_{n,t}}{A_{n,t}}\right)^{-\sigma_n} = \beta R_{n,t} E_t\left(\frac{C_{n,t+1}}{A_{n,t+1}}\right)^{-\sigma_n}\frac{P_{n,t}A_{n,t}}{P_{n,t+1}A_{n,t+1}}.$$

– Consumption, $C_{n,t}$:

$$(\frac{C_{n,t}}{A_{n,t}})^{-\sigma_n} = \Lambda_{n,t}P_{n,t}A_{n,t}.$$

– Labor Supply, $L_{n,t}$:

$$\omega_n L_{n,t}^{\eta_n}\left(\frac{C_{n,t}}{A_{n,t}}\right)^{\sigma_n} = \frac{W_{n,t}}{A_{n,t}P_{n,t}}.$$

Aggregate Price Index:

$$1 = (1-\theta)\left(\frac{P^*_{n,t}}{P_{n,t}}\right)^{1-\varepsilon_{n,t}} + \theta\left((1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}(1+\pi_{n,t})^{-1}\right)^{1-\varepsilon_{n,t}}.$$

Optimal Sticky Price:

$$p_t^* = \frac{1}{1+\tau_n}\frac{E_t\sum_{k=0}^{\infty}\beta^k\theta^k\varepsilon_{n,t+k}\left(\frac{P_{n,t+k}}{P_{n,t}\prod\limits_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}}\right)^{\varepsilon_{n,t+k}-1} mc_{n,t+k}\mathcal{C}_{n,t+k}C_{n,t+k}^{-\sigma}A_{n,t+k}^{\sigma-1}}{E_t\sum_{k=0}^{\infty}\beta^k\theta^k(\varepsilon_{n,t+k}-1)\left(\frac{P_{n,t+k}}{P_{n,t}\prod\limits_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}}\right)^{\varepsilon_{n,t+k}-2}\mathcal{C}_{n,t+k}C_{n,t+k}^{-\sigma}A_{n,t+k}^{\sigma-1}}.$$

Trade Shares:

$$\varpi_{nn',t} = \frac{\left(\kappa_{nn'}\mathcal{E}_{n',t}W_{n',t}\right)^{-\varphi}}{\sum_{n''}\left(\kappa_{nn''}\mathcal{E}_{n'',t}W_{n'',t}\right)^{-\varphi}}.$$

Trade Flows:

$$\mathcal{E}_{n,t}W_{n,t}L_{n,t}=\sum_{n'}\mathcal{E}_{n',t}\varpi_{n'n,t}MC_{n',t}\Delta_{n',t}\mathcal{C}_{n',t}.$$

Domestic Absorption:

$$\mathcal{C}_{n,t}=C_{n,t}+A_{n,t}E_t\left[\Theta_n\left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{n,t}P_{n,t}A_{n,t}}+\frac{1}{\tau}\xi_{n,t}\right)\right].$$

Marginal Cost of Production for Sticky Price Goods:

$$MC_{n,t}=\frac{P^Q_{n,t}}{A_{n,t}}=\frac{1}{A_{n,t}}\Gamma\left(\xi\right)^{\frac{1}{1-\gamma}}\left(\sum_{n'}\left(\kappa_{nn'}\frac{\mathcal{E}_{n',t}}{\mathcal{E}_{n,t}}W_{n',t}\right)^{-\varphi}\right)^{-\frac{1}{\varphi}}.$$

No Arbitrage in Currency Markets:

$$\mathcal{E}_{1,t}=1.$$

Definition of $\Delta_{n,t}$:

$$\Delta_{n,t}=(1-\theta)\left(\frac{P^*_{n,t}}{P_{n,t}}\right)^{-\varepsilon_{n,t}}+\theta\left((1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}(1+\pi_{n,t})^{-1}\right)^{-\varepsilon_{n,t}}.$$

Monetary Policy Rule:

$$R_{n,t}=R^{\rho_n}_{n,t-1}R^{*1-\rho_n}_{n,t}e^{\nu_{n,t}}\text{ where }R^*_{n,t}=R_n\left(\frac{1+\pi_{n,t}}{1+\pi_n}\right)^{\phi_{n,\pi}}\left(\frac{Y_{n,t}}{\overline{Y}_{n,t}}\right)^{\phi_{n,Y}}.$$

The objective from here on is to deflate this system of equations and express it in detrended form with a well defined steady state around which it may be linearized. It is straightforward to show that in a world steady state where inflation in country n is $1+\pi_n$, $\Delta_{n,t}=1$. Moreover, in a linearization around that steady state, $\widehat{\Delta}_{n,t}=0$ where the 'hat' notation denotes 'percent deviation from steady state.' Hence, in the remainder of the analysis, the equation defining $\Delta_{n,t}$ is not needed and, without loss of generality with respect to the model linearization, we set $\Delta_{n,t}=1$.

B.1 Deflated system

To deflate the system of equation described in the previous section, we define the following variables: real wages, $w_{n,t}=W_{n,t}/P_{n,t}$, real marginal cost, $mc_{n,t}=MC_{n,t}/P_{n,t}$, the quantity of financial securities held by country n in real terms (i.e., units of country 1 final goods), $s_{n,t}=\frac{S_{n,t}}{P_{1,t}}$, real exchange rates that convert real final goods in country n into units of country 1's final goods, $e_{n,t}=\mathcal{E}_{n,t}\frac{P_{n,t}}{P_{1,t}}$, and $\lambda_{n,t}=\Lambda_{n,t}P_{n,t}$. In addition, $1+\pi_{n,t}=P_{n,t}/P_{n,t-1}$ as defined above.

The system of equations becomes:

Household Budget Constraint:

$$mc_{n,t}\mathcal{C}_{n,t} + \frac{1}{e_{n,t}} E_t\left[(1+\pi_{1,t+1})V_{t+1}s_{n,t+1}\right] = w_{n,t}L_{n,t} + \frac{1}{e_{n,t}} s_{n,t}.$$

Household Optimality Conditions:

– International Financial Securities, $S_{n,t+1}$:

$$\left(\frac{C_{n,t}}{A_{n,t}}\right)^{-\sigma_n} V_{t+1}\left[1+\Theta_n'\left(\frac{V_{t+1}s_{n,t+1}(1+\pi_{1,t+1})}{e_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right]$$
$$= \beta \frac{e_{n,t}A_{n,t}}{e_{n,t+1}(1+\pi_{1,t+1})A_{n,t+1}} \left(\frac{C_{n,t+1}}{A_{n,t+1}}\right)^{-\sigma_n}.$$

– Domestic Bonds, $B_{n,t}$:

$$\left(\frac{C_{n,t}}{A_{n,t}}\right)^{-\sigma_n} = \beta R_{n,t} E_t \left(\frac{C_{n,t+1}}{A_{n,t+1}}\right)^{-\sigma_n} \frac{A_{n,t}}{(1+\pi_{n,t+1})A_{n,t+1}}.$$

– Consumption, $C_{n,t}$:

$$(\frac{C_{n,t}}{A_{n,t}})^{-\sigma_n} = \lambda_{n,t}A_{n,t}.$$

– Labor Supply, $L_{n,t}$:

$$\omega_n L_{n,t}^{\eta} \left(\frac{C_{n,t}}{A_{n,t}}\right)^{\sigma_n} = \frac{w_{n,t}}{A_{n,t}}.$$

Aggregate Price Index:

$$1 = (1-\theta)\left(p_{n,t}^{*}\right)^{1-\varepsilon_{n,t}} + \theta\left((1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}(1+\pi_{n,t})^{-1}\right)^{1-\varepsilon_{n,t}}.$$

Optimal Sticky Price:

The equation to be deflated is:

$$p_t^{*} = \frac{1}{1+\tau_n} \frac{E_t \sum_{k=0}^{\infty} \beta^k \theta^k \varepsilon_{n,t+k} \left(\frac{P_{n,t+k}}{P_{n,t}\prod\limits_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}}\right)^{\varepsilon_{n,t+k}-1} mc_{n,t+k}\mathcal{C}_{n,t+k}C_{n,t+k}^{-\sigma}A_{n,t+k}^{\sigma-1}}{E_t \sum_{k=0}^{\infty} \beta^k \theta^k (\varepsilon_{n,t+k}-1) \left(\frac{P_{n,t+k}}{P_{n,t}\prod\limits_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}}\right)^{\varepsilon_{n,t+k}-2} \mathcal{C}_{n,t+k}C_{n,t+k}^{-\sigma}A_{n,t+k}^{\sigma-1}}.$$

This yields:

$$p_t^* = \frac{1}{1+\tau_n}$$

$$\frac{E_t \sum_{k=0}^{\infty} \beta^k \theta^k \varepsilon_{n,t+k} (\prod_{s=1}^{k} (1+\pi_{n,t+s}))^{\varepsilon_{n,t+k}-1} \left(\frac{1}{\prod_{s=1}^{k} (1+\pi_{n,t+s-1})^{\varrho} (1+\pi_n)^{1-\varrho}} \right)^{\varepsilon_{n,t+k}-1} mc_{n,t+k} \mathcal{C}_{n,t+k} C_{n,t+k}^{-\sigma} A_{n,t+k}^{\sigma-1}}{E_t \sum_{k=0}^{\infty} \beta^k \theta^k (\varepsilon_{n,t+k}-1) (\prod_{s=1}^{k} (1+\pi_{n,t+s}))^{\varepsilon_{n,t+k}-1} \left(\frac{1}{\prod_{s=1}^{k} (1+\pi_{n,t+s-1})^{\varrho} (1+\pi_n)^{1-\varrho}} \right)^{\varepsilon_{n,t+k}-2} \mathcal{C}_{n,t+k} C_{n,t+k}^{-\sigma} A_{n,t+k}^{\sigma-1}}.$$

Trade Shares:

$$\varpi_{nn',t} = \frac{\left(\kappa_{nn'} e_{n',t} w_{n',t}\right)^{-\varphi}}{\sum_{n''} \left(\kappa_{nn''} e_{n'',t} w_{n'',t}\right)^{-\varphi}}.$$

Trade Flows:

$$e_{n,t} w_{n,t} L_{n,t} = \sum_{n'} e_{n',t} \varpi_{n'n,t} mc_{n',t} \mathcal{C}_{n',t}.$$

Domestic absorption:

$$\mathcal{C}_{n,t} = C_{n,t} + A_{n,t} E_t \left[\Theta_n \left(\frac{V_{t+1} s_{n,t+1} (1+\pi_{1,t+1})}{e_{n,t} A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right].$$

Marginal Cost of Production for Sticky Price Goods:

$$mc_{n,t} = \frac{1}{A_{n,t}} \Gamma(\xi)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\kappa_{nn'} \frac{e_{n',t}}{e_{n,t}} w_{n',t} \right)^{-\varphi} \right)^{-\frac{1}{\varphi}}.$$

No Arbitrage:

$$e_{1,t} = \mathcal{E}_{1,t} \frac{P_{1,t}}{P_{1,t}} = 1.$$

Monetary Policy Rule:

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^* = R_n \left(\frac{1+\pi_{n,t}}{1+\pi_n} \right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\overline{Y}_{n,t}} \right)^{\phi_{n,Y}}.$$

B.2 Detrended System

In each country n, $g_n = \frac{A_{n,t}}{A_{n,t-1}}$ $\forall t$ defines the rate of technical progress along a non-stochastic steady state growth path. We assume that each country evolves along its own balanced growth path in the long-run. Thus, in each country n, consumption, real wages and the real price of traded intermediate varieties all grow at rate g_n along its steady state growth path while, under the maintained preferences and technologies, labor supply and the real marginal cost of sticky-price goods are constant. In addition, we study the case where no country comes to asymptotically dominate or become insignificant as a consumer of another country's output. Said differently, the world as a whole features balanced growth in the long run.

For any variable $x_{n,t}$ that grows at rate g_n along country n's balanced growth path, we define its detrended counterpart as $\widetilde{x}_{n,t} = \frac{x_{n,t}}{A_{n,t}}$. For example, detrended wages in country n are defined as $\widetilde{w}_{n,t} = w_{n,t}/A_{n,t}$. These normalized variables will then be constant for each country in their detrended steady state. Similarly, shadow prices, $\lambda_{n,t}$, grow at rate $\frac{1}{g_n}$ so that we define $\widetilde{\lambda}_{n,t} = \lambda_{n,t}A_{n,t}$. As different countries may grow at different rates in the long-run, world balanced growth requires that the real exchange rate, $e_{n,t}$, grow at a rate that reflects the difference in technical progress between country n and the reference country in the long run. Specifically, we define the detrended real exchange rate, $\widetilde{e}_{n,t} = \frac{e_{n,t}A_{n,t}}{A_{1,t}}$, which will be constant along each country n's balanced growth path.

In detrended form, the system becomes:

Household Budget Constraint:

$$mc_{n,t}\widetilde{C}_{n,t} + \frac{1}{\widetilde{e}_{n,t}} E_t\left[(1+\pi_{1,t+1})V_{t+1}\widetilde{s}_{n,t+1}g_{1,t+1}\right] = \widetilde{w}_{n,t}L_{n,t} + \frac{1}{\widetilde{e}_{n,t}}\widetilde{s}_{n,t}.$$

Household Optimality Conditions:

– International Financial Securities, $S_{n,t+1}$:

$$\begin{aligned}\left(\widetilde{C}_{n,t}\right)^{-\sigma_n} V_{t+1}\left[1+\Theta_n'\left(\frac{V_{t+1}\widetilde{s}_{n,t+1}g_{1,t+1}(1+\pi_{1,t+1})}{\widetilde{e}_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right]& \\ = \beta\frac{\widetilde{e}_{n,t}}{\widetilde{e}_{n,t+1}g_{1,t+1}(1+\pi_{1,t+1})}\left(\widetilde{C}_{n,t+1}\right)^{-\sigma_n}&.\end{aligned}$$

– Domestic Bonds, $B_{n,t}$:

$$\left(\widetilde{C}_{n,t}\right)^{-\sigma_n} = \beta R_{n,t}E_t\left(\widetilde{C}_{n,t+1}\right)^{-\sigma_n}\frac{1}{(1+\pi_{n,t+1})g_{n,t+1}}.$$

– Consumption, $C_{n,t}$:

$$(\widetilde{C}_{n,t})^{-\sigma_n} = \widetilde{\lambda}_{n,t}.$$

– Labor Supply, $L_{n,t}$:

$$\omega L_{n,t}^{\eta}\widetilde{C}_{n,t}^{\sigma_n} = \widetilde{w}_{n,t}.$$

Aggregate Price Index:

$$1 = (1-\theta)\left(p_{n,t}^*\right)^{1-\varepsilon_{n,t}} + \theta\left((1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}(1+\pi_{n,t})^{-1}\right)^{1-\varepsilon_{n,t}}.$$

Optimal Sticky Price:

$$p_t^* = \frac{1}{1+\tau_n}$$

$$\frac{E_t \sum_{k=0}^{\infty} \beta^k \theta^k \varepsilon_{n,t+k} (\prod_{s=1}^{k}(1+\pi_{n,t+s}))^{\varepsilon_{n,t+k}-1} \left(\frac{1}{\prod\limits_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}}\right)^{\varepsilon_{n,t+k}-1} mc_{n,t+k} \widetilde{C}_{n,t+k}^{-\sigma_n} \widetilde{\mathcal{C}}_{n,t+k}}{E_t \sum_{k=0}^{\infty} \beta^k \theta^k (\varepsilon_{n,t+k}-1) (\prod_{s=1}^{k}(1+\pi_{n,t+s}))^{\varepsilon_{n,t+k}-1} \left(\frac{1}{\prod\limits_{s=1}^{k}(1+\pi_{n,t+s-1})^{\varrho}(1+\pi_n)^{1-\varrho}}\right)^{\varepsilon_{n,t+k}-2} \widetilde{C}_{n,t+k}^{-\sigma_n} \widetilde{\mathcal{C}}_{n,t+k}}.$$

Trade Shares:

$$\varpi_{nn',t} = \frac{\left(\widetilde{\kappa}_{nn'} \widetilde{e}_{n',t} \widetilde{w}_{n',t}\right)^{-\varphi}}{\sum_{n''} \left(\widetilde{\kappa}_{nn''} \widetilde{e}_{n'',t} \widetilde{w}_{n'',t}\right)^{-\varphi}}.$$

Trade Flows:

$$\widetilde{e}_{n,t} \widetilde{w}_{n,t} L_{n,t} = \sum_{n'} \widetilde{e}_{n',t} \varpi_{n'n,t} mc_{n',t} \widetilde{\mathcal{C}}_{n',t}.$$

Domestic Absorption:

$$\widetilde{\mathcal{C}}_{n,t} = \widetilde{C}_{n,t} + E_t \left[\Theta_n \left(\frac{V_{t+1} \widetilde{s}_{n,t+1} g_{1,t+1} (1+\pi_{1,t+1})}{\widetilde{e}_{n,t}} + \frac{1}{\tau} \xi_{n,t}\right)\right].$$

Marginal Cost of Production for Sticky Price Goods:

$$mc_{n,t} = \Gamma(\xi)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\widetilde{\kappa}_{nn'} \frac{\widetilde{e}_{n',t}}{\widetilde{e}_{n,t}} \widetilde{w}_{n',t}\right)^{-\varphi}\right)^{-\frac{1}{\varphi}}.$$

No Arbitrage:

$$\widetilde{e}_{1,t} = \frac{e_{1,t} A_{1,t}}{A_{1,t}} = 1.$$

Monetary Policy Rule:

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^* = R_n \left(\frac{1+\pi_{n,t}}{1+\pi_n}\right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\overline{Y}_{n,t}}\right)^{\phi_{n,Y}}.$$

C Steady-State

From here on, we omit the 'tildes' over the variables for notational convenience under the understanding that our final system of equations is expressed in deflated and detrended form.

Our modeling of portfolio adjustment costs does not provide a theory of countries' long-run net asset positions. As such, we proceed as in Eaton and Kortum (2002) and adopt as a benchmark a steady state with balanced trade (one that countries haven't yet reached). In practice, however, countries' net asset positions have switched from positive to negative or vice versa over our sample period, 2004Q2-2019Q4. Therefore, to carry out quantitative assessments of the effects of productivity or other disturbances, we recover counterfactual steady state allocations corresponding to balanced trade from a world in which imbalances exist even over long periods. That is, we adapt the approach first developed in Dekle, Eaton, and Kortum (2007) to the steady state of this dynamic setting. Specifically, we first write a variant of the economic environment whose steady state can be matched to observed average regional trade imbalances over our sample. This can be done by setting $\xi_n \neq 0$.[8] Then, for any implied set of parameters, we calculate counterfactual steady state allocations and prices consistent with a world without trade imbalances. Put another way, for any set of parameters, we compute counterfactual steady state allocations consistent with $\xi_n = 0 \ \forall n$ in which case real net financial assets are zero in every country, $s_n = \frac{S_n}{P_1} = 0 \ \forall n$.[9] The resulting counterfactual world steady state without imbalances is then used to produce a linearized model amenable to Bayesian estimation and to assessing the effects of productivity and other disturbances.

C.1 Steady State Equations with Unbalanced Trade

The unknowns are: $\mathcal{C}_n, s_n, C_n, mc_n, L_n, w_n, e_n, V, r_n, \lambda_n, p_n^*, \varpi_{nn'}$. The steady state equations are:

Household Budget constraint:

$$mc_n \mathcal{C}_n = w_n L_n + (1 - (1 + \pi_1) V g_1) \frac{1}{e_n} s_n.$$

Household Optimality Conditions:

– International Financial Securities, $S_{n,t+1}$:

$$V \left(1 + \Theta_n' \left(\frac{V s_n g_1 (1 + \pi_1)}{e_n} + \frac{1}{\tau} \xi_n \right) \right) = \beta \frac{1}{g_1 (1 + \pi_1)}.$$

– Domestic Bonds, $B_{n,t}$:

$$1 = \beta R_n \frac{1}{g_n (1 + \pi_n)} \Rightarrow R_n = \beta^{-1} g_n (1 + \pi_n).$$

– Consumption, $C_{n,t}$:

$$(C_n)^{-\sigma_n} = \lambda_n.$$

– Labor Supply, $L_{n,t}$:

$$\omega L_n^{\eta} C_n^{\sigma_n} = w_n.$$

Aggregate Price Index:

[8] The fact that $\xi_n \neq 0$ is exogenous captures the sense in which our modeling of portfolio adjustment costs provides no clear guidance regarding long-run net asset positions

[9] Recall that financial assets are denoted in units of the reference country's currency.

$$1 = (1-\theta)\left(p_n^*\right)^{1-\varepsilon_n} + \theta \Rightarrow p_n^* = 1.$$

Optimal Sticky Price:[10]

$$p_n^* = \frac{1}{1+\tau_n}\frac{\varepsilon_n}{\varepsilon_n - 1} mc_n \Rightarrow mc_n = (1+\tau_n)\frac{\varepsilon_n - 1}{\varepsilon_n}.$$

Trade Shares:

$$\varpi_{nn'} = \frac{\left(\kappa_{nn'} e_{n'} w_{n'}\right)^{-\varphi}}{\sum_{n''}\left(\kappa_{nn''} e_{n''} w_{n''}\right)^{-\varphi}}.$$

Trade Flows:

$$e_n w_n L_n = \sum_{n'} \varpi_{n'n} mc_{n'} e_{n'} \mathcal{C}_{n'} \Rightarrow e_n w_n L_n = \sum_{n'} (1+\tau_{n'})\frac{\varepsilon_{n'} - 1}{\varepsilon_{n'}} \varpi_{n'n} e_{n'} \mathcal{C}_{n'}.$$

Domestic Absorption:

$$\mathcal{C}_n = C_n.$$

Marginal Cost of Production for Sticky Price Goods:

$$e_n mc_n = \Gamma\left(\xi\right)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\kappa_{nn'} e_{n'} w_{n'}\right)^{-\varphi}\right)^{-\frac{1}{\varphi}}$$

$$\Rightarrow e_n \frac{\varepsilon_n - 1}{\varepsilon_n}(1+\tau_n) = \Gamma\left(\xi\right)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\kappa_{nn'} e_{n'} w_{n'}\right)^{-\varphi}\right)^{-\frac{1}{\varphi}}.$$

No Arbitrage:

$$e_1 = 1.$$

C.2 Algorithm for Steady State with Unbalanced Trade, $s_n \neq 0$

We solve for the steady-state (deflated-detrended) values of e_n, w_n, and s_n given parameters and calibrated values of nominal GDP, $\mathcal{E}_n W_n L_n$, employment, L_n and trade shares, $\varpi_{nn'}$. These calibrated values imply parameter values for ξ_n, ω_n and $\kappa_{nn'}$. For any country n, $\mathcal{E}_n W_n L_n = P_1 A_1 e_n w_n L_n$.

Nominal GDP, labor supply, and the ϖ matrix are calibrated to the 2004-2019 sample period. All other endogenous variables and parameters, namely ω, $\kappa_{nn'}$, and ξ_n, are output from this algorithm.

1. Optimality conditions for domestic bonds imply that $R_n = \beta^{-1} g_n (1+\pi_n)$. The aggregate

[10] In steady state, setting the subsidy to firms producing sticky-price goods to the inverse of the markup eliminates the monopolistic price distortion. Said differently, when $\tau_n = \frac{\varepsilon_n}{\varepsilon_n - 1} - 1$, $mc_n = 1$.

price index equation implies that $p_n^* = 1$, and the optimal sticky price equation implies that $mc_n = (1+\tau_n)\frac{\varepsilon_n - 1}{\varepsilon_n}$.

2. By assumption, $\Theta' = 0$ in steady state so that all optimality conditions for international financial securities hold so long as $V = \frac{\beta}{g_1(1+\pi_1)}$.

3. Combining the marginal cost of production for sticky price goods with the expression for trade shares, and imposing the normalization $\kappa_{nn} = 1$, we have

$$e_n mc_n = (\varpi_{nn})^{\frac{1}{\varphi}} e_n w_n \Rightarrow \frac{\varepsilon_n - 1}{\varepsilon_n}(1+\tau_n) = (\varpi_{nn})^{\frac{1}{\varphi}} w_n,$$

which we solve for w_n.

4. Given $\mathcal{E}_1 W L_1$ and L_1 in the data, as well as w_1 from the previous step, calculate $(P_1 A_1) = \frac{\mathcal{E}_1 W_1 L_1}{w_1 L_1}$ (using $e_1 = 1$).

5. Calculate $e_n = \frac{\mathcal{E}_n W_n L_n}{(P_1 A_1) w_n L_n}$.

6. From the trade-flow equations together with $\varpi_{nn'}$ and $e_n w_n L_n$, obtain $e_n mc_n \mathcal{C}_n$:

$$emc\mathcal{C} = (\varpi^T)^{-1} ewL$$

7. Find $\mathcal{C}_n$ from $\frac{e_n mc_n \mathcal{C}_n}{e_n mc_n}$.

8. From domestic absorption, $C_n = \mathcal{C}_n$.

9. From the household budget constraint, and substituting the steady-state values for V, solve for s_n: $s = ((emc\mathcal{C}) - ewL)/(1 - Vg_1(1+\pi_1))$.

10. Find ξ_n: from the equations for international financial securities, in order for $\Theta' = 0$ in steady-state, $\frac{1}{\tau}\xi_n = -\beta s_n / e_n$.

11. Determine ω_n using the labor supply equations: $\omega = L^{-\eta} C^{-\sigma} w$.

12. Determine $\kappa_{nn'}$ using the trade share equations and the normalization $\kappa_{nn} = 1$: $\kappa_{nn'} = \varpi_{nn'}^{-1/\varphi} e_n mc_n / e_{n'} w_{n'}$.

C.3 Algorithm for Counterfactual Steady State with Balanced Trade, $s_n = 0$

Conditional on the parameters either obtained or calibrated in Section C.2, $\{\beta, \sigma, \omega, \eta, \varepsilon, g_n, \varphi, \tau, \kappa_{nn'}\}$, we now follow Dekle et al. (2007) and solve for counterfactual allocations consistent with $\xi_n = 0$ which then implies $s_n = 0$. We arrive at a solution by guessing and iterating on a vector of prices.

1. We start with an initial guess for $(e_n w_n)^0$ (in this case, the solution in C.2 consistent with $\xi_n \neq 0$).

2. Using mc_n, we obtain the real (productivity adjusted) exchange rates, e_n^0, associated with $(e_n w_n)^0$ from:

$$e_n^0 mc_n = \Gamma(\xi)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\kappa_{nn'} (e_{n'} w_{n'})^0 \right)^{-\varphi} \right)^{-\frac{1}{\varphi}}.$$

3. Calculate w_n^0 using $w_n^0 = (e_n w_n)^0 / (e_n^0)$.

4. Given, $s_n = 0$, domestic absorption, $\mathcal{C}_n = C_n$, and the household budget constraint implies $mc_n C_n = w_n L_n$. Use this expression to substitute for C_n in the labor supply equation, $\omega_n L_n^\eta C_n^\sigma = w_n$, to obtain L_n^0 given w_n^0.

5. Calculate the new trade matrix, $\varpi_{nn'}^0$, using $\varpi_{nn'} = (\kappa_{nn'} e_{n'} w_{n'} / (e_n mc_n))^{-\varphi}$ evaluated at e_n^0 and $(e_n w_n)^0$.

6. Obtain a new value for $e_n w_n$, $(e_n w_n)^{\text{new}}$, using $(e_n w_n)^{\text{new}} = \sum_{n'} \varpi_{n'n}^0 ((e_{n'} w_{n'})^0 (L_{n'}^0 / L_n^0)$, and imposing the normalization $(e_1 w_1)^{\text{new}} = w_1^0$. Use $(e_n w_n)^{\text{new}}$ to update the initial guess, $(e_n w_n)^0$, using an adjustment factor, $\delta \in (0, 1]$, such that $(e_n w_n)^1 = (1 - \delta)(e_n w_n)^0 + \delta(e_n w_n)^{\text{new}}$ and iterate until convergence.

D Linearized System

We now log-linearize the (deflated-detrended) system around its steady-state with balanced trade. We use a 'hat' over variables to denote log-deviations from steady-state (i.e., $\widehat{x}_{n,t} = \log(x_{n,t}) - \log(x_n)$, and dx to denote level deviations from steady state for some variables, x, (i.e., $dx_{n,t} = x_{n,t} - x_n$).

D.1 Derivation of Country-Specific New Keynesian Phillips Curves

We can write

$$p_{n,t}^* = \frac{1}{1 + \tau_n} \frac{N_{n,t}}{D_{n,t}},$$

where

$$N_{n,t} = mc_{n,t} \varepsilon_{n,t} (\frac{C_{n,t}}{A_{n,t}})^{-\sigma_n} (\frac{\mathcal{C}_{n,t}}{A_{n,t}}) + \beta\theta E_t (\frac{P_{n,t+1}}{(1 + \pi_t)^{\varrho_n} (1 + \pi)^{1-\varrho_n} P_{n,t}})^{\varepsilon_{n,t}-1} N_{n,t+1}$$

and

$$D_{n,t} = (\varepsilon_{n,t} - 1)(\frac{C_{n,t}}{A_{n,t}})^{-\sigma_n} (\frac{\mathcal{C}_{n,t}}{A_{n,t}}) + \beta\theta E_t (\frac{P_{n,t+1}}{(1 + \pi_t)^{\varrho_n} (1 + \pi)^{1-\varrho_n} P_{n,t}})^{\varepsilon_{n,t}-2} D_{n,t+1}.$$

In steady-state we have,

$$N_n = \frac{mc_n \varepsilon_n (\tilde{C}_n)^{-\sigma_n} (\tilde{\mathcal{C}}_n)}{1 - \beta\theta},$$

and

$$D_n = \frac{(\varepsilon_n - 1)(\tilde{C}_n)^{-\sigma_n}(\tilde{\mathcal{C}}_n)}{1-\beta\theta}.$$

Linearizing $N_{n,t}$ around the steady state gives

$$\begin{aligned} dN_{n,t} = &-\sigma_n mc_n \varepsilon_n (C_n)^{-\sigma_n - 1}(\mathcal{C}_n) d\tilde{C}_{n,t} + \varepsilon_n (C_n)^{-\sigma_n}(\mathcal{C}_n) dmc_{n,t} + mc_n (C_n)^{-\sigma_n}(\mathcal{C}_n) d\varepsilon_{n,t} \\ &+ mc_n \varepsilon_n (C_n)^{-\sigma_n} d\tilde{\mathcal{C}}_{n,t} + \beta\theta E_t[dN_{n,t+1}] + (\varepsilon_n - 1)\beta\theta N E_t[\widehat{\pi}_{n,t+1}] - (\varepsilon_n - 1)\varrho_n \beta\theta N \widehat{\pi}_{n,t}, \end{aligned}$$

where the latter half of the term associated with $d\varepsilon_t$ drops out since $\log\left(\frac{P_{n,t+1}}{(1+\pi_t)^{\varrho_n}(1+\pi)^{1-\varrho_n}P_{n,t}}\right) = 0$ in steady state. We re-write this expression in terms of percent-deviations from steady state,

$$\begin{aligned} \widehat{N}_{n,t} = &\frac{mc_n \varepsilon_n (C_n)^{-\sigma_n}(\mathcal{C}_n)}{N_n}(-\sigma_n \widehat{C}_{n,t} + \widehat{mc}_{n,t} + \widehat{\mathcal{C}}_{n,t} + \widehat{\varepsilon}_{n,t}) \\ &+ \beta\theta E_t[\widehat{N}_{n,t+1}] + (\varepsilon_n - 1)\beta\theta E_t[\widehat{\pi}_{n,t+1}] - (\varepsilon_n - 1)\varrho_n \beta\theta \widehat{\pi}_{n,t}. \end{aligned}$$

Similarly, linearizing $D_{n,t}$ around the steady state gives, in percent-deviations form,

$$\begin{aligned} \widehat{D}_{n,t} = &\frac{(\varepsilon_n - 1)(C_n)^{-\sigma_n}(\mathcal{C}_n)}{D_n}(-\sigma_n \widehat{C}_{n,t} + \widehat{\mathcal{C}}_{n,t} + \frac{\varepsilon_n}{\varepsilon_n - 1}\widehat{\varepsilon}_{n,t}) \\ &+ \beta\theta E_t[\widehat{D}_{n,t+1}] + (\varepsilon_n - 2)\beta\theta E_t[\widehat{\pi}_{n,t+1}] - (\varepsilon_n - 2)\varrho_n \beta\theta \widehat{\pi}_{n,t}. \end{aligned}$$

The optimal price for firms able to reset their prices implies

$$\widehat{p}^*_{n,t} = \widehat{N}_{n,t} - \widehat{D}_{n,t},$$

$$N_n = mc_n \frac{\varepsilon_n}{\varepsilon_n - 1} D_n,$$

and

$$(C_n^{-\sigma_n}\mathcal{C}_n)/D_n = \frac{1-\beta\theta}{\varepsilon_n - 1}.$$

Substituting these expressions for $\widehat{p}^*_{n,t}$, we obtain

$$\begin{aligned} \widehat{p}^*_{n,t} = &(1-\beta\theta)(-\sigma_n \widehat{C}_{n,t} + \widehat{mc}_{n,t} + \widehat{\mathcal{C}}_{n,t} + \widehat{\varepsilon}_{n,t}) \\ &+ \beta\theta E_t[\widehat{N}_{n,t+1}] + (\varepsilon_n - 1)\beta\theta E_t[\widehat{\pi}_{n,t+1}] - (\varepsilon_n - 1)\varrho_n \beta\theta \widehat{\pi}_{n,t} \\ &- (1-\beta\theta)(-\sigma_n \widehat{C}_{n,t} + \widehat{\mathcal{C}}_{n,t} + \frac{\varepsilon_n}{\varepsilon_n - 1}\widehat{\varepsilon}_{n,t}) \\ &+ \beta\theta E_t[\widehat{D}_{n,t+1}] + (\varepsilon_n - 2)\beta\theta E_t[\widehat{\pi}_{n,t+1}] + (\varepsilon_n - 2)\varrho_n \beta\theta \widehat{\pi}_{n,t} \end{aligned}$$

which simplifies to

$$\widehat{p}^*_{n,t} = (1-\beta\theta)(\widehat{mc}_{n,t} - \frac{1}{\varepsilon_n - 1}\widehat{\varepsilon}_{n,t}) + \beta\theta E_t\left[\widehat{p}^*_{n,t+1} + \widehat{\pi}_{n,t+1} - \varrho_n \widehat{\pi}_{n,t}\right].$$

Linearizing the Aggregate Price Index equation around the steady-state gives

$$\widehat{p}^*_{n,t} = \left(\frac{\theta}{1-\theta}\right)\widehat{\pi}_{n,t} - \varrho_n\left(\frac{\theta}{1-\theta}\right)\widehat{\pi}_{n,t-1}.$$

Further, note that

$$\widehat{\mu}_{n,t} = \frac{-1}{\varepsilon_n - 1}\widehat{\varepsilon}_{n,t}.$$

This enables us to rewrite the Phillips Curve in terms of marginal cost, lagged inflation, and disturbances to the mark up,

$$\begin{aligned}\frac{\theta}{1-\theta}\widehat{\pi}_{n,t} - \varrho_n\left(\frac{\theta}{1-\theta}\right)\widehat{\pi}_{n,t-1} &= (1-\beta\theta)(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) \\ &\quad + \beta\theta E_t\left[\frac{\theta}{1-\theta}\widehat{\pi}_{n,t+1} - \varrho_n\left(\frac{\theta}{1-\theta}\right)\widehat{\pi}_{n,t} + \widehat{\pi}_{n,t+1} - \varrho_n\widehat{\pi}_{n,t}\right]\end{aligned}$$

which can be rewritten as,

$$\begin{aligned}\widehat{\pi}_{n,t} &= \frac{1-\theta}{\theta}(1-\beta\theta)(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) \\ &\quad + \beta(1-\theta)E_t\left[\frac{\theta}{1-\theta}\widehat{\pi}_{n,t+1} + \widehat{\pi}_{n,t+1} - \varrho_n\frac{\theta}{1-\theta}\widehat{\pi}_{n,t} - \varrho_n\widehat{\pi}_{n,t}\right] + \varrho_n\pi_{n,t-1},\end{aligned}$$

or

$$\begin{aligned}(1+\beta\varrho_n)\widehat{\pi}_{n,t} &= \frac{1-\theta}{\theta}(1-\beta\theta)(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) \\ &\quad + \beta(1-\theta)\left(1 + \frac{\theta}{1-\theta}\right)E_t[\widehat{\pi}_{n,t+1}] + \varrho_n\pi_{n,t-1}.\end{aligned}$$

This expression reduces to

$$\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta\varrho_n)}(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) + \frac{\beta}{(1+\beta\varrho_n)}E_t[\widehat{\pi}_{n,t+1}] + \frac{\varrho_n}{(1+\beta\varrho_n)}\pi_{n,t-1}.$$

For the quantitative application, we assume that nominal prices are indexed to trend inflation only and omit lagged inflation in the Phillips curve, $\varrho_n = 0$. The New Keynesian Phillips curve then becomes,

$$\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t[\widehat{\pi}_{n,t+1}].$$

D.2 Linearization

Household Budget Constraint:

$$\begin{aligned}mc_n\mathcal{C}_n e_n\left(\widehat{mc}_{n,t} + \widehat{\mathcal{C}}_{n,t} + \widehat{e}_{n,t}\right) + Vs_n g_1(1+\pi_1)E_t\left[\widehat{\pi}_{1,t+1} + \widehat{V}_{t+1} + \widehat{g}_{1,t+1}\right] \\ + Vg_1(1+\pi_1)E_t[ds_{n,t+1}] = e_n w_n L_n\left(\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t}\right) + ds_{n,t}.\end{aligned}$$

Household Optimality Conditions:

– International Financial Securities, $S_{n,t+1}$:

$$-\sigma_n \widehat{C}_{n,t-1} + \widehat{V}_t + \beta \frac{s_n}{e_n} \tau \left(\widehat{V}_t + \frac{1}{s_n} ds_{n,t} + \widehat{g}_{1,t} + \widehat{\pi}_{1,t} - \widehat{e}_{n,t-1} \right) + d\xi_{n,t-1}$$
$$= \widehat{e}_{n,t-1} - \widehat{e}_{n,t} - \widehat{g}_{1,t} - \widehat{\pi}_{1,t} - \sigma_n \widehat{C}_{n,t}.$$

– Domestic Bonds, $B_{n,t}$:

$$-\sigma_n \widehat{C}_{n,t} + \sigma_n E_t \widehat{C}_{n,t+1} = \widehat{R}_{n,t} - E_t \widehat{\pi}_{n,t+1} - E_t \widehat{g}_{n,t+1}.$$

– Consumption, $C_{n,t}$:

$$-\sigma_n \widehat{C}_{n,t} = \widehat{\lambda}_{n,t}.$$

– Labor Supply, $L_{n,t}$:

$$\eta \widehat{L}_{n,t} + \sigma_n \widehat{C}_{n,t} = \widehat{w}_{n,t}.$$

Phillips Curve (from the optimal pricing equation of firms able to reset prices in Section D.1):

$$\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta} (\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t[\widehat{\pi}_{n,t+1}],$$

where

$$\mu_{n,t} = \frac{\varepsilon_{n,t}}{\varepsilon_{n,t} - 1}.$$

Trade Shares:

$$\widehat{\varpi}_{nn',t} = -\varphi(\widehat{e}_{n',t} + \widehat{w}_{n',t}) + \varphi \sum_{n''} \varpi_{nn''} (\widehat{e}_{n'',t} + \widehat{w}_{n'',t}).$$

Trade Flows:

$$\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t} = \sum_{n'} \psi_{n'n} \left(\widehat{e}_{n',t} + \widehat{\varpi}_{n'n,t} + \widehat{mc}_{n',t} + \widehat{\mathcal{C}}_{n',t} \right),$$

where

$$\psi_{n',n} \equiv \frac{\varpi_{n'n} e_{n'} mc_{n'} \mathcal{C}_{n'}}{\sum_{n''} \varpi_{n''n} e_{n''} mc_{n''} \mathcal{C}_{n''}}.$$

Domestic Absorption:

$$\widehat{\mathcal{C}}_{n,t} = \widehat{C}_{n,t}.$$

Marginal Cost of Production for Sticky Price Goods:

$$\widehat{e}_{n,t} + \widehat{mc}_{n,t} = \sum_{n'} \varpi_{nn'}(\widehat{e}_{n',t} + \widehat{w}_{n',t}).$$

No Arbitrage:

$$\widehat{e}_{1,t} = 0.$$

Monetary Policy Rule:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)(\phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t) + \widehat{\nu}_t.$$

D.3 System Reduction

From the Domestic Absorption Equation, $\widehat{\mathcal{C}}_n = \widehat{C}_n$ and $\mathcal{C} = C$. Then, we have,

$$mc_n C_n e_n \left(\widehat{mc}_{n,t} + \widehat{C}_{n,t} + \widehat{e}_{n,t}\right) + V s_n g_1 (1+\pi_1) E_t \left[\widehat{\pi}_{1,t+1} + \widehat{V}_{t+1} + \widehat{g}_{1,t+1}\right]$$
$$+ V g_1 (1+\pi_1) E_t[ds_{n,t+1}] = e_n w_n L_n \left(\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t}\right) + ds_{n,t},$$

$$-\sigma_n \widehat{C}_{n,t-1} + \widehat{V}_t + \beta \frac{s_n}{e_n} \tau \left(\widehat{V}_t + \frac{1}{s_n} ds_{n,t} + \widehat{g}_{1,t} + \widehat{\pi}_{1,t} - \widehat{e}_{n,t-1}\right) + d\xi_{n,t-1}$$
$$= \widehat{e}_{n,t-1} - \widehat{e}_{n,t} - \widehat{g}_{1,t} - \widehat{\pi}_{1,t} - \sigma_n \widehat{C}_{n,t},$$

$$-\sigma_n \widehat{C}_{n,t} + \sigma_n E_t \widehat{C}_{n,t+1} = \widehat{R}_{n,t} - E_t \widehat{\pi}_{n,t+1} - E_t \widehat{g}_{n,t+1},$$

$$-\sigma_n \widehat{C}_{n,t} = \widehat{\lambda}_{n,t},$$

$$\eta \widehat{L}_{n,t} + \sigma_n \widehat{C}_{n,t} = \widehat{w}_{n,t},$$

$$\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t[\widehat{\pi}_{n,t+1}],$$

$$\widehat{\varpi}_{nn',t} = -\varphi(\widehat{e}_{n',t} + \widehat{w}_{n',t}) + \varphi \sum_{n''} \varpi_{nn''}(\widehat{e}_{n'',t} + \widehat{w}_{n'',t}),$$

$$\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t} = \sum_{n'} \psi_{n'n} \left(\widehat{e}_{n',t} + \widehat{\varpi}_{n'n,t} + \widehat{mc}_{n',t} + \widehat{C}_{n',t}\right),$$

$$\widehat{e}_{n,t} + \widehat{mc}_{n,t} = \sum_{n'} \varpi_{nn'}(\widehat{e}_{n',t} + \widehat{w}_{n',t}),$$

$$\widehat{e}_{1,t} = 0.$$

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)(\phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t) + \widehat{\nu}_t.$$

Combining the equations describing the evolution of trade shares and that of the marginal cost of sticky-price goods gives us $\widehat{\varpi}_{nn',t} = \varphi(\widehat{mc}_{n,t} + \widehat{e}_{n,t} - \widehat{w}_{n',t} - \widehat{e}_{n',t})$, which we can then use to substitute for trade shares, $\widehat{\varpi}_{nn'}$. It follows that the system reduces to

$$
\begin{aligned}
mc_n C_n e_n \left(\widehat{mc}_{n,t} + \widehat{C}_{n,t} + \widehat{e}_{n,t}\right) + V s_n g_1(1+\pi_1) E_t \left[\widehat{\pi}_{1,t+1} + \widehat{V}_{t+1} + \widehat{g}_{1,t+1}\right] \\
+ V g_1(1+\pi_1) E_t[ds_{n,t+1}] = e_n w_n L_n \left(\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t}\right) + ds_{n,t},
\end{aligned}
$$

$$
\begin{aligned}
-\sigma_n \widehat{C}_{n,t-1} + \widehat{V}_t + \beta \frac{s_n}{e_n} \tau \left(\widehat{V}_t + \frac{1}{s_n} ds_{n,t} + \widehat{g}_{1,t} + \widehat{\pi}_{1,t} - \widehat{e}_{n,t-1}\right) + d\xi_{n,t-1} \\
= \widehat{e}_{n,t-1} - \widehat{e}_{n,t} - \widehat{g}_{1,t} - \widehat{\pi}_{1,t} - \sigma_n \widehat{C}_{n,t},
\end{aligned}
$$

$$-\sigma_n \widehat{C}_{n,t} + \sigma_n E_t \widehat{C}_{n,t+1} = \widehat{R}_{n,t} - E_t \widehat{\pi}_{n,t+1} - E_t \widehat{g}_{n,t+1},$$

$$-\sigma_n \widehat{C}_{n,t} = \widehat{\lambda}_{n,t},$$

$$\eta \widehat{L}_{n,t} + \sigma_n \widehat{C}_{n,t} = \widehat{w}_{n,t},$$

$$\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta} (\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t[\widehat{\pi}_{n,t+1}],$$

$$\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t} = \sum_{n'} \psi_{n'n} \left(\widehat{e}_{n',t} + \varphi(\widehat{mc}_{n',t} + \widehat{e}_{n',t} - \widehat{w}_{n,t} - \widehat{e}_{n,t}) + \widehat{mc}_{n',t} + \widehat{C}_{n',t}\right),$$

$$\widehat{e}_{n,t} + \widehat{mc}_{n,t} = \sum_{n'} \varpi_{nn'} (\widehat{e}_{n',t} + \widehat{w}_{n',t}),$$

$$\widehat{e}_{1,t} = 0.$$

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)(\phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t) + \widehat{\nu}_t.$$

D.4 Final System of Linearized Equations

The final system of equations in vector form is given by (with some abuse of notation):

$$
\begin{aligned}
mcCe \left(\widehat{mc}_t + \widehat{C}_t + \widehat{e}_t\right) + V s g_1(1+\pi_1) E_t \left[\widehat{\pi}_{1,t+1} + \widehat{V}_{t+1} + \widehat{g}_{1,t+1}\right] + V g_1(1+\pi_1) E_t[ds_{t+1}] \\
= ewL \left(\widehat{e}_t + \widehat{w}_t + \widehat{L}_t\right) + ds_t, \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
-\sigma \widehat{C}_{t-1} + \widehat{V}_t + \beta \frac{\tau}{e} \left(ds_t + s\left(\widehat{V}_t + \widehat{g}_{1,t} + \widehat{\pi}_{1,t} - \widehat{e}_{t-1}\right)\right) + d\xi_{t-1} \\
= \widehat{e}_{t-1} - \widehat{e}_t - \widehat{g}_{1,t} - \widehat{\pi}_{1,t} - \sigma \widehat{C}_t, \qquad (4)
\end{aligned}
$$

$$-\sigma \widehat{C}_t + \sigma E_t \widehat{C}_{t+1} = \widehat{R}_t - E_t \widehat{\pi}_{t+1} - E_t \widehat{g}_{t+1}, \qquad (5)$$

$$\eta\widehat{L}_t + \sigma\widehat{C}_t = \widehat{w}_t, \tag{6}$$

$$\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widehat{mc}_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t[\widehat{\pi}_{n,t+1}], \tag{7}$$

$$(1+\varphi)\left(\widehat{e}_t + \widehat{w}_t\right) + \widehat{L}_t = \psi^T((1+\varphi)(\widehat{mc}_t + \widehat{e}_t) + \widehat{C}_t), \tag{8}$$

$$\widehat{e}_t + \widehat{mc}_t = \varpi(\widehat{e}_t + \widehat{w}_t), \tag{9}$$

$$\widehat{e}_{1,t} = 0. \tag{10}$$

E Analysis of a Global New Keynesian Economy

This section recasts some of our basic equations in terms of expressions familiar from the international trade and finance literatures used in the main text.

In the analysis below, we set the subsidy rate to firms producing sticky-price goods, $1 + \tau_n$, to the inverse of the markup to get rid of any effects stemming from steady state distortions associated with monopolistic pricing. Thus, the long-run marginal cost of sticky price goods is 1 in all countries (i.e., $mc_n = 1\ \forall n$). Recall also that our benchmark steady state is such that $s_n = 0\ \forall n$. Before reframing our key equations, we provide a nomenclature of the variables herein.

E.1 Nomenclature

- $\widehat{w}_{n,t}$: deviations (from steady state) in the (marginal) cost of producing traded intermediate goods in real (detrended) units of country n final goods. This also represents changes in the trading gains index, defined as the unit cost of production divided by the price of domestic purchases, $\frac{W_{n,t}/A_{n,t}}{P_{n,t}}$.

- $\widehat{e}_{n,t} + \widehat{w}_{n,t}$: deviations in the real (marginal) cost of producing traded intermediate goods in units of the reference country's (i.e., country 1) final goods. This also represents country n' export price.

- $\widehat{mc}_{n,t}$: deviations in the real marginal cost of producing sticky-price goods in country n, in units of country n final goods. This is also the 'social' or efficient price of consumption goods in country n or the inverse of the mark up.

- $\widehat{e}_{n,t} + \widehat{mc}_{n,t} + \widehat{C}_{n,t}$: deviations in the (socially efficient) real value of (expenditures on) consumption goods in country n in units of the reference country's final goods.

- $\widehat{w}_{n,t} + \widehat{L}_{n,t}$: deviations in the real total value added of traded intermediate goods in units of country n final goods.

- $\widehat{L}_t$: deviations in total labor input employed in the production of traded intermediate goods in country n, here also deviations in real GDP.[11] When the subsidy rate, $1+\tau_n$, is set to the inverse of the markup, profits are zero and nominal GDP is simply the nominal value added of tradable intermediates. Real GDP, therefore, is nominal GDP, $W_{n,t}L_{n,t}$, deflated by the GDP price deflator (i.e., the price index of aggregate nominal value added in the economy) or $W_{n,t}/A_{n,t}$ (also nominal unit labor cost or the nominal marginal cost of traded intermediate goods). Therefore, it follows that real GDP is $A_{n,t}L_{n,t}$, and real detrended GDP, $L_{n,t}$. Thus, in the remainder of the analysis, we use $L_{n,t}$ and $Y_{n,t}$ interchangeably to denote real (detrended) output.

- $\left(diag(I-\varpi)^{-1}\right)(I-\varpi)(\widehat{e}_{n,t}+\widehat{w}_{n,t})$: deviations in the terms of trade (in vector form).[12]

- $ewL(\widehat{w}_t+\widehat{L}_t) - emcC(\widehat{mc}_t+\widehat{C}_t) - e(mcC-wL)\widehat{e}_t$: deviations in countries' trade balance (in units of the reference country's final goods and in vector form). This follows from log-linearizing the expression for countries' trade balance (in real units of country 1), $(e_tw_tL_t - e_tmc_tC_t)$. When this expression is negative (positive) in country n, the country has a trade deficit (surplus).

  Note: The log-linearization around a steady-state with zero trade balance means that the expression reduces to $ewL\left(\widehat{w}_t+\widehat{Y}_t-\widehat{mc}_t-\widehat{C}_t\right)$.

- $Vg_1(1+\pi_1)E_t[ds_{t+1}] - ds_t$: financial flows (in vector form) where a negative (positive) expression indicates financial inflows (outflows). It is equal to the trade balance in that trade deficits are financed by financial inflows and trade surpluses result in financial outflows. In addition, $Vg_1(1+\pi_1)$ can also be expressed as $Vg_1(1+\pi_1)=\frac{g_1(1+\pi_1)}{1+r_1}$ where the RHS of the equation is country 1's real discount factor which brings the face value of assets with payoffs in period $t+1$ to their period t value.[13]

E.2 Interpretation and Mechanics of the Model

The New Keynesian Block: The first block of equations is standard New Keynesian framework and reflects intertemporal mechanisms at home along with a description of domestic labor supply

[11] The nominal value added of tradable intermediate goods is given by $W_{n,t}L_{n,t}$. The nominal value added of sticky-price goods is profits of firms producing those goods, Π_t.

[12] The terms of trade is the ratio of a country's export prices to import prices (in common units, here the reference country's final goods). Thus (changes) in country n's terms of trade are given by (after detrending) $\widehat{e}_{n,t}+\widehat{w}_{n,t}-\frac{1}{1-\varpi_{nn}}\sum_{n'\neq n}\varpi_{nn'}(\widehat{e}_{n',t}+\widehat{w}_{n',t}) = \frac{1}{1-\varpi_{nn}}\left(\widehat{e}_{n,t}+\widehat{w}_{n,t}-\sum_{n'}\varpi_{nn'}(\widehat{e}_{n',t}+\widehat{w}_{n',t})\right)$, or in matrix form, $\left(diag(I-\varpi)^{-1}\right)(I-\varpi)(\widehat{e}_{n,t}+\widehat{w}_{n,t})$.

[13] Because agents trade state contingent asset, the definition of interest rate income associated with those assets is ambiguous. For that reason, we incorporate any such income into asset valuations and as part of financial flows rather than deducting it from financial flows and adding it to the current account.

(expressed here in vector form for every country):

$$
\begin{aligned}
\widehat{C}_t &= E_t\widehat{C}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{g}_{t+1}\right),\\
\widehat{\pi}_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widehat{mc}_t + \widehat{\mu}_t) + \beta(E_t[\widehat{\pi}_{t+1}]),\\
\widehat{R}_t &= \rho\widehat{R}_{t-1} + (1-\rho)(\phi_\pi\widehat{\pi}_t + \phi_Y\widehat{Y}_t) + \widehat{\nu}_t,\\
\widehat{w}_t &= \eta\widehat{L}_t + \sigma\widehat{C}_t,
\end{aligned}
$$

The International Trade Block. The second block of equations relates to relationships that arise under multilateral international trade, namely a set of equations describing the marginal cost of producing sticky-price goods in each country, a set of equations describing the balance of payments in each country, a set of equations describing the evolution of countries' financial assets, and a set of equations describing market clearing conditions for traded goods.

i) Marginal Cost of Producing Sticky-Price Goods

Deviations in the marginal cost of producing sticky-price goods in any country n (following a disturbance), expressed in units of the reference country's goods across all countries, are given by $\widehat{e}_{n,t} + \widehat{mc}_{n,t}$. When country n purchases intermediate goods from abroad, this marginal cost of production is now a weighted average of trading partner's export prices, which themselves reflect marginal cost conditions abroad, $\widehat{e}_{n',t} + \widehat{w}_{n',t}$, and with weights given by country n's import shares, $\varpi_{nn'}$. Thus, $\widehat{e}_{n,t} + \widehat{mc}_{n,t} = \sum_{n'} \varpi_{nn'}(\widehat{e}_{n',t} + \widehat{w}_{n',t})$, or in vector form,

$$
\widehat{e}_t + \widehat{mc}_t = \varpi(\widehat{e}_t + \widehat{w}_t).
$$

ii) Balance of Payments

Re-arranging the household budget constraint, linearized around a steady state with balanced trade and no monopolistic trade distortions, gives

$$
C\left(\widehat{w}_t + \widehat{Y}_t - \widehat{mc}_t - \widehat{C}_t\right) = \frac{Vg_1(1+\pi_1)E_t[ds_{t+1}] - ds_t}{e},
$$

where the LHS of the above expression is a vector of trade balances for each country and the RHS a vector of capital flows, either inflows financing trade deficits or capturing outflows resulting from trade surpluses. Using the fact that $\widehat{mc}_t - \widehat{w}_t = (\varpi - I)(\widehat{e}_t + \widehat{w}_t)$ (by re-arranging the marginal cost of sticky-price goods above) allows countries' trade-balance to be decomposed into two components, one related to real quantities consumed and produced, $\widehat{C}_t - \widehat{Y}_t$, and another related to the terms of trade, $\left(diag(I-\varpi)^{-1}\right)(I-\varpi)(\widehat{e}_{n,t} + \widehat{w}_{n,t})$. Specifically, the vector describing deviations in countries' balance of payments is given by,

$$\underbrace{C}_{\text{s.s. consumption}}\left[\underbrace{\widehat{Y}_t-\widehat{C}_t}_{\text{real trade balance}}+\underbrace{(I-\varpi)(\widehat{e}_t+\widehat{w}_t)}_{\substack{\text{terms of trade}\\ \times\text{ s.s import shares}}}\right]=\underbrace{Vg_1(1+\pi_1)E_t\left[\frac{ds_{t+1}}{e}\right]-\frac{ds_t}{e}}_{\text{financial flows}}.$$

All else equal, an appreciation of country n's currency, $\widehat{e}_{n,t}>0$, improves its trade balance.

iii) Financial Asset Allocations

In a steady state with balanced trade, the evolution of countries' financial asset position is given by

$$-\sigma\widehat{C}_{t-1}+\widehat{V}_t+\beta\frac{\tau}{e}ds_t+d\xi_{t-1}=\widehat{e}_{t-1}-\widehat{e}_t-\widehat{g}_{1,t}-\widehat{\pi}_{1,t}-\sigma\widehat{C}_t.$$

Subtracting the the analogous expression written in terms of the reference country gives

$$\underbrace{\widehat{C}_{1,t}-\widehat{C}_t}_{\text{relative consumption}}=\frac{1}{\sigma}\underbrace{\widehat{e}_t}_{\substack{\text{real}\\ \text{exchange rate}}}+\underbrace{\beta\frac{\tau}{\sigma}\overbrace{\left(\frac{1}{e}dS_t-dS_{1,t}\right)}^{\substack{\text{cumulative financial}\\ \text{asset position}}}+\overbrace{d\Xi_t-d\Xi_{1,t}}^{\substack{\text{cumulative portfolio}\\ \text{adjustment shocks}}}}_{\text{asset pricing wedge}},$$

$$\text{where } dS_t=\sum_{k=-\infty}^{t}ds_k;\ d\Xi_t=\frac{1}{\sigma}\sum_{k=-\infty}^{t-1}d\xi_k.$$

In a world with no frictions in global financial markets, $\tau=0$ and $d\xi_{t-k}=0\ \forall k$, this equation reduces to that first emphasized by Backus and Smith (1993) that underlies the consumption-exchange rate disconnect puzzle.

iv) Market Clearing Condition for Traded Goods

The market clearing condition for traded goods, $(1+\varphi)(\widehat{e}_t+\widehat{w}_t)+\widehat{Y}_t=\psi^T((1+\varphi)(\widehat{mc}_t+\widehat{e}_t)+\widehat{C}_t)$, can be re-written to describe the make-up of the real trade balance:

$$\underbrace{(\widehat{Y}_t-\widehat{C}_t)}_{\text{real trade balance}}=\underbrace{(\psi^T-I)\widehat{C}_t}_{\substack{\text{relative demand}\\ \text{in export markets}}}+\underbrace{(1+\varphi)}_{\substack{\text{price elasticity}\\ \text{of trade}}}\underbrace{\left(\overbrace{(\psi^T-I)\varpi(\widehat{e}_t+\widehat{w}_t)}^{\substack{\text{relative cost}\\ \text{of consumption bundles}}}-\overbrace{(I-\varpi)(\widehat{e}_t+\widehat{w}_t)}^{\text{terms of trade}}\right)}_{\text{relative price at destination}}.$$

First, the real trade balance (i.e., $\widehat{Y}_t-\widehat{C}_t$) moves towards a surplus when consumption of home goods increases abroad, $\psi^T\widehat{C}_t$, and towards a deficit when domestic consumption increases, $-I\widehat{C}_t$[14]. Second, the real trade balance also depends on the relative price of exports in foreign markets.

[14]Recall that ψ^T is a matrix of export shares. Thus each foreign country's consumption of home-produced goods is weighted by the share of exports it receives from the home country.

This relative price in turn consists, for each country, of the (export-share weighted) relative cost of consumption bundles relative to the reference country, $(\psi^T - I)\varpi(\widehat{e}_t + \widehat{w}_t)$, less a terms-of-trade adjustment that converts the cost of consumption goods in the domestic market to its output cost. In that sense, the Fréchet parameter, $\varphi > 0$, ends up playing the role of a price elasticity of trade.

E.3 Deviations from Uncovered Interest Rate Parity

Our goods and asset trading equations imply an uncovered interest parity condition. Specifically, starting with

$$E_t[\sigma\widehat{C}_{t+1} - \sigma\widehat{C}_t] + E_t[\widehat{V}_{t+1}] + \beta\frac{\tau}{e}E_t[ds_{t+1}] + d\xi_t = E_t[\widehat{e}_t - \widehat{e}_{t+1}] - E_t[\widehat{g}_{1,t+1} - \widehat{\pi}_{1,t+1}],$$

and substituting

$$E_t[\sigma\widehat{C}_{t+1} - \sigma\widehat{C}_t] = \widehat{R}_t - E_t[\widehat{g}_{t+1} - \widehat{\pi}_{t+1}],$$

yields

$$\widehat{R}_t - E_t[\widehat{g}_{t+1} - \widehat{\pi}_{t+1}] + E_t[\widehat{V}_{t+1}] + \beta\frac{\tau}{e}E_t[ds_{t+1}] + d\xi_t = E_t[\widehat{e}_t - \widehat{e}_{t+1}] - E_t[\widehat{g}_{1,t+1} - \widehat{\pi}_{1,t+1}].$$

After subtracting the same equation written for the reference country, we have that,

$$\begin{aligned}(E_t\widehat{e}_{t+1} - \widehat{e}_t) = -\widehat{R}_t + E_t[\widehat{\pi}_{t+1} + \widehat{g}_{t+1}] + \left(\widehat{R}_{1,t} - E_t[\widehat{\pi}_{1,t+1} + \widehat{g}_{1,t+1}]\right) \\ - \beta\tau E_t\left(\frac{1}{e}ds_{t+1} - ds_{1,t+1}\right) - (d\xi_t - d\xi_{1,t}),\end{aligned} \tag{11}$$

where endogenous costs associated with financial portfolio adjustments, $\beta\tau E_t\left(\frac{1}{e}ds_{t+1} - ds_{1,t+1}\right)$, and exogenous disturbances to global financial markets, $(d\xi_t - d\xi_{1,t})$, drive a wedge in the Uncovered Interest Rate Parity (UIP) condition.

F An International System of Phillips Curves: Spillovers from Foreign Shocks to Domestic Inflation

F.1 The Global Phillips Curve

It follows from the labor supply equation, (6), and the definition of marginal cost, (9), that

$$\widehat{mc}_t = \varpi(\eta\widehat{Y}_t + \sigma\widehat{C}_t) + (\varpi - I)\widehat{e}_t.$$

Given a real exchange rate, marginal cost at home now reflects production costs of imported goods, that is wages in trading partner countries. These are in turn determined by real activity embodied in the determination of labor supply abroad, $\widehat{w}_t = \eta\widehat{L}_t + \sigma\widehat{C}_t$. The Global Phillips Curve is then given by (in vector form),

$$\widehat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\left[\varpi(\eta\widehat{Y}_t + \sigma\widehat{C}_t) + (\varpi - I)\widehat{e}_t + \widehat{\mu}_{n,t}\right] + \beta(E_t[\widehat{\pi}_{t+1}]). \tag{12}$$

Thus, spillovers from real activity in country j to inflation in country i are in this case exactly proportional to the import share of country i from country j, ϖ_{ij}. To be sure, real exchange rates are not given and, as the UIP condition (11) makes clear, will generally reflect frictions in international financial markets.

F.2 Spillovers from Idiosyncratic Demand Shocks: An Analytical Example

This section considers a special case that abstracts from international financial frictions, $\tau = 0$ and $d\xi_t = 0\ \forall t$. In addition, shocks to TFP growth and markups are *i.i.d* across all countries and over time while monetary policy is given by a simple forward looking inflation centric rule in every country from Clarida, Gali, and Gertler (2002),

$$\widehat{R}_t = \phi E_t\widehat{\pi}_{t+1} + \sigma\nu_t. \tag{13}$$

System Reduction: Under the maintained assumptions, $E_t\widehat{g}_{t+1} = 0$, and the New Keynesian block of the model reduces to,

$$\begin{aligned}
-\sigma\widehat{C}_t &= -\sigma E_t\widehat{C}_{t+1} + \widehat{R}_t - E_t\widehat{\pi}_{t+1},\\
\mathcal{A}_{YY}\widehat{Y}_t &= \mathcal{A}_{YC}\widehat{C}_t + \mathcal{A}_{Ye}\widehat{e}_t,\\
\widehat{\pi}_t &= \mathcal{A}_{\pi C}\widehat{C}_t + \mathcal{A}_{\pi e}\widehat{e}_t + \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{\mu}_t,\\
\sigma(\widehat{C}_{1,t} - \widehat{C}_t) &= \widehat{e}_t,
\end{aligned}$$

where $\mathcal{A}_{YY}$, $\mathcal{A}_{YC}$ and $\mathcal{A}_{Ye}$ are matrices of coefficients obtained from using the labor supply equation, (6), to substitute out $\widehat{w}_t$ in the the market clearing condition for traded goods, (8). Similarly, $\mathcal{A}_{\pi C}$ and $\mathcal{A}_{\pi e}$ are matrices obtained by using the resulting market clearing condition for traded goods to eliminate $\widehat{Y}_t$ from the GPC (12). Specifically, we have that

$$\begin{aligned}
\mathcal{A}_{YY} &= I + (1+\varphi)\left(I - \psi^T\varpi\right)\eta,\\
\mathcal{A}_{YC} &= \psi^T - (1+\varphi)\left(I - \psi^T\varpi\right)\sigma,\\
\mathcal{A}_{Ye} &= -(1+\varphi)\left(I - \psi^T\varpi\right),\\
\mathcal{A}_{\pi C} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(\eta\varpi\mathcal{A}_{YY}^{-1}\mathcal{A}_{YC} + \sigma\varpi),\\
\mathcal{A}_{\pi e} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(\eta\varpi\mathcal{A}_{YY}^{-1}\mathcal{A}_{Ye} + \varpi - I),
\end{aligned}$$

where $\mathcal{A}_{ij}$ refers to the effect of variable j on variable i. Individual elements of the $\mathcal{A}$ matrices capture the effects of a country n' on another country n. Thus, for example, the $(1,2)$ element of $\mathcal{A}_{\pi e}$ refers to the effect of country 2's exchange rate on inflation in country 1.

We now use the monetary policy rule to substitute for the nominal rates, $\widehat{R}_t$, and use the Backus and Smith (1993) equation to substitute for the vector of exchange rates, $\widehat{e}_t$, to obtain

$$\begin{aligned}
-\sigma\widehat{C}_t &= -\sigma E_t\widehat{C}_{t+1} + \phi E_t\widehat{\pi}_{t+1} + \sigma\nu_t - E_t\widehat{\pi}_{t+1}, \\
\widehat{Y}_t &= \mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\widehat{C}_t, \\
\widehat{\pi}_t &= (\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\widehat{C}_t + \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{\mu}_t,
\end{aligned}$$

where $\mathcal{A}_{\pi e}\mathbf{1} = \mathbf{0}$ is used to eliminate the scalar $\widehat{C}_{1,t}$.[15] Observe also that

$$\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye} = \psi^T,$$

and

$$\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\eta\varpi\mathcal{A}_{YY}^{-1}\left(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye}\right) + \sigma I\right).$$

We are now in a position to solve this system of linear rational expectations equations analytically by way of undetermined coefficients.

Model Solution and Equilibrium: We conjecture that in equilibrium, endogenous variables are linear functions of the states, in this case the exogenous shocks.[16] Thus, let the solutions for consumption, employment, and inflation be given by

$$\begin{aligned}
\widehat{C}_t &= \alpha_{C\nu}\nu_t + \alpha_{C\mu}\mu_t, \\
\widehat{Y}_t &= \alpha_{Y\nu}\nu_t + \alpha_{Y\mu}\mu_t, \\
\widehat{\pi}_t &= \alpha_{\pi\nu}\nu_t + \alpha_{\pi\mu}\mu_t,
\end{aligned}$$

where $\alpha_{C\nu}, \alpha_{C\mu}, \alpha_{Y\nu}, \alpha_{Y\mu}, \alpha_{\pi\nu}$, and $\alpha_{\pi\mu}$ are unknown coefficients. To solve for these coefficients, we substitute these solutions into the model equations. It is then straightforward to show that the coefficients describing the solutions for consumption, $\widehat{C}_t$, output, $\widehat{Y}_t$, and inflation, $\widehat{\pi}_t$, as functions

[15] In the framework developed by Eaton and Kortum (2002), the matrices of import and export shares, ϖ and ψ^T, are derived as stochastic matrices. Therefore, $(\varpi - I)\mathbf{1} = (I - \psi^T)\mathbf{1} = \mathbf{0}$. The fact that $\mathcal{A}_{\pi e}\mathbf{1} = \mathbf{0}$ then follows immediately from the definition of $\mathcal{A}_{\pi e}$ and the fact that the product of stochastic matrices is also a stochastic matrix.

[16] In this example, we do not allow for equilibria that depend on sunspots. See Galí (2008) for a discussion of the conditions on ϕ, beyond the Taylor principle, that guarantee a unique equilibrium.

of monetary policy and markup shocks satisfy

$$
\begin{aligned}
-\alpha_{C\nu} &= 1, \\
-\sigma\alpha_{C\mu} &= 0, \\
\alpha_{Y\mu} &= \mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\alpha_{C\mu}, \\
\alpha_{Y\nu} &= \mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\alpha_{C\nu}, \\
\alpha_{\pi\nu} &= \left(\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e}\right)\alpha_{C\nu}, \\
\alpha_{\pi\mu} &= \left(\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e}\right)\alpha_{C\mu} + \frac{(1-\theta)(1-\beta\theta)}{\theta}I.
\end{aligned}
$$

Put another way, the model solution is such that

$$
\begin{aligned}
\widehat{C}_t &= -\nu_t, \\
\widehat{Y}_t &= -\mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\nu_t, \\
\widehat{\pi}_t &= -(\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\nu_t + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{\mu}_t.
\end{aligned}
$$

Therefore, $\widehat{C}_t$ depends only on its own-country monetary policy shocks, or alternatively its own demand shocks, and is *i.i.d.* across countries. As expected, higher rates in a given country lower consumption relative to trend. Furthermore, because all shocks are uncorrelated across countries, endogenous spillovers from foreign demand shocks, ν_t, to domestic inflation, $\widehat{\pi}_t$, arise solely by way of trade rather than global shocks.

International Comovement Properties in a Global New Keynesian Economy: In this section, for simplicity, we let all elements of ν_t and μ_t have unit variances. However, this is without loss of generality in that the proposition below holds even when shock variances are away from 1 and country-specific.

To characterize comovement in the global New Keynesian economy, observe first that we can write the vector of inflation across countries, $\widehat{\pi}_t$, as a function of countries' output gap, $\widehat{Y}_t$ and exogenous disturbances. In particular, since $\widehat{\pi}_t = -(\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\nu_t + \kappa\widehat{\mu}_t$ and $\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e} = \kappa(\eta\varpi\mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye}) + \sigma I)$ above, where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, it follows that

$$
\widehat{\pi}_t = \kappa\eta\varpi\widehat{Y}_t - \kappa\sigma\nu_t + \kappa\mu_t.
$$

Then, we obtain the following proposition.

Proposition 1. *For any matrix M, let $M \geq 0$ if and only if $m_{ij} \geq 0$ $\forall ij$. Then, under the maintained assumptions, $E[\widehat{Y}_t\widehat{Y}_t^T] \geq 0$, $E[\widehat{\pi}_t\widehat{\pi}_t^T] \geq 0$ and $E[\widehat{\pi}_t\widehat{Y}_t^T] \geq 0$.*

Proof. The variance-covariance matrix of output across countries is given by,

$$E\left[\widehat{Y}_t\widehat{Y}_t^T\right] = \mathcal{A}_{YY}^{-1}\psi^T\psi\left(\mathcal{A}_{YY}^{-1}\right)^T.$$

Since $\psi^T \geq 0$, it immediately follows that $E\left[\widehat{Y}_t\widehat{Y}_t^T\right] \geq 0$ whenever $\mathcal{A}_{YY}^{-1} \geq 0$.

To show that $\mathcal{A}_{YY}^{-1} \geq 0$, recall that $\mathcal{A}_{YY} = a(I - b\psi^T\varpi)$ where a and b are scalars such that $a = 1 + (1+\varphi)\eta$ and $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$. Because the matrices of import and export shares, ϖ and ψ^T respectively, are stochastic matrices, so is their product, $\psi^T\varpi$. It follows that $\rho(\psi^T\varpi) = 1$ where $\rho(\psi^T\varpi) = \max_{\Lambda(\psi^T\varpi)}\{|\lambda|\}$ such that $\Lambda(\psi^T\varpi) = \{\lambda|\lambda$ is an eigenvalue of $\psi^T\varpi\}$, i.e., the spectral radius of $\psi^T\varpi$ is 1. In addition, since $b < 1$, the spectral radius of $b\psi^T\varpi$ is strictly less than 1, $\rho(b\psi^T\varpi) < 1$. We can then write $\mathcal{A}_{YY}^{-1}$ as

$$\mathcal{A}_{YY}^{-1} = \{a(I - b\psi^T\varpi)\}^{-1} = \frac{1}{a}\sum_{m=0}^{\infty} b^m(\psi^T\varpi)^m.$$

Given that $\psi^T\varpi \geq 0$, $\mathcal{A}_{YY}^{-1} \geq 0$ and $E\left[\widehat{Y}_t\widehat{Y}_t^T\right] \geq 0$.

The solution for inflation above is given by $\widehat{\pi}_t = \kappa\eta\varpi\widehat{Y}_t - \kappa\sigma\nu_t + \kappa\mu_t$. Therefore, $E[\widehat{\pi}_t\widehat{\pi}_t^T] = \kappa\eta\varpi E[\widehat{Y}_t\widehat{Y}_t^T]\varpi^T\ \eta\kappa + 2\kappa^2\eta\sigma\varpi\mathcal{A}_{YY}^{-1}\psi^T I + \kappa^2\sigma^2 I + \kappa^2 I$ since $E[\nu_t\mu_t^T] = 0$. Moreover, given that $E[\widehat{Y}_t\widehat{Y}_t^T] \geq 0$, it then follows that $E[\widehat{\pi}_t\widehat{\pi}_t^T] \geq 0$. It also follows that $E[\widehat{\pi}_t\widehat{Y}_t^T] = \kappa\eta\varpi E[\widehat{Y}_t\widehat{Y}_t^T] + \kappa\sigma\psi(\mathcal{A}_{YY}^{-1})^T \geq 0$. $\square$

G Estimation and Calibration

G.1 Calibrated Steady States

We calibrate the steady state import trade matrix, ϖ, using data on import shares for each country from the World Integrated Trade Solution (WITS), compiled by the World Bank. In particular, we construct the trade matrix for each quarter and calibrate ϖ to be the trade matrix averaged over the sample period. We take the diagonal to include only domestically-produced consumption and distribute the remaining imports proportionally across the four foreign countries in the sample. Specifically, let $\varpi^*_{nn',t}$ and $\varpi_{nn',t}$ be the (n, n') elements of the trade matrices in period t for the whole world and for our sample, respectively. We set $\varpi_{nn,t} = \varpi^*_{nn,t}$ and define

$$\varpi_{nn',t} = \frac{\varpi^*_{nn',t}}{\sum_{n'\in\mathcal{S},n'\neq n}\varpi^*_{nn',t}}\left(1 - \varpi_{nn,t}\right),$$

where $\mathcal{S}$ is the set of countries that we use for the estimation. Note that each row of ϖ_t thus sums to one. Finally, we calibrate the steady state $\varpi \equiv \frac{1}{T}\sum_{t=1}^{T}\varpi_t$.

The steady state wage bill $\mathcal{E}_n W_n L_n$ and labor L_n also need to be calibrated to compute the steady state. We calibrate the wage bill to match average GDP in current U.S. dollars over the sample period, taken from the World Bank and OECD National Accounts data files. We define labor as the product of employment and average hours, with data taken from the Penn World Tables. While these are potentially non-stationary variables, only their relative values matter for our estimation. Thus the growth rates are immaterial so long as the ratio of the variables across countries remains fairly constant over the sample period.

G.2 Estimation Data and Measurement Equations

We estimate the remaining variables using Bayesian methods. We compute the likelihood using the Kalman filter with data on the following observables:

1. **Per Capita Real Output Growth.** Denote the population and real gross domestic product of country n in period t by $GDP_{n,t}$ and $POP_{n,t}$, respectively. We define

$$\text{Per Capita Real Output Growth} \equiv 100 \left[\log \frac{GDP_{n,t}}{POP_{n,t}} - \log \frac{GDP_{n,t-1}}{POP_{n,t-1}} \right].$$

2. **Annualized Inflation.** Denote CPI price level of country n in period t by $CPI_{n,t}$. We define

$$\text{Annualized Inflation} \equiv 400 \log \frac{CPI_{n,t}}{CPI_{n,t-1}}.$$

3. **Interest Rate.** The interest rate is the call money or interbank rate from Federal Reserve Economic Data (FRED) and the OECD.

4. **Nominal Exchange Rate Growth.** Denote the nominal exchange rate between country n and the U.S. by $EX_{n,t}$. We define

$$\text{Nominal Exchange Rate Growth} \equiv 100 \log \frac{EX_{n,t}}{EX_{n,t-1}}.$$

All data are from Haver and aggregation for the E.U. follows Eurostat.

The measurement equations in the Kalman filter are as follows:

$$\begin{aligned}
\text{Per Capita Real Output Growth} &= 100(\widehat{L}_{n,t} - \widehat{L}_{n,t-1} + \widehat{g}_{n,t}), \\
\text{Annualized Inflation} &= 400\widehat{\pi}_{n,t}, \\
\text{Interest Rate} &= 400\widehat{R}_{n,t}, \\
\text{Nominal Exchange Rate Growth} &= 100(\widehat{e}_{n,t} - \widehat{e}_{n,t-1} + \widehat{\pi}_{n,t} - \widehat{\pi}_{n,t-1} + \widehat{g}_{n,t} - \widehat{g}_{n,t-1}).
\end{aligned}$$

We demean each of the variables, so that we only use information on their variations. In particular, we abstract from the steady state relationships that the model implies across variables, as they may not accurately reflect, for instance, the exchange rate and interest rate regime in China.

G.3 Sequential Monte Carlo Algorithm

This section summarizes the adaptive sequential Monte Carlo (SMC) algorithm that we use to estimate the model. The algorithm is taken directly from Cai, Del Negro, Herbst, Matlin, Sarfati, and Schorfheide (2021). As with Markov chain Monte Carlo methods, the end goal is to obtain a set of draws from the posterior.

The SMC algorithm begins with draws from the prior, referred to as *particles*, then iteratively constructs particle approximations to *bridge distributions* that serve as intermediate steps between the prior and posterior. In stage $n-1$, we have particles $\{\theta^i_{n-1}, W^i_{n-1}\}^N_{i=1}$, where θ^i_{n-1} denotes the parameter draw and W^i_{n-1} denotes the weight. To move to stage n, we follow three steps:

1. **Correction:** Reweighting the stage $n-1$ particles to reflect the stage n distributions.

2. **Selection:** Resampling the particles to avoid an uneven distribution of weights.

3. **Mutation:** Propogating particles forward using an MCMC algorithm.

The algorithm is initialized with θ^i_0 drawn from the prior and $W^i_0 = 1$. It concludes when the (weighted) distribution of particles approximates the posterior.

Details of Sequential Monte Carlo Steps: We now describe the SMC steps in more detail.

1. **Correction.** The correction step is importance sampling between the stage $n-1$ and stage n distributions. It starts with particles $\{\theta^i_{n-1}, W^i_{n-1}\}^N_{i=1}$ and updates the weights W^i_{n-1} to

$$\overline{W}^i_n \equiv \frac{w^i_n W^i_{n-1}}{\frac{1}{N}\sum^N_i w^i_n}.$$

The incremental weights w^i_n are defined by:

$$w^i_n = \frac{p_n(\theta^i_{n-1})}{p_{n-1}(\theta^i_{n-1})},$$

where $p_n(\theta^i_{n-1})$ is the pdf of the stage n bridge distribution at θ^i_{n-1}, which we discuss below. This step yields particles $\{\theta^i_{n-1}, \overline{W}^i_n\}^N_{i=1}$.

2. **Selection.** The selection step resamples particles $\{\theta^i_{n-1}, \overline{W}^i_n\}^N_{i=1}$ based on weights $\overline{W}^i_n$ to obtain $\{\hat{\theta}^i_{n-1}, W^i_{n-1}\}^N_{i=1}$, resetting the weights to $W^i_{n-1} = 1$ for all i. The selection step faces a trade-off—it adds noise through resampling but equalizes particle weights, which increases the accuracy of importance sampling. Hence, following Herbst and Schorfheide (2015) and

Cai et al. (2021), we only carry out the selection step if the effective sample size,

$$\widehat{ESS}_n \equiv N/\left[\frac{1}{N}\sum_{i=1}^{N}\left(\overline{W}_n^i\right)^2\right], \tag{14}$$

falls below a threshold $\underline{N}$. The effective sample size measures how uneven the weights are. If the weights are all equal to one, then $\widehat{ESS}_n = N$. If the weight on all but one particle is zero, then $\widehat{ESS}_n = 1$.

3. **Mutation.** The mutation step changes the parameter values $\hat{\theta}_n^i$ so that they do not remain in the inital set of values. In particular, we generate N independent MCMC chains, typically using Metropolis-Hastings. These can be parallelized. Herbst and Schorfheide (2015) discuss how to select proposal densities. The weights remain unchanged. This yields particles $\{\theta_n^i, W_n^i\}_{i=1}^N$.

Adaptively Choosing Bridge Distributions: What remains is the choice of bridge distributions p_n. We use a likelihood tempering approach, in which the stage n bridge distribution is:

$$p_n(\theta) \propto \pi(\theta)\mathcal{L}(\theta|Y)^{\phi_n},$$

where $\pi(\cdot)$ is the prior, $\mathcal{L}(\cdot|Y)$ is the likelihood given data Y, and ϕ_n is an exponent that we choose adaptively. The algorithm begins with $\phi_0 = 0$, which corresponds to the prior, and terminates when $\phi_n = 1$, which corresponds to the posterior.

To choose ϕ_n, we rely on the effective sample size, defined analogously to (14). In particular, define:

$$\widehat{ESS}(\phi) \equiv N/\left[\frac{1}{N}\sum_{i=1}^{N}\left(\widetilde{W}_n^i\right)^2\right], \tag{15}$$

where

$$\widetilde{W}_n^i \equiv \frac{w^i(\phi)W_{n-1}^i}{\frac{1}{N}\sum_{i=1}^{N} w^i(\phi)W_{n-1}^i} \tag{16}$$

$$w^i(\phi) \equiv \pi(\theta)\mathcal{L}(\theta|Y)^{\phi-\phi_{n-1}}. \tag{17}$$

We choose ϕ to obtain a target effective sample size, so that $\widehat{ESS}(\phi) = \alpha\widehat{ESS}_{n-1}$. The tuning parameter $\alpha \in (0, 1)$ determines how fast the effective sample size decays. A larger value of α improves accuracy at the cost of computational time.

Tuning Parameter Choices: We take $N = 20,000$ particles, set $\alpha = 0.96$, and use the selection threshold $\underline{N} = N/2$. These are all in line with values used in Cai et al. (2021).

H Supplementary Figures

Figure 1: Prior and Posterior of Country-Specific Parameters

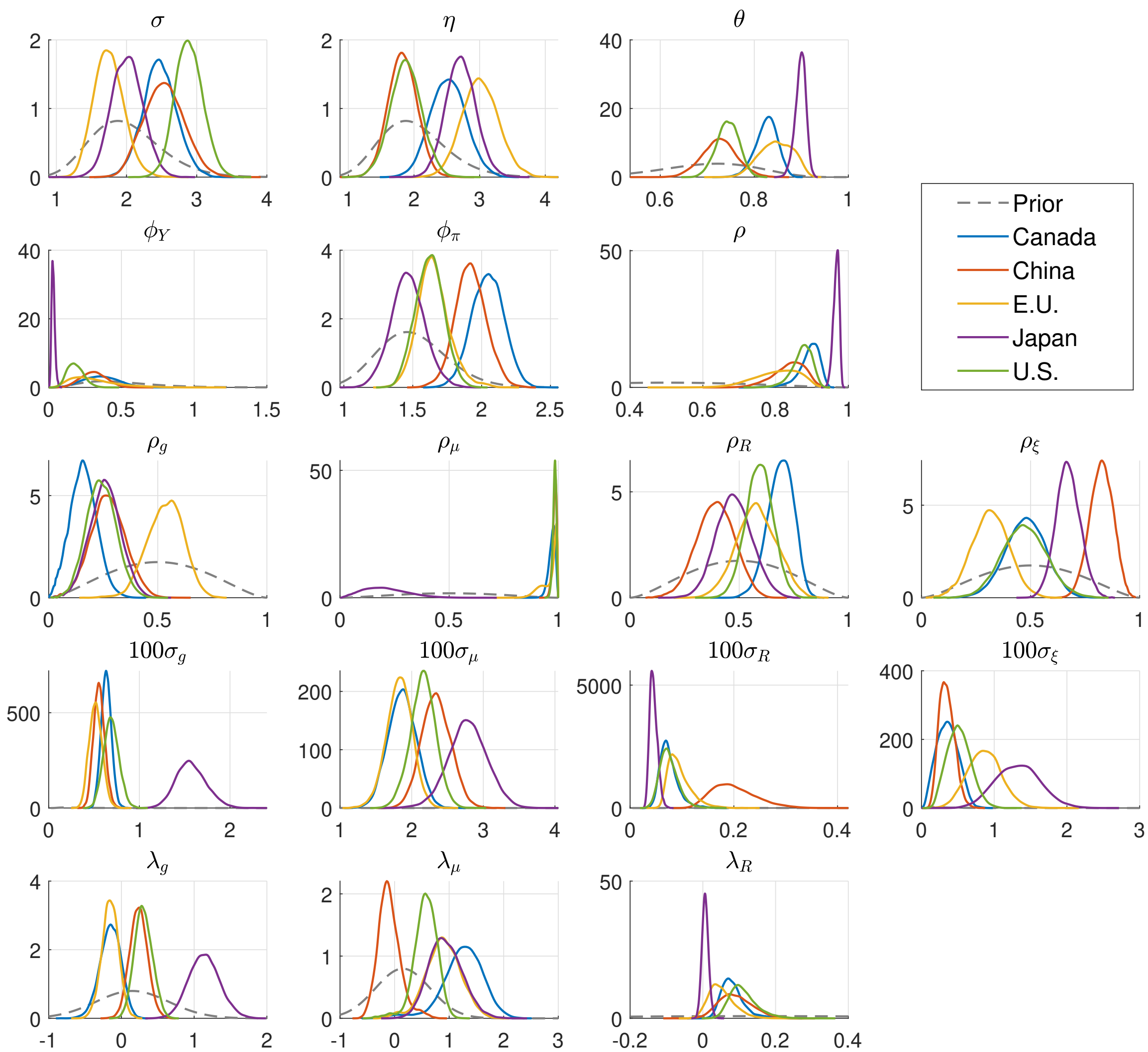


Figure 2: Prior and Posterior of Global Parameters

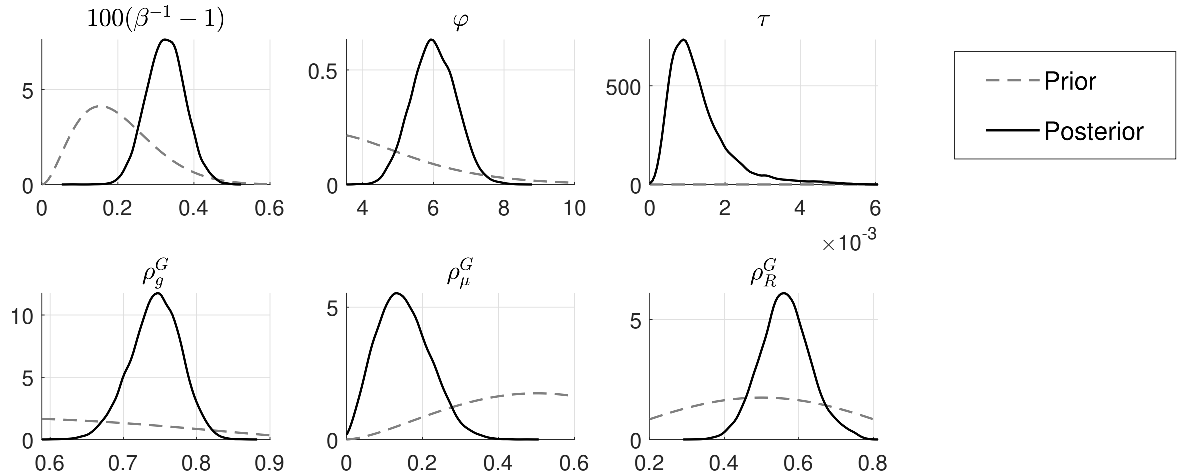
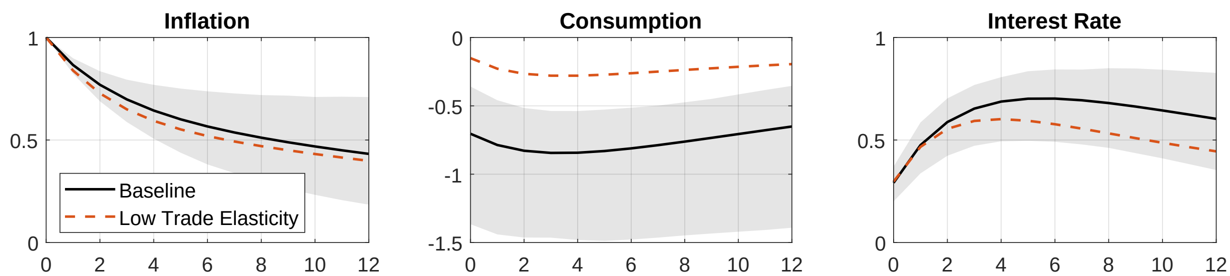
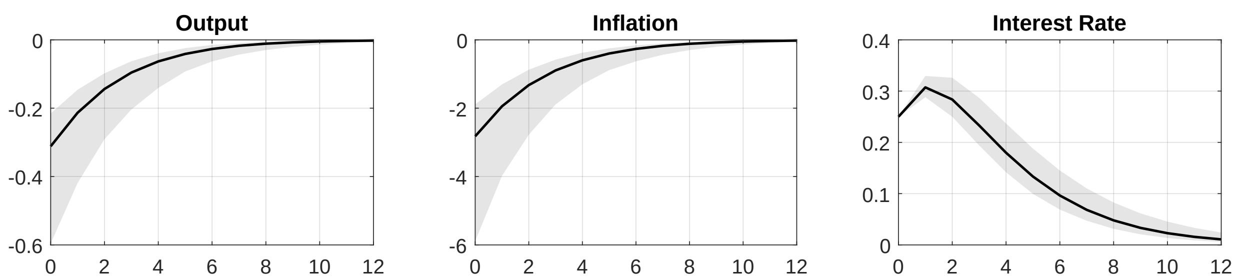


Figure 3: Response of E.U. Variables to an E.U. Markup Shock



Notes: Impulse responses scaled to generate a 1 percent increase in E.U. inflation on impact. Solid black lines correspond to estimated impulse response at posterior mean; gray shaded regions correspond to 68 percent posterior credible intervals; red dashed lines correspond to impulse response at posterior mean with $\varphi = 0.03$. Horizontal axis is in quarters; vertical axis is in percent. Inflation and interest rate are annualized.

Figure 4: Response of U.S. Variables to a Tightening of U.S. Monetary Policy



Notes: Impulse responses correspond to a 25 basis point increase in U.S. nominal interest rates on impact. Solid black lines correspond to impulse response at posterior mean for parameters; gray shaded regions correspond to 68 percent posterior credible intervals. Horizontal axis is in quarters; vertical axis is in percent. Inflation and interest rate are annualized.

I Derivations of the Price Index for Traded Goods and Trade Shares

I.1 The Price Index for Traded Goods

The price index of traded goods in Section A.3 is

$$P^Q_{n,t} = \left[\int P_{n,t}(\mathbf{z})^{1-\gamma} d\Phi(\mathbf{z}) \right]^{\frac{1}{1-\gamma}}.$$

This price index takes the form of an expected value. Since the derivation of this expected value does not depend on t, we omit the time subscript in the derivations below.

Recall that

$$P_n(\mathbf{z}) = \min_{n'} \left\{ \frac{\kappa_{nn'} \mathcal{E}_{nn'} W_{nn'}}{z_{n'}} \right\}.$$

To begin, let's find an expression for the distribution of prices charged by different countries for variety $\mathbf{z}$ in local currency, $\Pr(\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}} < p)$. Since z_n is distributed Fréchet with shape parameter φ, we have that

$$\begin{aligned} \Pr(\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}} < p) &= \Pr(z_{n'} \geq \frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{p}) \\ &= 1 - e^{-\lambda_{nn'} p^{\varphi}}, \end{aligned}$$

where $\lambda_{nn'} = [\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}]^{-\varphi}$.

Now, consider a sequence of independently distributed random variables, $(x_1, x_2, ..., x_n)$, and observe that by the law of complements,

$$\begin{aligned} \Pr(\min_i \{x_i\} < x) &= 1 - \Pr(x_1 \geq x \cap x_2 \geq x \cap ... x_n \geq x) \\ &= 1 - \Pr(x_1 \geq x) \Pr(x_2 \geq x) ... \Pr(x_n \geq x). \end{aligned}$$

Alternatively,

$$\begin{aligned} \Pr(\min_i \{x_i\} < x) &= 1 - \Pr(\min_i \{x_i\} \geq x) \\ &= 1 - \Pr(x_1 \geq x) \Pr(x_2 \geq x) ... \Pr(x_n \geq x). \end{aligned}$$

So then,

$$\begin{aligned}
\Pr\left(\min_{n'}\left\{\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}}\right\}<p\right) &= 1-\Pr\left(\left\{\frac{\kappa_{n1}\mathcal{E}_{n1}W_{n1}}{z_1}\right\}\geq p\right)...\Pr\left(\left\{\frac{\kappa_{nn}\mathcal{E}_{nn}W_{nn}}{z_n}\right\}\geq p\right)\\
&= 1-\Pr\left(\left\{\frac{\kappa_{n1}\mathcal{E}_{n1}W_{n1}}{p}\right\}\geq z_1\right)...\Pr\left(\left\{\frac{\kappa_{nn}\mathcal{E}_{nn}W_{nn}}{p}\right\}\geq z_n\right)\\
&= 1-e^{-\lambda_{n1}p^{\varphi}}...e^{-\lambda_{nn}p^{\varphi}}\\
&= 1-e^{-\Phi_n}p^{\varphi},
\end{aligned}$$

where $\Phi_n = \sum_{n'}\lambda_{nn'} = \sum_{n'}\left[\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}\right]^{-\varphi}$. Hence, we now have an explicit probability distribution for the random varilable $\min_{n'}\left\{\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}}\right\}$, namely $F(p) = 1-e^{-\Phi_n}p^{\varphi}$. We can, therefore, take an expectation of $P_n(\mathbf{z})^{1-\gamma}$.

Since $F(p) = 1-e^{-\Phi_n}p^{\varphi}$, we have the corresponding *p.d.f.* $f(p) = \Phi_n\varphi p^{\varphi-1}e^{-\Phi_n}p^{\varphi}$. It follows that

$$\left(P_n^Q\right)^{1-\gamma} = \int p^{1-\gamma}\Phi_n\varphi p^{\varphi-1}e^{-\Phi_n}p^{\varphi}dp.$$

We can simplify things by working with $y = g(p) = p^{\varphi}$ (instead of p) with corresponding *p.d.f.* $f(y)$. The change of variables implies that

$$\begin{aligned}
f(y) &= f(g^{-1}(y))\left|\frac{dg^{-1}(y)}{dy}\right|\\
&= \Phi_n\varphi(y^{\frac{1}{\varphi}})^{\varphi-1}e^{-\Phi_n y}\frac{1}{\varphi}y^{\frac{1-\varphi}{\varphi}}\\
&= \Phi_n e^{-\Phi_n y}.
\end{aligned}$$

Therefore, we can write

$$\left(P_n^Q\right)^{1-\gamma} = \int y^{\frac{1-\gamma}{\varphi}}\Phi_n e^{-\Phi_n y}dy.$$

Let's carry out one last change of variables, $u = \Phi_n y$, so that $du = \Phi_n dy$. Then, we have that

$$\left(P_n^Q\right)^{1-\gamma} = \Phi_n^{\frac{-(1-\gamma)}{\varphi}}\int u^{\frac{1-\gamma}{\varphi}}e^{-u}du.$$

Recall that $\Phi_n = \sum_{n'}\lambda_{nn'} = \sum_{n'}\left[\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}\right]^{-\varphi}$ and given the definition of the Gamma function, $\Gamma(\alpha) = \int x^{\alpha-1}e^{-x}dx$, we obtain,

$$P_n^Q = \Gamma\left(1+\frac{1-\gamma}{\varphi}\right)^{\frac{1}{1-\gamma}}\left(\sum_{n'}\left[\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}\right]^{-\varphi}\right)^{-\frac{1}{\varphi}}.$$

I.2 Trade Shares

Recall that trade shares are given by

$$\varpi_{nn'} = \frac{X_{nn'}}{X_n},$$

and observe that

$$X_{nn'} = \Pr\left(\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}} < \min_{m\neq n'}\left\{\frac{\kappa_{nm}\mathcal{E}_{nm}W_{nm}}{z_m}\right\}\right) X_n.$$

Recall above that the term $\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}}$ is distributed according to $1 - e^{-\lambda_{nn'}p^{\varphi}}$. The derivations above also imply that the term $\min_{m\neq n'}\left\{\frac{\kappa_{nm}\mathcal{E}_{nm}W_{nm}}{z_m}\right\}$ is distributed according to $1 - e^{-\widetilde{\Phi}_n p^{\varphi}}$, where $\widetilde{\Phi}_n = \sum_{m\neq n'}\lambda_{nm} = \sum_{m\neq n'}\left[\kappa_{nm}\mathcal{E}_{nm}W_{nm}\right]^{-\varphi}$.

We have the following result: if x is distributed according to $\exp(\mu_x)$ and y is distributed according to $\exp(\mu_y)$, and x and y are independent, then $\Pr(x < y) = \frac{\mu_x}{\mu_x+\mu_y}$. Given this result, it immediately follows that

$$\begin{aligned}
\varpi_{nn'} &= \Pr\left(\frac{\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}}{z_{n'}} < \min_{m\neq n'}\left\{\frac{\kappa_{nm}\mathcal{E}_{nm}W_{nm}}{z_m}\right\}\right) \\
&= \frac{\lambda_{nn'}}{\lambda_{nn'} + \widetilde{\Phi}_n} \\
&= \frac{\lambda_{nn'}}{\Phi_n} \\
&= \frac{\left[\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}\right]^{-\varphi}}{\sum_{n'}\left[\kappa_{nn'}\mathcal{E}_{nn'}W_{nn'}\right]^{-\varphi}}.
\end{aligned}$$

References

Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*, 297–316.

Cai, M., M. Del Negro, E. Herbst, E. Matlin, R. Sarfati, and F. Schorfheide (2021). Online estimation of DSGE models. *Econometrics Journal 24*(1), C33–C58.

Clarida, R., J. Gali, and M. Gertler (2002). A simple framework for international monetary policy analysis. *Journal of Monetary Economics 49*(5), 879–904.

Dekle, R., J. Eaton, and S. Kortum (2007). Unbalanced trade. *American Economic Review 97*, 351–355.

Eaton, J. and S. Kortum (2002). Technology, geography, and trade. *Econometrica 70*, 1741–1779.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*.

Herbst, E. P. and F. Schorfheide (2015). *Bayesian Estimation of DSGE Models*. Princeton University Press.